

## RETAS model software to identify the best fit model version of aftershock temporal decay

D. Gospodinov\*

*Paisii Hilendarski University of Plovdiv, 24 Tsar Asen Str., BG-4000 Plovdiv, Bulgaria*

The options and the applicability of a FORTRAN 95 software program to model aftershock rate decay in time have been demonstrated in this paper. The program was designed to perform computations of the maximum likelihood estimates of the restricted epidemic type aftershock sequence (RETAS) model parameters. This is a trigger model, which provides a stochastic description of aftershocks temporal evolution in a sequence. The advantage of RETAS is that it offers a number of model versions, including the Modified Omori formula (MOF) and the epidemic type aftershock sequence (ETAS) models as limit cases. In that way RETAS offers a wider variety of model versions to fit best the temporal distribution of aftershocks in a sequence. The author has provided examples on simulated data to demonstrate the use of the software to different types of aftershock sequences and the interpretation of the results obtained.

**Key words:** RETAS model, stochastic modelling, aftershock decay, ETAS model, FORTRAN software

### INTRODUCTION

The intensive study of earthquake clustering, especially aftershocks, is motivated by several key objectives. First, aftershocks shed light on the physical processes occurring in the fault zone [1-3]. Strong earthquakes change the stress in the surrounding crust, leading to the occurrence of new earthquakes and aftershocks [4-5]. Exploring aftershock sequences of mainshocks presents another way to examine faults geometry and non-linear features of seismicity [6-7].

Second, random process modeling of aftershocks sequences allows identifying and interpreting of the actual process deviations from the model prior strong aftershocks or new earthquakes [8-10]. Stochastic short-term models that describe the phenomenon of earthquake clustering are achieving increasing success in the seismological community [11-14]. These models delineate seismicity as a random point-process, for which a continuous space-time density distribution of the earthquake occurrence can be defined. Except for its scientific importance, stochastic modeling of aftershocks sequence evolution could be important for operational activity after a strong earthquake. After such an event, the possibility of the occurrence of either significant aftershocks or an even stronger mainshock is a continuing hazard that hinders the resumption of critical services and reoccupation of partially damaged structures. A stochastic

parametric model allows determination of probabilities for aftershocks and larger mainshocks during intervals following the mainshock [15-16, 8]. Because these probabilities are estimated through a particular model, the closeness between the formulation of the model and the reality is the essentially important factor influencing the output. The closer the model is to the real data, the more reliable the output is. That is why it is important to consider many models fitted to the same set of data and to apply some model selection procedures to choose the best model among them.

The Modified Omori formula (MOF) and the epidemic type aftershock sequence (ETAS) model are among the ones of greatest successes in empirical studies [17-18]. They, however, consider only two limit cases, the MOF model with only one parent event and the ETAS model in which every event shares in the generation of the subsequent ones. Gospodinov and Rotondi [12] proposed the restricted epidemic type aftershock sequence (RETAS), a kind of trigger model, which is based on the assumption that only aftershocks with magnitudes larger than or equal to a threshold  $M_{th}$  can induce secondary seismicity. By varying  $M_{th}$  between the cut-off  $M_0$  and the maximum magnitude  $M_{max}$  one considers a number of RETAS versions, including MOF and ETAS as limit cases. In that way RETAS offers a wider variety of model versions from which to choose the one that fits best the data.

Following a quasi-Newton method, Gospodinov & Rotondi [12] have developed a program in

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\* To whom all correspondence should be sent:  
drago@uni-plovdiv.bg

FORTRAN 95, which exploits subroutines of the IMSL library to obtain the maximum likelihood estimates (MLEs) of the RETAS model parameters. The purpose of this paper is to demonstrate the options of this software and its applicability for modelling aftershocks temporal distribution (the RETAS model software and simulated data are available at <http://geo.physics.uni-plovdiv.bg/download.htm>).

#### RETAS MODEL FORMULATION

Each stochastic model is developed on the basis of some basic physical assumptions. For the Modified Omori Formula (MOF) it is supposed that the total relaxation process is controlled by the stress field changes caused by a main shock. The aftershocks are conditionally independent and follow a nonstationary Poisson process. By the MOF model the decaying frequency of aftershocks per unit time is described to follow a negative power law [17]

$$n(t) = \frac{K}{(t+c)^p} \quad (1)$$

where  $t$  is the time elapsed from the occurrence of the main shock,  $K$  is a parameter related to the magnitude of the main shock and to the magnitude cut-off,  $p$  is a coefficient of attenuation and  $c$  is a constant.

The frequency  $n(t)$  in Eq. (1) corresponds to the intensity function of a point process, i.e.

$$n(t) \approx \lambda(t) \quad (2)$$

where  $\lambda(t)$  is the conditional intensity function

$$\begin{aligned} Pr\{\text{an event occurs in } (t, t+dt)\} = \\ = \lambda(t|H_t)dt + o(dt) \end{aligned} \quad (3)$$

Formula (3) reveals the probability of an event to occur in the time interval  $(t, t+dt)$ . Here  $H_t$  is the history of the process which for the MOF is only given by the time and the magnitude of the main shock.

The ETAS model [18] assumes that every aftershock in a certain zone can trigger further shocks and the contribution of any previous earthquake to the occurrence rate of the subsequent events is decomposable into separate terms representing the time and magnitude distribution as:

$$\lambda_j(t, m) = h(t-t_j|m_j) = k(m_j)g(t-t_j) \quad (4)$$

The temporal decay rate follows the MOF  $g(t) \propto (t+c)^{-p}$  and the functional form of  $k(m_j)$  is chosen

to be exponential on the basis of the linear correlation between the logarithm of the aftershock area and the main shock's magnitude, studied extensively by Utsu and Seki [19]. Hence, the expected resultant rate of aftershocks is given by Ogata [18]:

$$\lambda(t/H_t) = \mu + \sum_{t_j < t} \frac{K_0 e^{\alpha(m_j - M_0)}}{(t-t_j+c)^p} \quad (5)$$

where  $\mu$  (shocks/day) is the rate of background activity, the history  $H_t$  consists of the times  $t_j$  (days) and magnitudes  $m_j$  of all the events which occurred before  $t$  and the summation is taken over every  $j$ -th aftershock with a magnitude stronger than the cut-off  $m_j \geq M_0$  i.e. over all events in the sample. The main shock in this case is indicated by  $j=1$ . In probabilistic terminology, the first term on the right-hand side of (5) stands for the "independent" seismicity and in the second the "induced" seismicity is represented by a superposition of the modified Omori functions shifted in time. In formula (5) the coefficient  $\alpha$  measures the magnitude efficiency of a shock in generating its aftershock activity and  $K_0$  is a multiplier, common to all aftershocks, which has an impact on the total aftershock productivity. The MOF and the ETAS model are two limit cases, the former with only one parent-event and the latter with all events sharing in the generation of the subsequent ones. Gospodinov and Rotondi [12] offer the Restricted Epidemic Type Aftershock Sequence (RETAS) model, which is based on the assumption that not all events in a sample but only aftershocks with magnitudes larger than or equal to a threshold  $M_{th}$  can induce secondary seismicity. Then the conditional intensity function for the model is formulated as:

$$\lambda(t/H_t) = \mu + \sum_{\substack{t_j < t \\ m_j \geq M_{th}}} \frac{K_0 e^{\alpha(m_j - M_0)}}{(t-t_j+c)^p} \quad (6)$$

An advantage of the RETAS model is that this gap is not fixed and by varying  $M_{th}$  all RETAS versions between the MOF and the ETAS model can be examined on the basis of the Akaike criterion [20], given by

$$AIC = -2 \max \log L(\theta) + 2k \quad (7)$$

where  $k$  is the number of parameters of the model and  $\log L$  is the logarithm of the likelihood function, given by

$$\log L(\theta) = \sum_{i=1}^N \log \lambda_{\theta}(t_i|H_{t_i}) - \int_S^T \lambda_{\theta}(t|H_t) dt \quad (8)$$

The smallest value of the Akaike criterion recognizes the best fit model. Gospodinov and Rotondi [12] have developed a program in Fortran 95 which exploits subroutines of the IMSL library to maximize the likelihood of the RETAS model following a quasi-Newton method and we apply the same software in this study.

One can see how well or poorly the estimated model is fitted to an earthquake sequence by comparing the cumulative number of earthquakes with the predicted rate offered by the estimated models. If a series of events  $\{t_0, t_1, \dots, t_N\}$  is simulated based on a statistical model  $\lambda(t)$ , which is the predicted occurrence rate of events per unit time, then by integrating:

$$\Lambda(S, t) = \int_S^t \lambda(u) du \quad (9)$$

we obtain the expected cumulative number of events over the time interval  $(S, t)$ . To estimate quantitatively the significance of deviations of observed to model cumulative numbers we can first focus on formula (1), which presents the MOF model. The basic assumption behind this model, as revealed above, is that the events in an aftershock sequence follow a nonstationary Poisson process. Then the process, offered by formula (6), being a superposition of such models ( $\mu = 0$ ), is a nonstationary Poisson process itself. For such processes the common mean and variance are both given by formula (9) for  $S = 0$  [21]. Thus, we may use this formula to calculate the standard deviation of the expected cumulative number and use it as error bounds for quantitative estimation of the possible departure between real data and model.

Relation (9) also produces a transformation of time from  $t$  to  $\tau = \Lambda(t)$  [9] so that the occurrence times  $t_j$  are transformed 1:1 into  $\tau_j$  and the earthquakes follow the standard stationary Poisson process on the new axis if the intensity function is the true one for the data.

#### INPUT DATA FILES

Two types of input data files are used in the RETAS model analysis. One should contain the real catalog data in two columns; left column values are events' magnitudes and the right column values are the time periods [days] after the first earthquake occurrence (see example of part of *retas297.txt* file in Table 1).

The second input file contains parameter values, necessary for the program implementation (see the

Table 1. Example of a catalog data file (part of *retas297.txt* file). Left column – magnitudes, right column – the time periods [days] after the first earthquake occurrence.

7.2	0
5.1	0.004166664
4	0.008333331
3.8	0.010416664
3.8	0.013194442
3.8	0.013888887
3.7	0.017361109
...	.....

contents of *dataretas297.txt* file in Table 2). This file's name should be input interactively after running the program. The parameters in the file are as follows:

1<sup>st</sup> row – name of the catalog data file, which will be analyzed by the program (*retas63.txt* in our example)

2<sup>nd</sup> row – lower magnitude cut-off of the data sample. For computational reasons, if this value is 3.7 (as in the example), it should be written down as 3.695. In fact this magnitude is usually the magnitude of completeness of your broader data sample and its value should be estimated before analyzing the data through the RETAS software.

3<sup>rd</sup> row – number of runs for each triggering value  $M_{th}$ . Usually the program converges quickly.

4<sup>th</sup> row – number of runs, after which there is an onscreen printing of the maximum likelihood function value, the corresponding threshold magnitude, and the triggering magnitude, which yields lowest AIC for the moment. If the value in this row is 1, the program will print the mentioned parameters for each run. If the value is equal to the one in the 3<sup>rd</sup> row (as in the example in Table 2) then onscreen printing will be only once for all runs of a corresponding triggering magnitude.

Table 2. Parameter values, necessary for the program implementation (contents of *dataretas297.txt* file; see details in text)

retas297.txt
3.695
10
10
1.d-6 20.d0
1.d-6 5.d0
1.d-7 6.d0
6.d-1 3.d0

The next four rows – from 5<sup>th</sup> to 8<sup>th</sup>, contain the value ranges (lower and upper cut-offs), which the initial model parameter values are taken from. For each run the program takes an initial set of random parameter values (within these ranges) for  $K$ ,  $\alpha$ ,  $c$ ,  $p$  (formula 6;  $\mu$  in the formula is accepted to be 0 and not estimated) and after that evaluates the maximum likelihood estimates (MLE) of these parameters for the corresponding  $M_{th}$ .

The package contains three input data sets for different types of aftershock sequences: one, for which the best fit model is RETAS with  $M_{th} = 4.4$  (retas297.txt and dataretas297.txt); a second one, for which the best fit model is ETAS with  $M_{th} = M_0 = 3.8$  (etas215.txt and dataetas215.txt); the third data set is best fitted by the MOF model with  $M_{th} = M_{main} = 6.6$  (mof215.txt and datamof215.txt).

#### RETAS ANALYSIS SOFTWARE

The executable program file is *retasanalysis.exe*. It evaluates the maximum likelihood parameters estimates (MLE) for the RETAS model, which best fits the examined data set (minimum AIC). The program was developed in Fortran 95 which exploits subroutines of the IMSL library to maximize the likelihood of the RETAS model following a quasi-Newton method. As usual, the program runs after double clicking over its icon. After starting, a window opens (see Fig. 1a) and the user is invited to input a file name (the one which contains parameter values, necessary for the program implementation; *dataretas215.txt* in our example). File extension should also be included (Fig. 1b) and then 'ENTER' should be pressed.

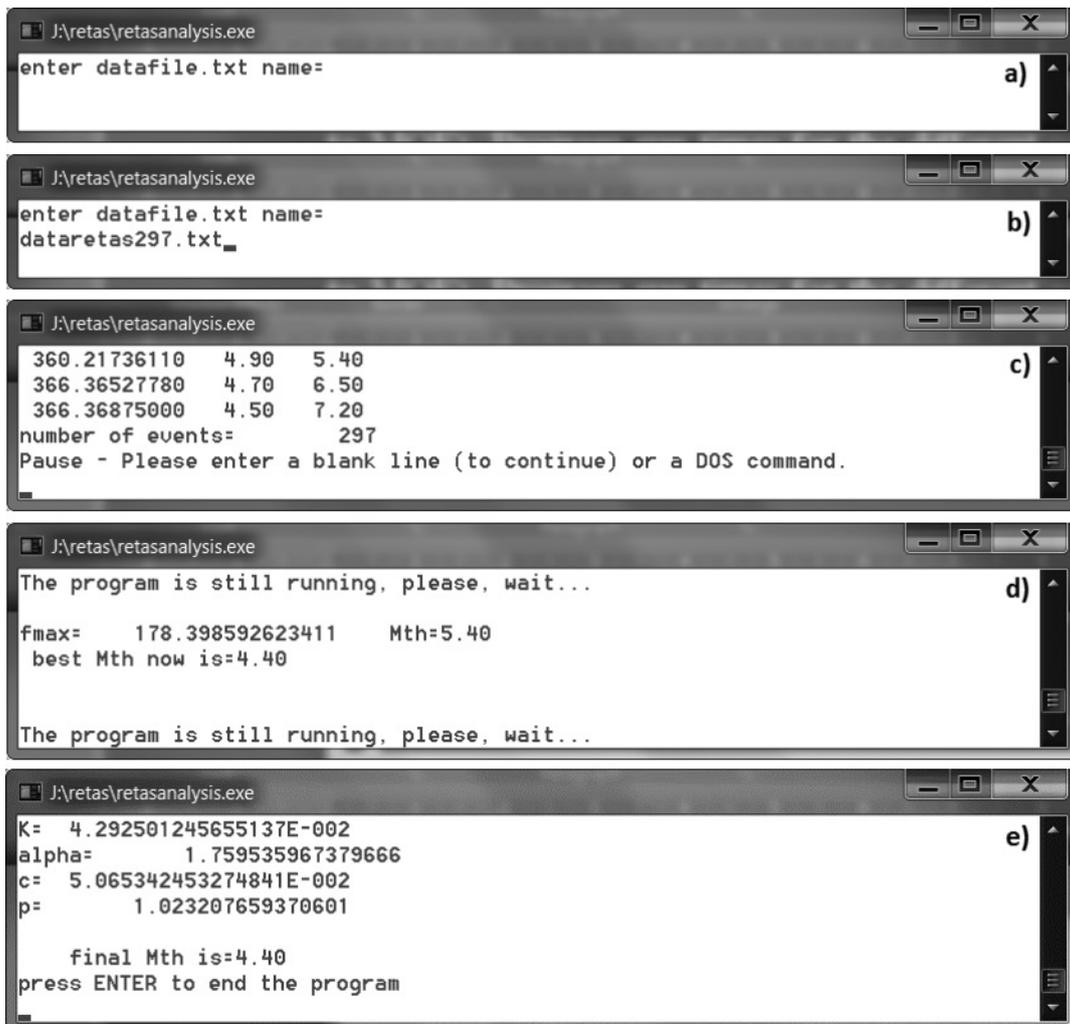


Fig. 1. Subsequent windows of the RETAS software implementation.

The program reads the catalog data file (*retas297.txt*) and the other parameters, and the catalog data is printed onscreen - time periods after the first shock in days (left), magnitudes (center) and again magnitudes in an increasing order (right). The last row presents the number of events (Fig. 1c). The purpose of this operation is that the user verifies he is analyzing the right data set. After pressing 'ENTER', code execution continues and the program issues an announcement 'The program is still running. Please, wait ...'. Different times for program completion are needed depending on the number of events. This is so because the program verifies all RETAS model versions for different  $M_{th}$ . It starts from the lower magnitude cut-off ( $M_{th} = M_0$  - RETAS degenerates into ETAS; see Fig. 1d) and then  $M_{th}$  is increased step by step to the case when it is equal to main shock magnitude (RETAS turns to MOF).

Program run times for the different catalog data files are presented in Table 6. At the end the program prints on the screen the parameter MLEs ( $K, \alpha, c, p$ ) for the identified best fit model version (Fig. 1e).

#### OUTPUT FILES

The RETAS software generates three output files in the same directory as follows:

*output.txt* - The file includes the catalog data file name (*retas297.txt*), the lower magnitude cut-off (usually magnitude of completeness), number of events, ranges for initial model parameter values, parameter MLEs, number of estimated parameters, the  $M_{th}$  value for which the best fit model was recognized and the corresponding value of the Akaike criterion (see Table 3).

*aichist.txt* (Table 4) - The file keeps the AIC values and model parameter estimates ( $K, \alpha, c, p$ ) for the different values of the threshold magnitude  $M_{th}$ . The two left columns ( $M_{th}, AIC$ ) allow to plot the Akaike criterion versus the triggering magnitude  $M_{th}$  and thus to recognize the prevailing clustering pattern for the studied sequence. The graphic interpretation not only reveals the best fit model (its parameter estimates are also printed in *output.txt*), but provides the possibility to analyze secondary minima of the curve, too.

When using the AIC criterion to select between competing models, individual AIC values are not suitable as they are less interpretable and contain arbitrary constants. It is more reasonable to rescale  $AIC$  or  $AIC_i$  to the differences

$$dAIC_i = AIC_i - AIC_{\min} \quad (10)$$

Table 3. Contents of the *output.txt* file

Data set : retas297.txt		
Magnitude cut-off : [3.69, ...]		
Number of data = 297		
parameter	inf	sup
k	.0000010	20.0000000
alpha	.0000010	5.0000000
c	.0000001	6.0000000
p	.6000000	3.0000000
parameter	estimate	
k	.042925012457	
alpha	1.759535967380	
c	.050653424533	
p	1.023207659371	
Number of parameters = 4    final Mth = 4.40		
AIC = -376.067754063429200		

Table 4. Example of the AIC values and model parameter estimates (formula (6)) for the different values of the threshold magnitude  $M_{th}$ . This information is included in *aichist.txt*

$M_{th}$	AIC	K	$\alpha$	c	p
3.7	-368.254	0.0144	2.068	0.049	1.036
3.8	-368.225	0.015	2.058	0.049	1.03
—	—	—	—	—	—
6.5	-350.308	0.009	2.277	0.057	0.964
7.2	-312.726	10.945	0.284	0.057	0.926

$dAIC_i^{\min}$  where  $AIC_{\min}$  is the minimum of the different  $AIC_i$  values for a dataset. After this transformation the best model has  $dAIC_i = 0$ , while the rest of the models have positive values.

The  $dAIC_i$  allow meaningful interpretation without the unknown scaling constants and sample size issues that enter into AIC values. These differences are easy to interpret and allow comparison and ranking of candidate models [22]. The larger the  $dAIC_i$ , the less probable the model  $i$  is as being the best approximating model in the candidate set. It is generally important to know which model is the second best, as well as some measure of its standing with respect to the best model. There are some simple rules of thumb, which are useful in measuring the relative strength of models in the set: Models having  $dAIC_i \leq 2$  have substantial support for being alternatives of the best fit

Table 5. Example of real and expected cumulative number of events in time according to the best fit model. Standard deviation stands for the error bounds. This information is included in *outcum.txt*

Time [days]	M	Cum.N	Cum.Nm	Cum.Nm + st.d.	Cum.Nm - st.d.
0	7.2	1	1	1	1
0.00416666	5.1	2	2.73239	4.3853844	1.0793958
0.00833333	4	3	4.378021	6.4703936	2.2856492
0.01041666	3.8	4	5.156669	7.427499	2.8858389
—	—	—	—	—	—

model, those in which  $4 \leq dAIC_i \leq 7$  have considerably less support, and models having  $dAIC_i > 10$  have essentially no support [22].

*outcum.txt* (Table 5) - By substituting the ML estimates of the best fit model parameters in relation (6) we may calculate the expected cumulative number of events, using the standard deviation (formula 9) as error bounds. These are presented in the above mentioned output file. The columns in the file represent:

- Time [days] — Time periods after the occurrence of the first event in the sequence in days
- M — Events magnitudes
- Cum.N — Real cumulative number of earthquakes in time
- Cum.Nm — Expected cumulative number of earthquakes for the best fit model, calculated by formula (6).
- Cum.Nm + st.d. — upper error bound (after calculating the variance by formula (6))
- Cum.Nm - st.d. - lower error bound (after calculating the variance by formula (6))

The results of *outcum.txt* can be plotted to enable analysis of earthquakes temporal distribution in the sequence.

#### CHARTS AND RESULTS INTERPRETATION

The RETAS software only provides the results from the RETAS analysis in the output files, revealed above. If one wants to plot the results, any adequate software can be applied. Below we present the results from the RETAS model analysis for the three input data sets:

*retas297.txt* - the file contains a sequence of  $N = 297$  aftershocks, starting with a main shock of magnitude  $M_{main} = 7.2$

The model, for which the minimum  $AIC$  ( $dAIC_i = 0$ ) value was obtained, turned out to be RETAS with  $M_{th} = 4.4$  (see Table 3 and Fig. 2a). A highly competitive model is also RETAS with  $M_{th} = 4.5$  ( $dAIC_i = 0.599$ , much less than 2). That points to a clustering type, where predominantly aftershocks with magnitudes equal to or bigger than 4.4 cause secondary activity. If the data is for a defined seismic region, these results could be correlated to its geotectonic features like faults etc. Some small deviations between actual and expected numbers are observed from the 50<sup>th</sup> to the 200<sup>th</sup> day (Fig. 2b), but they are not significant as the real values stay within the error bounds for the entire period.

*etas215.txt* - this is a sequence of  $N = 215$  aftershocks, the first being the strongest of magnitude  $M = 6.6$ . The  $dAIC_i$  values on Fig. 3a identify the best fit model for the data to be ETAS ( $M_{th} = M_0 = 3.8$ ).

The ETAS model is clearly identified as the RETAS version, which best describes the temporal evolution of this sequence, as the  $dAIC$  value to the second best fit model (RETAS;  $M_{th} = 3.9$ ) is more than 5. The  $dAIC$  curve on Fig. 3a has a secondary minimum for  $M_{th} = 4.9$ .

This result reveals that if we analyze only stronger aftershocks ( $M_0 = 4.4$ ), the clustering type would be different. Gospodinov and Rotondi [12] obtained similar results for the Umbria-Marche region in Italy).

Table 6. MLEs of the RETAS model parameters (Formula (6)) for the different data sets. Right most column provides information about the time needed for the program implementation on a PC (Intel®Core™i3 CPU, 4.00GB, 32-bit OS)

Data sets	K	$\alpha$	c	p	Time needed for the program implementation [min]
<i>retas297.txt</i>	0.042	1.759	0.05	1.023	~ 15
<i>etas215.txt</i>	0.058	1.562	0.042	1.22	~ 10
<i>mof215.txt</i>	0.007	2.495	0.002	0.871	~ 15

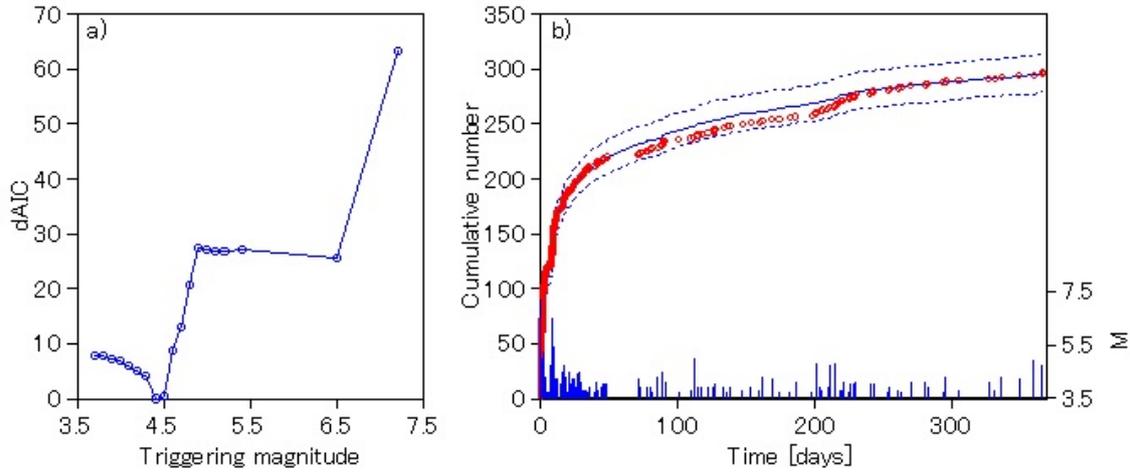


Fig. 2. a) Values of  $dAIC_i$  versus the triggering magnitude  $M_{th}$  from the analysis of the data in *retas297.txt*. The model for which  $dAIC_i = 0$  is the one, best fitting the data ( $M_{th} = 4.4$ ); b) Cumulative real (circles) and expected (continuous line) numbers in time (days). The expected values were calculated by relation (6) for the best fit model. Standard deviation stands for the error bounds (dashed curves) and the vertical lines are magnitudes.

Aftershocks with magnitudes above this cutoff seem to be more clustered temporally to events of magnitudes  $M = 4.9$  or stronger, while weaker shocks are more scattered in time. All this comes to support that the RETAS model analysis ( $dAIC$  curve) throws light not only on the best fit model, but also about more details of aftershock clustering features in a sequence. The results on Fig. 3b point out that there is a relative deviation of actual numbers, compared to model, for the time span between the 7<sup>th</sup> and the 17<sup>th</sup> day, which probably is due to the occurrence of several aftershocks of magnitude  $M = 5$  or higher, which have

rate productivity higher than the one predicted by the model.

*mof215.txt* - this data set is composed of  $N = 215$  earthquakes. The strongest of them is the first one of magnitude  $M = 6.6$ . The  $dAIC_i$  values on Fig. 4a identify the best fit model for the data to be MOF ( $M_{th} = M_{main} = 6.6$ ). The MOF model does not have a competing alternative as  $dAIC$  to the second best fit model (RETAS;  $M_{th} = 5.4$ ) is more than 5. No significant divergence between real and model cumulative numbers is observed on Fig. 4b (actual values stay within the error bound for the total time period).

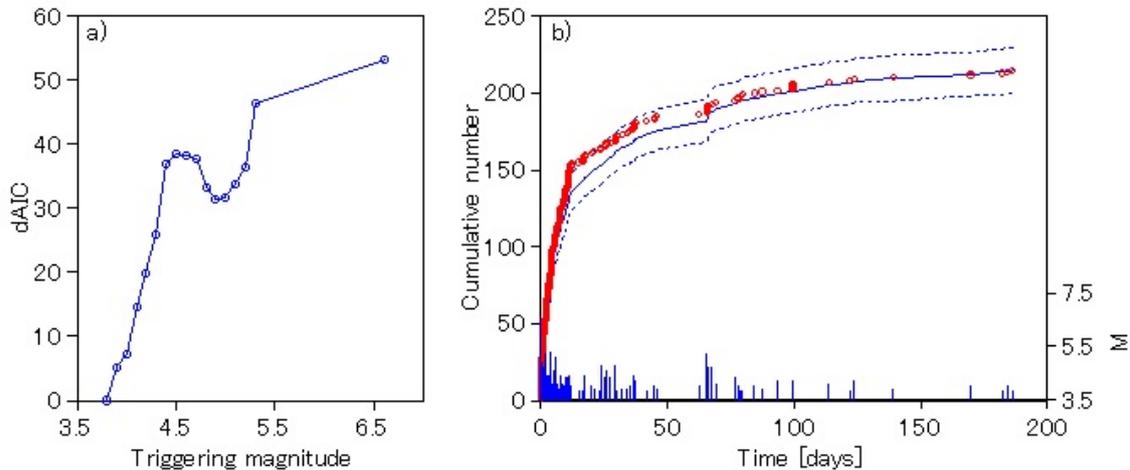


Fig. 3. The results from the RETAS software analysis for *etas215.txt* (best fit model is ETAS for  $M_{th} = M_0 = 3.8$ ). Notation as in Fig. 2

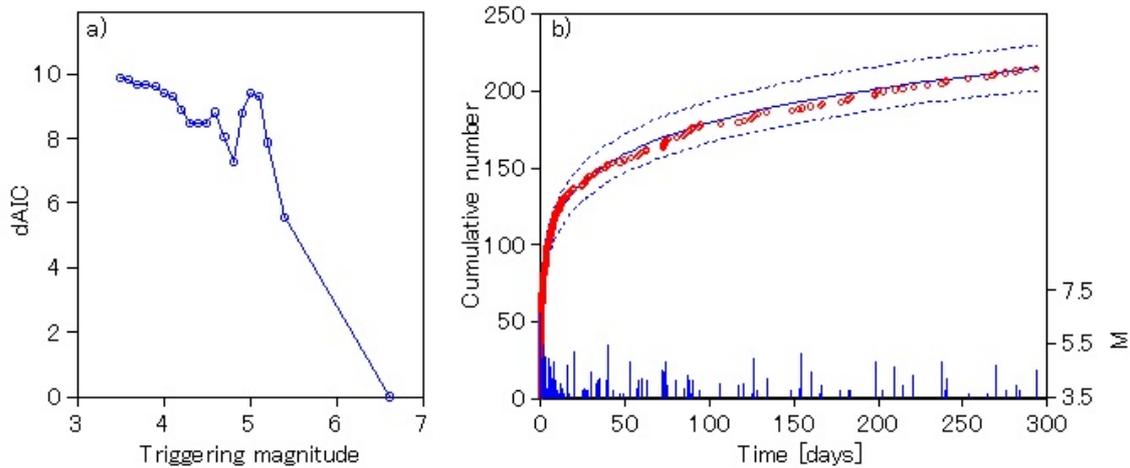


Fig. 4. The results from the RETAS software analysis for *mof215.txt* (best fit model is MOF for  $M_{th} = M_{main} = 6.6$ ). Notation as in Fig. 2

## CONCLUSIONS

In fact, by RETAS we denote a number of stochastic model versions, defined by formula (6). The MOF and the ETAS model are two limit cases of these versions, the former with only one parent-event and the latter with all events sharing in the generation of the subsequent ones. In that case the advantage of using RETAS is that we estimate the MLEs for the ETAS and MOF models as limit cases (we lose nothing), but we also verify all intermediate cases (for the different  $M_{th}$  values), for which only aftershocks of magnitudes equal to or bigger than  $M_{th}$  can induce secondary aftershocks. We identify the model, best fitting our data, as the one with the minimum AIC value. In the present paper the author applied the RETAS software on three data sets, presenting completely different types of aftershock sequences. The software correctly identified the corresponding best fit model versions, providing the necessary files to plot the expected and real cumulative numbers in time. Except that RETAS gives us a wide range of model versions to depict the temporal evolution of a series of aftershocks, it also provides broader possibility to relate the results from the stochastic analysis to other information about a seismic region, as for example, the possible length distribution of faults, which have triggered secondary aftershocks.

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СОФТУЕРНА ПРОГРАМА ЗА ИДЕНТИФИЦИРАНЕ НА НАЙ-ДОБРАТА ВЕРСИЯ  
НА RETAS МОДЕЛА ЗА ПРЕДСТАВЯНЕ ЗАТИХВАНЕТО НА АФТЕРШОКОВАТА АКТИВНОСТ ВЪВ ВРЕМЕТО

Др. Господинов

Физически факултет, Пловдивски университет "Паисий Хилендарски",  
ул. "Цар Асен" №24, 4000 Пловдив, България

(Резюме)

В тази статия са демонстрирани възможностите и приложимостта на софтуерна програма, написана на FORTRAN 95, за моделиране спада на афтершоковата активност във времето. Програмата е предназначена за извършване изчисления на параметрите на ограничен епидемичен тип модел на афтершокова активност (RETAS) за описание времевото разпределение на афтершоците. Това е тригериращ модел, който осигурява статистическо описание на времевата еволюция на вторичните трусове в серията. Предимство на RETAS е, че моделът предлага редица версии, като едновременно включва най-използваните модели - модифицираната формула на Омори (MOF) и епидемичния тип модел на афтершокова активност (ETAS) като гранични случаи. По този начин RETAS предлага по-голямо разнообразие на модели за подбиране на най-добрия от тях с цел описание времевото разпределение на вторични трусове в една сеизмична последователност. Авторът е предоставил примери на симулирани данни, за да демонстрират използването на софтуера за различни типове афтершокови серии, като е посочил и начините за интерпретация на получените резултати.