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Numerical analysis of mass and momentum transfer in co-axial cylinders with rotating inner cylinder

H. Ahmari[1], S. Zeinali Heris[2*]

[1]Ph.D. Student, Department of Chemical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
[2] Associated Professor, Department of Chemical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

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Bearing is one of the most commonly used tools in the industry that the distance between the two cylinders is filled with a fluid. Given the possibility of oil pollution, the emission behavior of mass and momentum transfer within the fluid appears to be necessary. In this paper, the coaxial cylinders with rotating inner cylinder and a non-Newtonian fluid is simulated. Differential equations of mass and momentum transfer with simple problem situation are introduced and the boundary conditions for solving these equations are obtained. The results showed that there was insignificant radial velocity. Even if there was an appendage of the outer cylinder, the change of radial velocity was very low, but, it did not affect diffusion. Also, variations of the Reynolds number, the radial velocity and diffusion as well as concentration are studied.

Keywords: Bearings, Co-axial cylinders, Momentum, Mass transfer, Non-Newtonian fluid

INTRODUCTION

Coaxial cylinders are used in many experimental analyses [1, 2] and industrial applications such as medicine filter industries. Some studies have focused on momentum transfer and behavior of system and some have addressed problems of heat flow in composite cylinders, as mentioned in the literatures [3-8]. However, mass transfer in coaxial cylinders has rarely been investigated. On the contrary, mass transfer in the laminar flow of non-Newtonian fluids between rotating coaxial cylinders is used in most industrial processes, such as the catalytic chemical reactors, filtration devices, plant cell bioreactors, liquid–liquid extractors [4], cylinder extraction columns, high efficiency batch distillation columns, cyclone chambers, Journal bearings and rotating electrical machinery [9].

Molki et al. [10]investigated convective heat/mass transfer characteristics of the laminar flow in a circular annulus with a rotating inner cylinder in the presence of a laminar axial flow. Sung et al. [11] measured the local mass transfer from a cylinder placed in a pulsating free stream. Flowers et al. [9] did an experimental study on the transport properties of a system in which a stream of air flows axially in the long annulus between a rotating cylinder and a stationary coaxial outer tube. Also, they compared mass and momentum transfer in streams containing secondary flows [12]. Zeraibi et al. [13] presented a numerical investigation of the thermal convection for a thermo-dependent non-Newtonian fluid in an annular space between the two coaxial rotating cylinders. Jeng et al. [14]investigated the effects of jet flow and flow outlet configuration on the fluid flow in an annulus between co-axis cylinders. Laminar conjugate heat transfer by natural convection and conduction in a vertical annulus formed between co-axial cylinders was studied by Shahi et al. [15].Ogawa et al studied the relationship between the wave length of Taylor vortices and the lapse of time related to the spin-up time in the wide gap between coaxial cylinders in details[2]. Sedahmed and Nirdosh [16] measured the free convection mass transfer rates at the outer surface of the inner duct of a horizontal square annular duct by an electrochemical technique. Cheng presented a modeling of the heat transfer of upward annular flow in a smooth tube and a spirally internally ribbed tube[17]. Various method of predicting the transition to annular flow in an upward, two-phase, gas-liquid flow in a vertical

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* To whom all correspondence should be sent:
E-mail: zeinali@um.ac.ir

tube were tested against a large set of experimental data by Rezkallah and Sims[18].

A bearing is a specific device that allows a relative motion, typically rotational or linear, between two or more pieces. Bearings can be widely classified. A layer of fluid bearings with a lubricant or compressed gas can prevent the collision of two surfaces and reduce the vertical forces between the two surfaces [19]. Also, the bearing rotational speed is generally low.

Dust and dirt usually find their way to a bearing and mix with the oil. However, sometimes these dust and dirt come from inside a bearing. Therefore, the bearing fluid should be able to take out this matter to protect itself against rubbing. The fluid must be able to pass through the material as quickly as possible.

In this study, the ways of transferring mass within the fluid bearing and the interaction between mass and momentum transfer are studied. Also, the effect of baffles on the fluid flow and diffusion is investigated. The variation of velocity, Reynolds number, concentration of diffuser and diffusion intensity between two walls vs. destination is plotted. The equations of mass transfer are solved in transient conditions, while momentum transfer is in stationary conditions.

Theory

The two dimensional geometry of the study is illustrated in Fig. 1. The inner cylinder moves with constant angular velocity. The outer cylinder is at rest, as the material on the wall is diffused into the fluid. The liquid is in the annular gap between the two vertical coaxial cylinders, by which the concentration difference across the thin liquid layer is supplied. Since the velocity of inner cylinder is low, the flow remains laminar. It is because the shear flow is replaced by an actually symmetric three-dimensional secondary flow when the rotational speed of the inner cylinder exceeds a certain value [9].

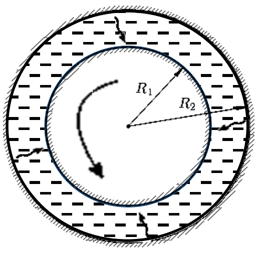


Fig. 1. Geometry of the system.

In the first geometry, Navies–Stock's (Momentum) equation is simplified and solved to provide more details about the velocity distribution of the system. There is no radial velocity ($v_r = 0$) and from continuity equation:

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = 0 \qquad (1)$$

Results:

$$\frac{\partial v_\theta}{\partial \theta} = 0 \qquad (2)$$

Then:

$$v_\theta = v_\theta(r) \qquad (3)$$

From momentum equations for r:

$$\rho \frac{{v_\theta}^2}{r} = \frac{dp}{dr} \qquad (4)$$

And from momentum equation for θ:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_\theta}{dr}\right) - \frac{v_\theta}{r^2} = 0 \overset{or}{\Rightarrow} \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right) = 0 \qquad (5)$$

$$v_\theta = \frac{A}{2}r + \frac{B}{r} \qquad (6)$$

$$v_\theta\big|_{r=r_1} = r_1\omega \qquad (7)$$

$$v_\theta\big|_{r=r_2} = 0 \qquad (8)$$

Then:

$$v_\theta = \frac{1}{{r_1}^2 - {r_2}^2}\left({r_1}^2\omega r - \frac{{r_1}^2{r_2}^2\omega}{r}\right) \qquad (9)$$

As can be seen v_θ is a function of r_1, r_2 and ω and it is time independent. Exact solutions of Navies–Stock's equation are of paramount importance in terms of both theoretical and practical values [20].

To investigate the concentration of fluid, the concentration equation is solved with $\frac{\partial C}{\partial \theta} = 0$:

$$\frac{\partial C}{\partial t} = D\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)\right) \qquad (10)$$

$$\left.\frac{\partial C}{\partial r}\right|_{r=r_1} = 0 \qquad (11)$$

$$C\big|_{r=r_2} = C_0 \qquad (12)$$

Equation (10) does not have an analytical solution. It shows that the concentration and momentum equations do not exhibit the expected analogous relationship [12].The concentration of fluid between coaxial cylinders changes relative to time and radial velocity. Concentration and diffusion are not dependent on the angular velocity of fluid. Angular velocity homogenizes fluid in a certain angular line. This results in radial diffusion, which is not affected by the fluid flow. On the other hand, both phenomena, i.e., mass and momentum transfer, are independent and need to be investigated separately.

In the geometry “with baffles”, the radial velocity affects the diffusion phenomena. Hence, the equation (10) is rewritten by v_r.

$$v_r \frac{\partial C}{\partial r} + \frac{\partial C}{\partial t} = D\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)\right) \quad (13)$$

Radial velocity creates convective mass transfer in the system. To solve the equations of mass and momentum transfer, the finite element method is used.

To ensure the presence of the laminar flow, it should be tested with Taylor number. A definition of Taylor number is [21]:

$$Ta = \frac{\rho^2 {v_\theta}^2 R_i (R_o - R_i)^3}{\mu^2} \qquad (14)$$

Here, the critical Taylor number is about 1700. In this study, given the assumed properties, the maximum permissible angular velocity is 0.77-0.98.

In the first geometry, r_1= 6 cm and r_2= 8 cm, the inner cylinder rotates with a speed of 0.005 m/sec, and for mass transfer analysis, the concentration of outer cylinder is10 mol/m^3. In the second geometry, 4 baffles with 0.1×0.1 dimensions are used. The fluid is non-Newtonian based on the power law model with a consistency index (k) and behavior index (n) of 0.5 and 0.7, respectively. The density of fluid is 1000 kg/m^3. Finally, the value of diffusion coefficient of fluid (irrespective of its variation) is constant and equal to 1×10^5 m/sec.

RESULTS AND DISCUSSION

The grid independency of the numerical solutions was investigated meticulously to ensure the accuracy of the numerical solution [22]. Order velocity was studied on a number of different quantities mesh elements. The results are shown in Fig. 2.In accordance with the increase of the number of mesh to 1200, velocity is independent of the number of mesh elements. As a result, the same mesh was used for the calculations.

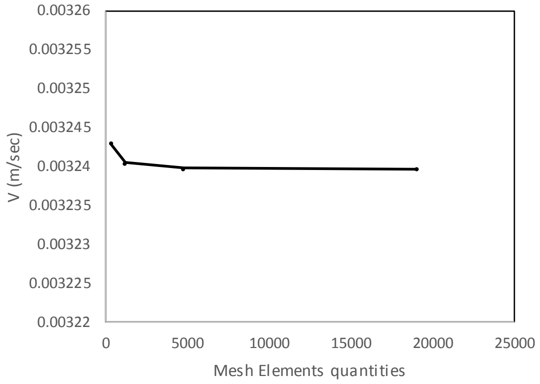
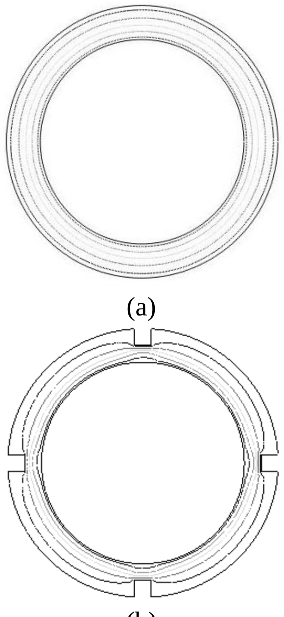


Fig. 2. Velocity vs. mesh quantity in certain point.

Momentum Transfer

Fig. 3 shows contour lines in both geometries. In the first geometry, the fluid layers turn in a constant circular line, meaning that the fluid is not mixed with the system. In the second geometry, contour lines turn uniformly but at a closer distance than the first geometry, except near the baffles where lines are closer together.


(a)

(b)

Fig. 3. Velocity contours in Both Geometry, without (a) and with Baffle (b).

As shown in Fig.4, in the absence of baffles, radial velocity is not created, while, in present of bafflesit is created and changed by destination. The velocity reaches its maxium point near the inner cylinder. Maximum rate is about 5.5e-6 with an average of 2.42e-6, which is very low. Unlike the first geometery, in the presence of baffle, the concentartion is incresed up to 3%in certain points. In other words, the fluid motion in the radial direction is negligible in the presence of baffles.

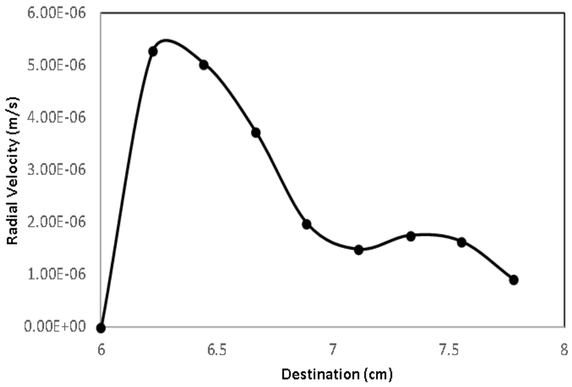


Fig. 4. Radial velocity vs. Destination in Geometry with the Baffle.

Fig. 5 ilustrates the variation of Re number vs. destination. Re number is defined as [5]:

$$\mathrm{Re} = \frac{\rho v R_o (R_o - R_i)}{\mu} \quad (15)$$

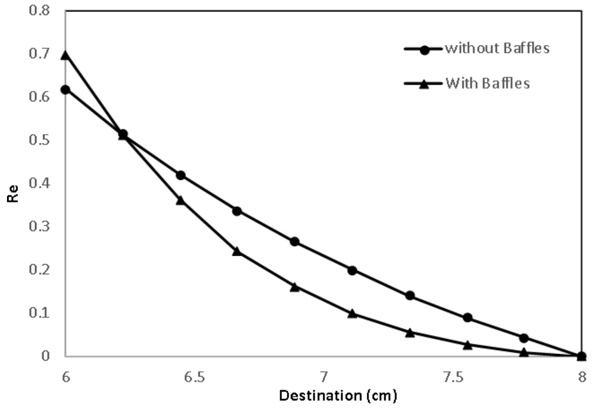


Fig. 5. Re number vs. Destination in both Geometries.

According to the above equation, Re number is the direct velocity magnitude-dependent and reversed apparent viscosity-dependent. In all points with baffle, velocity is lowerthan the points without baffle, which is due to the dramatic increase of surface after the baffle.The apparent viscosity in “with baffle” case is divided into two parts: areasnear the inner cylinder and the areas outside it . In the former, the viscosity of points“with baffles” is less than the points“without baffles”. whereas in the latter, the viscosity is greaterinthe case “without baffle”.As a result, as shown in Fig. 5, Reynolds number in the case with baffles is less than the case without baffles, but in the vicinity of the inner cylinder, the slowdown is greaterthan the increasedviscosity. Therefore, the Reynolds number is greater in the case without baffle.

Mass transfer

In this section, the system diffusion is investigated in both geometries. The radial velocity, which only appears in the first geometry, is generated in the system by baffles. However, the magnitude of velocity is negligible and in effective in the radial mass transfer. As a result, the radial convection mass transfer is not produced in system. Mass transfer is carried out through diffusion mechanism.

Fig. 6 shows concentration in the system vs. destination at several points after the inception of diffusion. The numerical analysis of Equation (10) results variation of concentration vs. radius, which is nonlinear. As the time goes by, curves show a tendency toward nonlinearity.

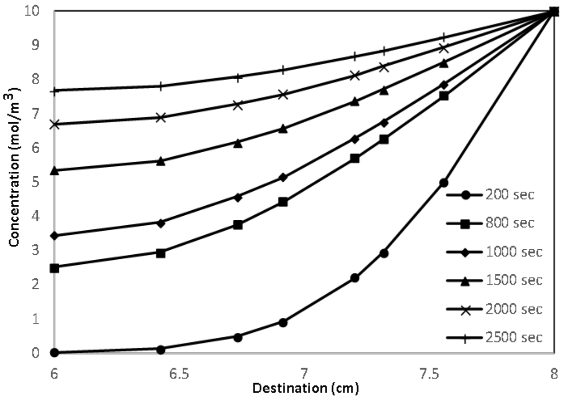


Fig. 6. Concentration vs. Destination at different times.

In certain points, concentration changes vs. time, as plotted in Fig. 7. In the first one, the range of curve is high because of the significant concentration difference and diffusion rate. However, the concentration difference reduces with time, the scope decreases and gives a linear curve.

Fig. 8 shows the variation of diffusion against the destination. At the initial time, the rate of diffusion is greater near the outer cylinder, but it decreases dramatically as the distance from the outer cylinder is furthered, because at that point its concentration is equal to zero. In other words, in the initial times, the substance is either non-existent or its value is very low for diffusion.

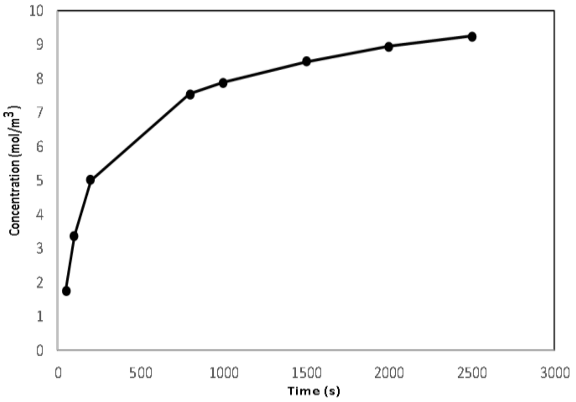


Fig. 7. Concentration vs. Time in r= 7.56 cm.

Over time, the radial diffusion appears in these areas and the concentration changes will be the same, though curves shrink gradually. This happens because concentration changes relative to the distance and time, as shown in Fig. 9. As mentioned earlier, the intensity of diffusion decreases over time. Also, radial diffusion decreases with time due to the concentration difference. This decline is sharp at the beginning of the process but it gradually becomes smoother.

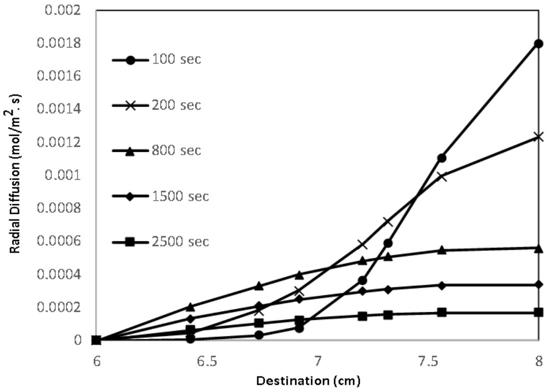


Fig. 8. Diffusion Flux vs. Destination in different times.

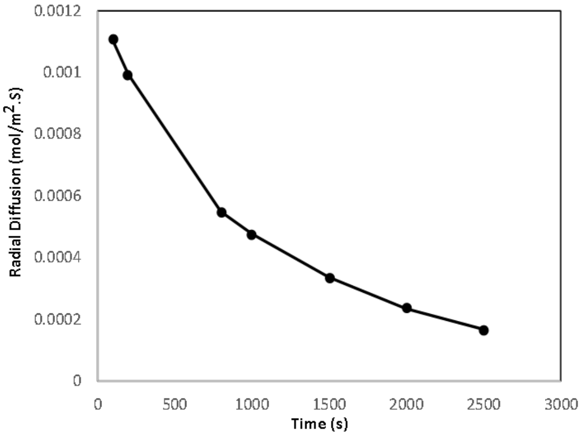


Fig. 9. Radial diffusion vs. time in r=7.56 cm.

CONCLUSION

In this research, mass and momentum transfer in co-axial cylinders was examined. The inner cylinder rotates at a constant velocity and the outer cylinder has a certain concentration. The simplification of the differential equation showed that both phenomena were distinct and could be solved separately. In this regard, the fluid turned uniformly and the radial velocity did not appear. The single mechanism of mass transfer is diffusion. The convection mass transfer is not achieved because the radial velocity is approximately zero. In the second case, baffles were used to change the flow model. At low velocities, the pattern of stream changed, but the creation of radial velocity was very low and it did not affect the radial mass transfer. Variations of concentration and diffusion mass transfer were also studied. The results showed that at initial times, the starting point of diffusion had high rate of diffusion, but the rate reduced over the time.

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ЧИСЛЕН АНАЛИЗ НА ПРЕНОСА НА МАСА И ЕНЕРГИЯ ПРИ КОАКСИАЛНИ ЦИЛИНДРИ ПРИ ВЪРТЯЩ СЕ ВЪТРЕШЕН ЦИЛИНДРИ

Х. Ахмари, С. Зейнали Херис*

Департамент по инженерна химия, Инженерен факултет, Университет "Фирдоуси" в Машхад, Иран

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(Резюме)

Лагерите са едни от най-често срещаните средства в промишлеността, при които разстоянието между два цилиндъра е запълнено с флуид. Опасността от замърсяване на околната среда с масла налага оптимизирането на масопренасянето и преноса на енергия в тези флуиди. В настоящата работа се моделира не-Нютоновото течение между два коаксиални цилиндъра около въртящ се вътрешен цилиндър. Използвани са диференциалните уравнения за преноса на маса и енергия с необходимите гранични условия. Резултатите показват незначителна радиална компонента на скоростта с нищожно влияние върху дифузията. Изследвани са освен това измененията в числото на Рейнолдс и влиянието му върху радиалната компонента на скоростта, дифузията и концентрацията на пренасяното вещество.