

Unsteady non-Darcian flow between two stationary parallel plates in a porous medium with heat transfer subject to uniform suction or injection

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The unsteady non-Darcian flow in a porous medium of a viscous incompressible fluid bound by two stationary parallel porous plates is studied with heat transfer. A non-Darcy model that obeys the Forchheimer extension is assumed for the characteristics of the porous medium. A uniform and constant pressure gradient is applied in the axial direction whereas uniform suction and injection are applied in the direction normal to the plates. The two plates are kept at constant and different temperatures and the viscous dissipation is not ignored in the energy equation. The effects of porosity of the medium, inertial effects and uniform suction and injection velocity on both the velocity and temperature distributions are investigated.

Key words: Non-Darcian flow; parallel plates; Forchheimer equation; finite difference; numerical solution.

NOMENCLATURE:

x, y	Coordinates in horizontal and vertical directions, respectively
T_1, T_2	Temperature of lower and upper plates, respectively
dp/dx	Fluid pressure gradient
μ	Coefficient of viscosity
ρ	Density of the fluid
K	Darcy permeability of porous medium
β	Porosity parameter
λ	Inertial coefficient
S	Suction parameter
γ	Dimensionless non-Darcian parameter
c	Specific heat capacity of the fluid
k	Thermal conductivity of the fluid
T	Fluid temperature
u	Velocity component in the x -direction
v_o	Constant velocity component in the y -direction
Ec	Eckert number
Pr	Prandtl number
Re	Reynolds number
t	Time

INTRODUCTION

The flow of a viscous electrically conducting fluid between two parallel plates has important applications, e.g., in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays [1]. The flow between parallel plates of a Newtonian fluid with

heat transfer has been examined by many researchers in the hydrodynamic case [2, 3] considering constant physical properties. The extension of the problem to the MHD case has attracted the attention of many authors [4-8].

Fluid flow in porous media is now one of the most important topics due to its wide applications in both science and engineering [9, 10]. In most of the previous work, the Darcy model was adopted when studying porous flows. The Darcy law is sufficient in studying small rate flows where the Reynolds number is very small. For larger Reynolds numbers the Darcy law is insufficient and a variety of models have been implemented in

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studying flows in porous media. The Darcy–Forchheimer (DF) model is probably the most popular modification to Darcian flows utilized in simulating inertial effects [11-14]. It has been used extensively in chemical engineering analysis and in materials processing simulations. On the other hand, we may indicate the existence of non-Darcian flows (of different kinds) for very low velocity in low-permeability media [15-17].

In this paper, the transient unsteady non-Darcian flow with heat transfer through a porous medium of an incompressible viscous fluid between two infinite horizontal stationery porous plates is investigated and the DF model is used for the characteristics of the porous medium. A constant pressure gradient is applied in the axial direction and uniform suction from above and injection from below is imposed in the direction normal to the plates. The two plates are maintained at two different but constant temperatures. The non-Darcian flow in the porous medium deals with the analysis in which the partial differential equations governing the fluid motion are based on the non-Darcy law (Darcy-Forchheimer flow model) that accounts for the drag exerted by the porous medium [18-20] in addition to the inertial effect [14, 21-26]. The viscous dissipation is taken into consideration in the energy equation. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The governing momentum and energy equations are solved numerically using finite difference approximations. The inclusion of the porosity effect, inertial effects as well as the velocity of suction or injection leads to some interesting effects on both the velocity and temperature distribution.

DESCRIPTION OF THE PROBLEM

The two parallel insulating horizontal plates are located at the $y = \pm h$ planes and extend from $x = -\infty$ to ∞ and $z = -\infty$ to ∞ embedded in a DF porous medium where a high Reynolds number is assumed [11-14]. The lower and upper plates are kept at the two constant temperatures T_1 and T_2 , respectively, where $T_2 > T_1$ and a heat source is included, as shown in Fig (1). The fluid flows between the two plates in a porous medium where the non-Darcy law (Darcy-Forchheimer flow model) is assumed

[14, 21-26]. The motion is driven by a constant pressure gradient dp/dx in the x -direction, with uniform suction from above and injection from

below applied at $t = 0$ with velocity v_o in the positive y -direction. Due to the infinite dimensions in the x and z -directions, all quantities apart from the pressure gradient dp/dx which is assumed constant, are independent of the x and z -coordinates, thus the velocity vector of the fluid is given as

$$\vec{v}(y,t) = u(y,t) \vec{i} + v_o \vec{j}$$

with the initial and boundary conditions $u = 0$ at $t \leq 0$, and $u = 0$ at $y = \pm h$ for $t > 0$. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T=T_1$ at $t \leq 0$, $T=T_2$ at $y = +h$, and $T=T_1$ at $y = -h$ for $t > 0$. The fluid flow is governed by the momentum equation [27]

$$\rho \frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K} u - \frac{\rho \lambda}{K} u^2 \quad (1)$$

where ρ and μ are the density of the fluid and the coefficient of viscosity, respectively, K is the Darcy permeability of the porous medium [18-20] and λ is the inertial coefficient (i.e. the non-Darcian Forchheimer geometrical constant which is related to the geometry of the porous medium [14]). The last two terms in the right side of Eq. (1) represent the non-Darcy porosity forces. To find the temperature distribution inside the fluid we use the energy equation [28]

$$\rho c \frac{\partial T}{\partial t} + \rho c v_o \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\mu}{K} u^2 \quad (2)$$

where c and k are the specific heat capacity and the thermal conductivity of the fluid, respectively. The last two terms on the right side of Eq.(2) represent the viscous dissipation effect; the first term is the classical expression of the viscous dissipation for a clear fluid ($K \rightarrow \infty$), while the second term is the viscous dissipation in the Darcy limit ($K \rightarrow 0$) [29]. For a full discussion of modeling this form of viscous dissipation, see [30, 31].

Introducing the following non-dimensional quantities

$$\hat{x} = \frac{x}{h}, \quad \hat{y} = \frac{y}{h}, \quad \hat{z} = \frac{z}{h}, \quad \hat{u} = \frac{\rho h u}{\mu}, \quad \hat{P} = \frac{P \rho h^2}{\mu^2}, \quad \hat{t} = \frac{t \mu}{\rho h^2}, \quad \hat{T} = \frac{T - T_1}{T_2 - T_1}$$

$\$ = \rho v_o h / \mu$ (the suction parameter), $Pr = \mu c / k$ (the Prandtl number), $Ec = \mu^2 / \rho^2 c h^2 (T_2 - T_1)$ (the Eckert number), $\beta = h^2 / K$ (the porosity

parameter), $\gamma = \lambda h / K$ (the dimensionless non-Darcy parameter).

Equations (1), (2) are written as: (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t} + \$ \frac{\partial u}{\partial y} = - \frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \beta u - \gamma u^2 \tag{3}$$

$$\frac{\partial T}{\partial t} + \$ \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y} \right)^2 + \beta Ec u^2 \tag{4}$$

The initial and boundary conditions for the velocity become

$$u = 0, t \leq 0 \ \& \ u = 0, y = \pm 1, t > 0 \tag{5}$$

and the initial and boundary conditions for the temperature are given by

$$t \leq 0 : T = 0 \ \& \ t > 0 : T = 1, y = +1 \ \& \ t > 0 : T = 0, y = -1 \tag{6}$$

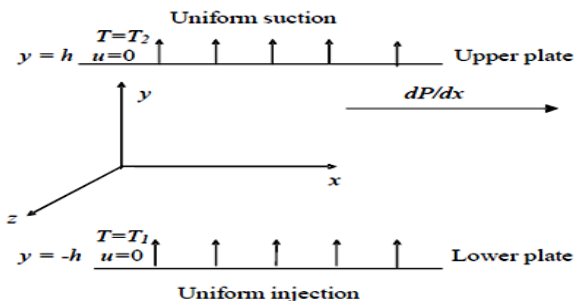


Fig. 1 The geometry of the problem

NUMERICAL SOLUTION OF THE GOVERNING EQUATIONS

Equations (3) and (4) are solved numerically using finite differences [32] under the initial and boundary conditions (5) and (6) to determine the velocity and temperature distributions for different values of the parameters β , γ and $\$$. The Crank-Nicolson implicit method [33] is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. Finally, the block tri-diagonal system is solved using Thomas algorithm. All calculations are carried out for $dP/dx = -5$, $Re = 1$, $Pr = 1$ and $Ec = 0.2$, while the results are obtained in a covering range for the non-Darcian parameter, $0.0 \leq \gamma \leq 2.0$ as [34]. It is found that the unsteady results reduce to those reported by Attia et al. [35] for the cases of Newtonian fluid and Darcian model. These comparisons lend

confidence in the accuracy and correctness of the solutions and, in turn, in the convergence of the two series defining the exact solution.

RESULTS AND DISCUSSION

Figures (2 – 4) show the time progression of the velocity profiles till the steady state for ($\$ = 1$) and various values of the porosity and non-Darcian parameters β and γ .

It is clear that the velocity charts are asymmetric about the $y = 0$ plane because of the suction. It is observed that the velocity component u increases monotonously with time. The porosity parameter β and the non-Darcian parameter γ have a marked effect on the time development of u . It is obvious that increasing β decreases u and its steady state time as a result of increasing the resistive porosity force on u , while increasing γ for each value of β decreases more the velocity u and its steady state time which reflects the expected resistance because of the inertial effects. For $\gamma = 0$ in figures (3-a) and (4-a) we mean a flow without additional inertial effects and the Darcian case is obtained to provide an easier quick path for the fluid flow. Fig (2-a) represents the simpler linear Newtonian case where the medium is non-porous with $\beta = \gamma = 0$ obtaining the highest velocity values.

Figures (5 – 7) show the time development of the temperature profiles for ($\$ = 1$) and various values of β and γ . It is observed that the temperature T increases monotonously with t . The parameters β and γ affect the time progression of the temperature T ; increasing γ decreases T and its steady state time, as increasing γ decreases u , which, in turn, decreases the viscous dissipation which decreases T . Increasing β in the non-Darcian case ($\gamma \neq 0$) increases the temperature and decreases its steady state time because of the viscous dissipation in the Darcy limit. In figures (6 – a) and (7 – a) where ($\gamma = 0$) we obtain the linear Darcian case with higher temperature values for each β , while fig (5-a) shows the linear Newtonian case where the medium is non-porous ($\beta = \gamma = 0$) in which the highest temperature values are reached. It is observed that the velocity component u and the temperature T reach the steady state monotonously and that u reaches the steady state faster than T . This is expected, since u acts as the source of temperature.

Figures (8) and (9) indicate the effect of suction and injection on the time progression of both the velocity u and the temperature T at the center of the channel, respectively, for various values of β and γ .

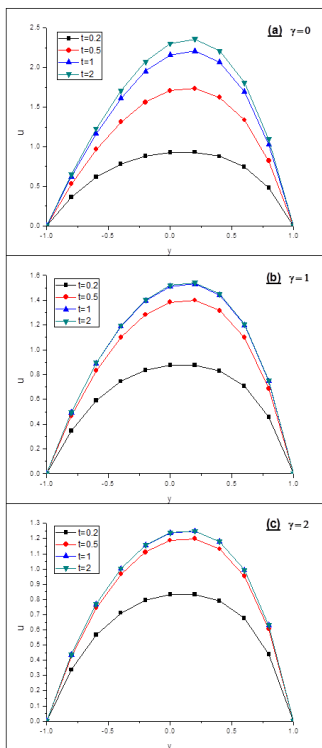


Fig. 2 Time development of the velocity u for $\beta = 0$, $S = 1$ and various values of γ

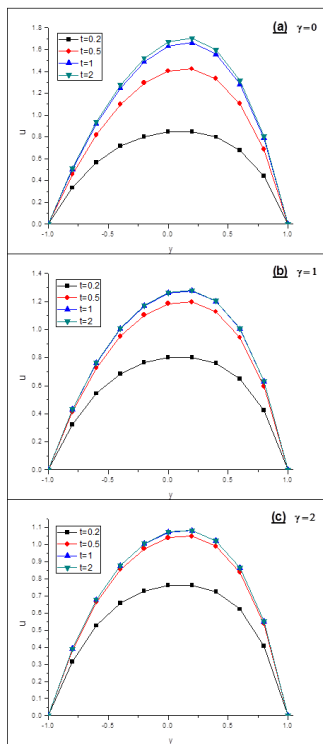


Fig. 3. Time development of the velocity u for $\beta = 1$, $S = 1$ and various values of γ

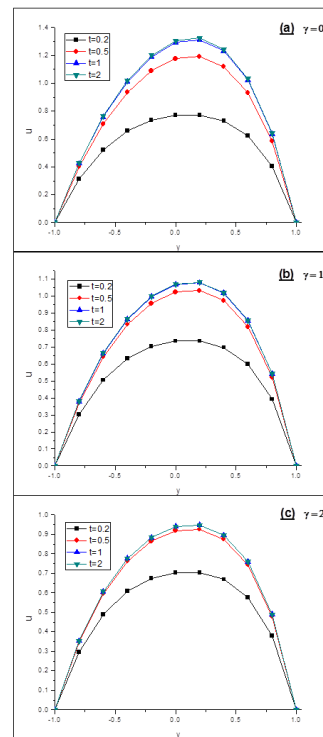


Fig. 4. Time development of the velocity u for $\beta = 2$, $S = 1$ and various values of γ

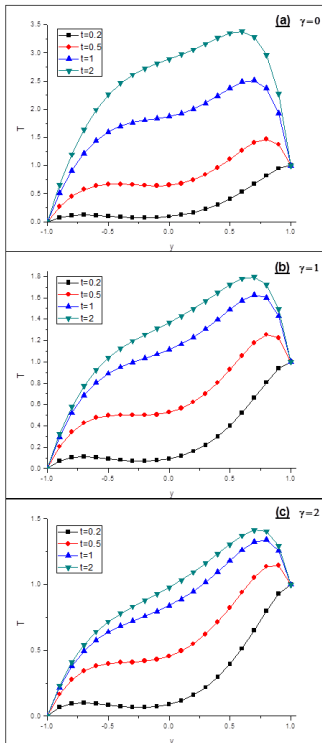


Fig. 5. Time development of the temperature T for $\beta = 0$, $S = 1$ and various values of γ

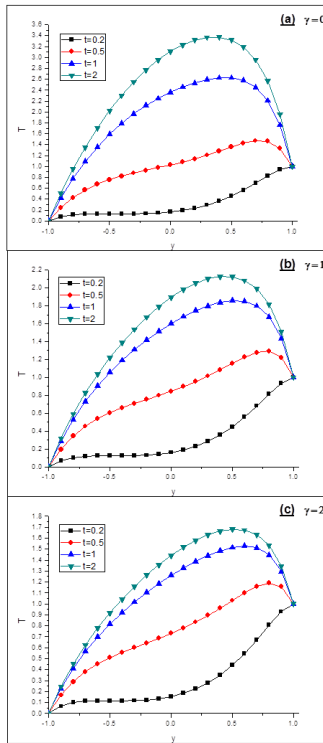


Fig. 6. Time development of the temperature T for $\beta = 1$, $S = 1$ and various values of γ

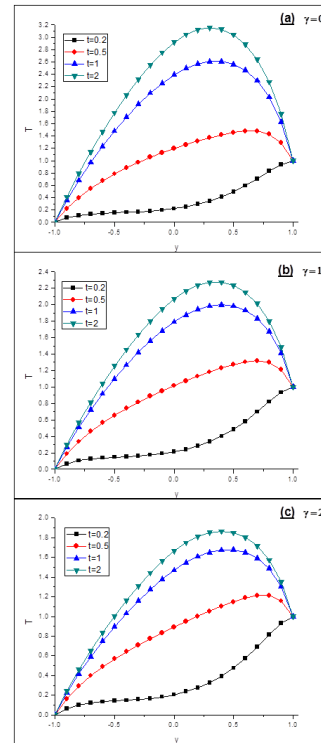


Fig. 7. Time development of the temperature T for $\beta = 2$, $S = 1$ and various values of γ

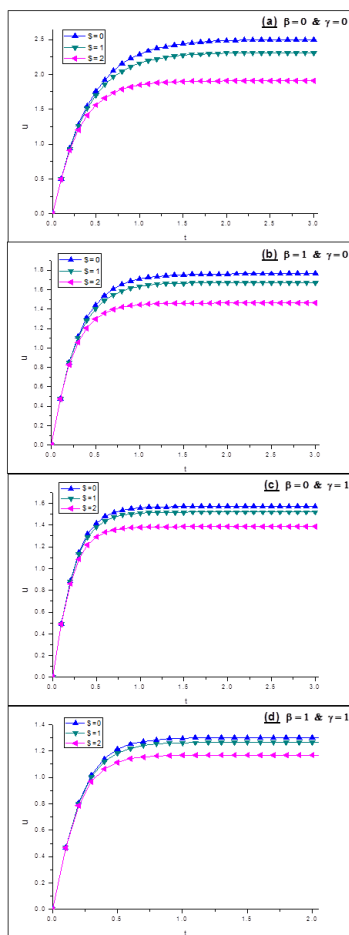


Fig. 8. Effect of the suction parameter S on the time development of the velocity u at the center of the channel ($y = 0$) for various values of the parameters β and γ

It is seen that increasing the suction parameter S decreases the velocity and its steady state time at the center of the channel due to the convection of the fluid from regions in the lower half to the center which has higher fluid speed. On the other hand, increasing the suction parameter S decreases the temperature T at the center of the channel which is influenced more by the convection term, which pushes the fluid from the cold lower half towards the center. Figures (8-a) and (9-a) indicate the linear Newtonian case where the plates are non-porous ($\beta = 0$) and there are no inertial effects ($\gamma = 0$) to obtain the highest velocity and temperature distributions. Figures (8-b) and (9-b) show the Darcian case in which the velocity decreases more because of the porosity of the medium ($\beta = 1$) and the temperature profile shows an increase due to the viscous dissipation caused by the porosity drag. Figures (8-c) and (9-c) show that the inertial effects ($\gamma = 1$) decrease more the velocity and temperature considering non-porous medium ($\beta = 0$). Also,

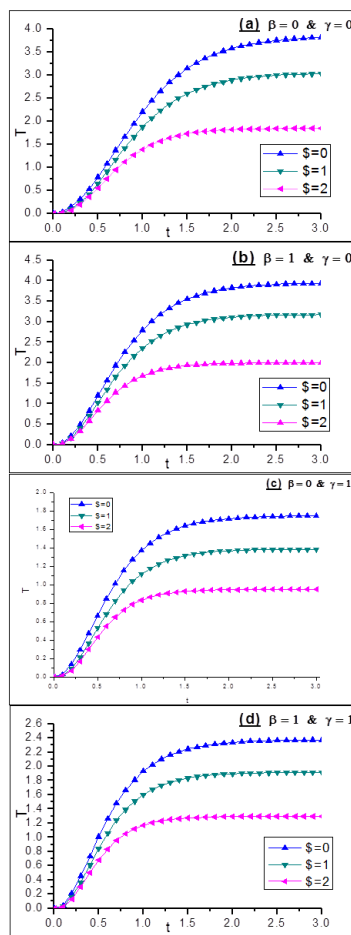


Fig. 9. Effect of the suction parameter S on the time development of the temperature T at the center of the channel ($y = 0$) for various values of the parameters β and γ

figures (8-d) and (9-d) represent the non-Darcian flow in a porous medium ($\beta = \gamma = 1$) which shows an obvious resistive effect in decreasing u and T where a noticeable similarity in the velocity profiles for the different values of the suction parameter S is achieved. Also, it can be seen from Figure (9) that T may exceed the value 1 which is the temperature of the hot plate and this is due to the viscous dissipation.

Tables (1), (2) and (3) summarize the variation of the steady state values of both the velocity u and the temperature T at the center of the channel ($y = 0$), respectively, for various values of β and γ and different values of the suction parameter ($S = 0, 1, 2$). The results confirm the inverse proportionality between the parameters β and γ and the velocity u and the temperature T (considering the viscous dissipation for a clear fluid, $K \rightarrow \infty$) reaching the steady state of both because the increase in the porosity resistance and the inertial effects reduce u and hence T .

Table 1. Variation of the steady state velocity u at the center of the channel ($y = 0$) for various values of β , γ and $\$$.

(a) $\$ = 0$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	2.499986	1.570278	1.266707
$\beta = 1$	1.761786	1.301599	1.09713
$\beta = 2$	1.354654	1.097677	0.957878
(b) $\$ = 1$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	2.312212	1.518816	1.238231
$\beta = 1$	1.671946	1.264911	1.07488
$\beta = 2$	1.304083	1.071377	0.9405971
(c) $\$ = 2$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	1.907495	1.384467	1.16059
$\beta = 1$	1.460687	1.168418	1.014149
$\beta = 2$	1.178782	1.001486	0.893262

Table 2. Variation of the steady state temperature T at the center of the channel ($y = 0$) for various values of β , γ and $\$$ (considering the viscous dissipation for a clear fluid, $K \rightarrow \infty$).

(a) $\$ = 0$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	3.84995	1.75015	1.2918
$\beta = 1$	2.115184	1.345975	1.087395
$\beta = 2$	1.430434	1.092352	0.9425454
(b) $\$ = 1$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	3.04365	1.386067	0.9881772
$\beta = 1$	1.665431	1.029702	0.8039784
$\beta = 2$	1.091862	0.8048577	0.673273
(c) $\$ = 2$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	1.844335	0.9489002	0.6783103
$\beta = 1$	1.078147	0.6951373	0.5387978
$\beta = 2$	0.7174322	0.5321741	0.4392434

Table 2. Variation of the steady state temperature T at the center of the channel ($y = 0$) for various values of β , γ and $\$$ (Considering the viscous dissipation in the Darcy limit, $K \rightarrow 0$).

(d) $\$ = 0$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	3.84995	1.75015	1.2918
$\beta = 1$	3.951613	2.376075	1.831821
$\beta = 2$	3.632962	2.570357	2.084983
(e) $\$ = 1$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	3.04365	1.386067	0.9881772
$\beta = 1$	3.178412	1.917231	1.455387
$\beta = 2$	2.957291	2.08949	1.677764
(f) $\$ = 2$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	1.844335	0.9489002	0.6783103
$\beta = 1$	1.99993	1.296159	0.9982081
$\beta = 2$	1.931794	1.423236	1.15748

Furthermore, a direct proportionality between the porosity parameter β and the temperature T is obtained for $\gamma \neq 0$ (considering the viscous dissipation in the Darcy limit, $K \rightarrow 0$); which gives an obvious proof that the viscous dissipation in the Darcy limit considering the quadratic drag formulation depends on the porosity of the medium. The results also showed that increasing the suction parameter $\$$ decreases the velocity and the temperature and that higher velocity and temperature values for various β and γ are obtained at $\$ = 0$.

CONCLUSIONS

The unsteady non-Darcian flow through a porous medium between two stationery parallel plates of a viscous incompressible fluid was studied with heat transfer in the presence of uniform suction and injection considering different modes of viscous dissipation. The effects of porosity of the medium, inertial effects, suction and injection velocity on the velocity and temperature distributions are investigated. It is found that porosity, inertial effects and suction or injection velocity have a marked effect on decreasing the velocity distribution in an inverse proportionality manner. Furthermore, increasing the porosity parameter increases the temperature, while

increasing the non-Darcian parameter decreases the temperature for each value of the porosity. Various cases were monitored passing through the Newtonian fluid flow in a non-porous medium, the Darcian flow model and the non-Darcian flow in a porous medium which showed the greatest flow resistance resulting in lower velocity and temperature values.

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НЕСТАЦИОНЕРЕН НЕ-DARCIAN-ОВ ПОТОК МЕЖДУ ДВЕ СТАЦИОНАРНИ
УСПОРЕДНИ ПЛОЧИ В ПОРЪОЗНА СРЕДА С ПРЕНОС НА ТОПЛИНА, ПРЕДМЕТ НА
ПОСТОЯННО ВСМУКВАНЕ ИЛИ ИНЖЕКТИРАНЕ

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(Резюме)

Изучен беше нестационарен не-Darcian-ов поток в поръозна среда на един вискозен несвиваем флуид между две стационарни успоредни плочи с пренос на топлина. Един не-Дарси модел, който се подчинява на разширението на Forchheimer се приема за характеристиките на порестата среда. Еднакъв постоянен градиент на налягането се прилага в аксиална посока, докато еднакво всмукване и инжектиране се прилагат в посока перпендикулярна на плочите. Двете плочи се съхраняват при постоянни и различни температури и дисипацията на вискозитета не се пренебрегва в енергийното уравнение. Изследвани са ефектите на поръозност на средата, инерционните ефекти и постоянни скорости на всмукване и на инжектиране върху скоростния профил и разпределението на температурата са изследвани.