Calculation of particle size field in hydrocyclones

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A study of particle motions in the hydrocyclone based on fluid rotational motion formula and fluid vortex rotation theory is proposed. The effects of radial flow on the flow field were included, as well as the Magnus lift force on particles at high rotational speeds. Also, equations for axial motion, radial motion and trajectory of particles were established and a regular distribution of particles was demonstrated. These were achieved by analysis of the motion of and forces on solid particles in the hydrocyclone. This study serves as a reference to future studies of hydrocyclone separation.

Keywords: Hydrocyclone, Particle size field, Calculation, Magnus lift force

INTRODUCTION

A hydrocyclone refers to a device that can realize heterogeneous separation based on the differences in particle size and density [1-4] .Upon entering the hydrocyclone, solid particles would experience rotational, axial and radial motions due to the effects of fluids and the velocity and direction of particle motion are dependent on the forces applied on the particles. As the separation of solid particles and fluids can be attributed to the relative sliding motion, it is of great significance to understand the interaction between solid particles and fluids. In this study, the effects of radial flow on the flow field and the Magnus lift force on particles in the hydrocyclone, which were neglected in previous studies, were considered. Based on the proposal that particles in the hydrocyclone were affected by various forces, including centrifugal force, gravity, centripetal buoyance, fluid resistance, and Magnus lift force [5-7], a more accurate force model of particles was developed and a regular distribution of particles was demonstrated.

DISTRIBUTION OF PARTICLE SIZE FIELD IN THE HYDROCYCLONE

To precisely predict the forces experienced by solid particles in the hydrocyclone, the effects of fluid radial motions on the particles should be included to develop an effective force model for the particles.

The equations of centrifugal and centripetal buoyance are as follows:

$$F_c = \frac{\pi}{6} d^3 \rho_s \frac{u_t^2}{r} \tag{1}$$

$$F_f = \frac{\pi}{6} d^3 \rho \left(\frac{u_t^2}{r} - u_r \frac{du_r}{dr} \right) \tag{2}$$

Where Fc and Ff are the centrifugal force and centripetal buoyance on the particle, respectively; d is the particle diameter; ρs and ρ are the densities of particles and fluid in a continuous phase, respectively; ur is the radial speed.

The drag force on particles by the fluid can be described by:

$$F_d = \xi A \frac{\rho u^2}{2} \tag{3}$$

Where ξ is the resistance coefficient, which is a function of the Reynolds number (Re); u is the relative speed of fluid and particle; A is the area of particle projection in the radial direction:

$$A = \frac{\pi d^2}{4}$$

However, the forces experienced by particles are more complicated in real cases. Besides the centrifugal force, the centripetal buoyance and the fluid resistance, revolution caused by internal frictions between different laminar flows was observed as the particles are in a vortex with high rotational speeds. The rotational directions of particles in forced vortex zone and semi-free vortex zone (with the radius corresponding to the maximum tangential velocity (rm) as the demarcation line) are opposite. The difference in the flow rate and pressure at the internal and external sides of the particle, which can be attributed to the gradient of particle velocity in the radial direction, results in a radial Magnus lift force towards the wall or the axis. In the semi-free vortex zone, the Magnus lift force is towards the axis as

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the flow rate at the internal side is higher than that at the external side; in the forced vortex zone, the Magnus lift force is towards the wall as the external side is higher than that at the internal side. The significant laminar frictions experienced by particles at high rotational speeds in the hydrocyclone lead to rapid revolution and the effects of the Magnus force should be included. Cen et al[8] proposed an equation of Magnus lift force:

$$F_M = \frac{1}{8}\pi\rho d^3 u\omega \tag{4}$$

Where ω is the rotational angular speed of particles.

In the field of radial centrifugal forces in the hydrocyclone, the directions of Magnus lift forces are different in different vortex zones. Therefore, the forces on particles in the semi-free vortex zone and the forced vortex zone shall be discussed separately.

For particles in the semi-free vortex zone, centrifugal force (Fc), centripetal force by the fluid in continuous phase (Ff), dragging force by the fluid (Fd) and Magnus lift force (FM) are observed. The radial sedimentation of particles in the centrifugal force field can be described by:

$$F_c - F_f - F_d - F_M = m\frac{du}{dt} \tag{5}$$

Namely

$$\frac{\pi}{6}d^3\rho_s\frac{u_t^{\,2}}{r} - \frac{\pi}{6}d^3\rho(\frac{u_t^{\,2}}{r} - u_r\frac{du_r}{dr}) - \xi A\frac{\rho u^2}{2} - \frac{1}{8}\pi\rho d^3 u\omega = m\frac{du}{dt} \tag{6}$$

Where m is the particle mass. In cases where forces on the particles are balanced, uniform radial motion ($\frac{du}{dt} = 0$) was observed and the size of particles in the centrifugal force field of the hydrocyclone particle can be described by:

$$d = \frac{3\xi\rho u^2 r}{4(\rho_s - \rho)u_t^{\,2} + 4r\rho u_r\frac{du}{dr} - 3r\rho u\omega} \tag{7}$$

Based on the tangential speed of the fluid $u_t r^n = c_1$ (constant), the following equation can be obtained:

$$\frac{u_t^{\,2}}{r} = \frac{c_1^{\,2}}{r^{2n+1}} \tag{8}$$

Based on the empirical formula of fluid radial speed $u_r r^m = -c_2$ (constant), m is the index number and $0 < m < 1$, the following equation can be obtained:

$$u_r\frac{du_r}{dr} = -\frac{mc_2^{\,2}}{r^{2m+1}} \tag{9}$$

By substituting Equation (8) and (9) into Equation (7), we can obtain:

$$d = \frac{3\xi\rho u^2 r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho - 3\rho u\omega r^{2m+1}} \tag{10}$$

Where $c_3 = mc_2^{\,2}$ (constant).

For particles in the semi-free vortex zone, the Magnus lift force is towards the wall and the radial sedimentation of particles in the centrifugal force field can be described by:

$$\frac{\pi}{6}d^3\rho_s\frac{u_t^{\,2}}{r} - \frac{\pi}{6}d^3\rho(\frac{u_t^{\,2}}{r} - u_r\frac{du_r}{dr}) - \xi A\frac{\rho u^2}{2} + \frac{1}{8}\pi\rho d^3 u\omega = m\frac{du}{dt} \tag{11}$$

In cases where forces on the particles are balanced, the size of particles in the centrifugal force field of the hydrocyclone particle can be described by:

$$d = \frac{3\xi\rho u^2 r}{4(\rho_s - \rho)u_t^{\,2} + 4r\rho u_r\frac{du}{dr} + 3r\rho u\omega} \tag{12}$$

By substituting Equation (8) and (9) into Equation (12), we can obtain:

$$d = \frac{3\xi\rho u^2 r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho + 3\rho u\omega r^{2m+1}} \tag{13}$$

ξ is a function of R_e, which can be described by:

$$R_e = \frac{ud\rho}{\mu} \tag{14}$$

Where μ is the fluid viscosity.

If the relative motion between particles and the medium is regarded as a laminar flow, $R_e < 1$. The viscosity force is the dominant resistance by the fluid and the Stoke's resistance coefficient is utilized as ξ. With $\xi = \frac{24}{R_e}$ and Equation (14) substituted into the Stoke's Equation, the following equation can be obtained:

$$\xi = \frac{24\mu}{ud\rho} \tag{15}$$

Herein, $d < 0.1$ mm.

If the relative motion between particles and the medium is regarded as a transition flow, $1 < R_e < 1000$. The Allende's resistance coefficient is utilized as ξ. With $\xi = \frac{10}{\sqrt{R_e}}$ and Equation (14) substituted into the Allende's Equation, the following equation can be obtained:

$$\xi = \frac{10}{\sqrt{\frac{ud\rho}{\mu}}} \quad (16)$$

Herein, 0.1 mm < d < 1.5 mm.

If the relative motion between particles and the medium is regarded as a turbulent flow, Re > 1000. The inertia resistance is the dominant resistance and the Newton's resistance coefficient is utilized as ξ (ξ = 0.44). Herein, d > 1.5 mm.

If Re of particles is within the laminar flow zone, we can obtain the particle size distribution in the semi-free vortex zone and forced vortex zone by substituting Equation (15) and (10) into Equation (13), as follows:

$$d = \left(\frac{72u\mu r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho - 3\rho u\omega r^{2m+1}} \right)^{\frac{1}{2}} \quad (17)$$

$$d = \left(\frac{72u\mu r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho + 3\rho u\omega r^{2m+1}} \right)^{\frac{1}{2}} \quad (18)$$

If Re of particles is within the transition flow zone, we can obtain the particle size distribution in the semi-free vortex zone and forced vortex zone by substituting Equation (16) and (10) into Equation (13), as follows:

$$d = \left[\frac{30\mu^{\frac{1}{2}}\rho^{\frac{3}{2}}u^{\frac{3}{2}}r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho - 3\rho u\omega r^{2m+1}} \right]^{\frac{2}{3}} \quad (19)$$

$$d = \left[\frac{30\mu^{\frac{1}{2}}\rho^{\frac{3}{2}}u^{\frac{3}{2}}r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho + 3\rho u\omega r^{2m+1}} \right]^{\frac{2}{3}} \quad (20)$$

If Re of particles is within the turbulent flow zone, we can obtain the particle size distributions in the semi-free vortex zone and forced vortex zone by substituting ξ = 0.44 into Equation (10) and (13), as follows:

$$d = \frac{1.32\rho u^2 r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho - 3\rho u\omega r^{2m+1}} \quad (21)$$

$$d = \frac{1.32\xi\rho u^2 r^{2m+1}}{4c_1^{\,2}(\rho_s - \rho)r^{2m-2n} - 4c_3\rho + 3\rho u\omega r^{2m+1}} \quad (22)$$

According to the equations obtained, the particle size at r is proportional to the gyration radius, although the specific relation is complicated. Briefly, the gyration radius of the particle increases with its size and a regular distribution of particles along the radial direction is observed in the hydrocyclone. As a result, coarse particles are directed towards the underflow opening and fine particles are directed towards the overflow opening.

Besides radial motions mentioned above, axial motions (upwards or downwards) of particles caused by internal vortex are also observed in actual cases. Gravity, buoyance, fluid resistance and Magnus lift force are applied on particles in the axial direction. The axial motion of particles can be described by:

$$\frac{\pi}{6}d^3 g(\rho_s - \rho) - \xi\frac{\rho u_b^{\,2}\pi d^2}{8} - \frac{\pi}{8}\rho d^3 u_b\omega = \frac{\pi}{6}d^3\rho_s\frac{du_b}{dt} \quad (23)$$

Where ub is the relative sliding speed of particle and fluid in the radial direction.

Assuming that the accelerating stage in sedimentation is negligible, uniform radial motion ($\frac{du}{dt} = 0$) was observed and size of particles in the axial gravity field can be described by:

$$\frac{\pi}{6}d^3 g(\rho_s - \rho) - \xi\frac{\rho u_b^{\,2}\pi d^2}{8} - \frac{\pi}{8}\rho d^3 u_b\omega = 0 \quad (24)$$

The size distribution of particles in the axial direction can be described by:

$$d = \frac{3\xi\rho u_b^{\,2}}{4g(\rho_s - \rho) - 3\rho u_b\omega} \quad (25)$$

According to Equation (25), the particle size in the axial direction is determined by the physical properties of particles and fluids and not affected by the flow rate in the hydrocyclone. The size distribution of particles is dependent on the field of radial centrifugal force.

TRAJECTORY OF PARTICLES IN THE HYDROCYCLONE

For solid particles in a hydrocyclone, gravity, buoyance, fluid resistance and Magnus lift force are observed in the axial gravity field, while centrifugal force, centripetal buoyance, fluid resistance and Magnus lift force are observed in the radial centrifugal field. The accelerating stage is negligible as the forces on particles reach an equilibrium state in a short time. Due to the low Re in the axial sedimentation process, this process can be regarded as a Stoke's sedimentation.

The particle motion was investigated in a cylindrical coordinate system by substituting Equ (15) into Equ (25). The relative axial sedimentation rate of particles can be described by:

$$u_b = \frac{4g(\rho_s - \rho)d^2 - 3\rho\omega d}{72\mu} \quad (26)$$

The sedimentation rate of the fluid in the field of axial gravity is defined as v and the sedimentation rate of particles in the fluid can be obtained by:

$$v + u_b = v + \frac{4g(\rho_s - \rho)d^2 - 3\rho\omega d}{72\mu} \qquad (27)$$

In cases where both axial and radial forces experienced by particles are in equilibrium, particles would rotate along the axis at ωr. At the same time, sedimentation along the axis at the speed described in Equation (27) is observed for these particles. Therefore, the location of a particle at time = t can be described by:

$$x = r\cos\omega_r t$$
$$y = r\sin\omega_r t$$
$$z = \left[v + \frac{4g(\rho_s - \rho)d^2 - 3\rho\omega d}{72\mu} \right] t \qquad (28)$$

Equation (28) are the combined spiral vortex parameter formula for particle sedimentation and equation of particle trajectory. At a certain pressure, particles would get into the hydrocyclone with the fluid and be distributed to their respective rotational tracks in a short time. A regular spatial distribution due to the effects of particle size on its rotational radius can be observed.

CONCLUSIONS

The separation of solid particles and fluids can be attributed to the relative sliding motion caused by forces experienced by the particles. Besides centrifugal force, gravity, centripetal buoyance and fluid resistance, Magnus lift force is also applied on the particles in a hydrocyclone. Based on conventional rotating flow theory, a force model of particles in a hydrocyclone was established with effects of radial flows and the Magnus lift force taken into consideration. Additionally, equations for the trajectory of particles were established and a regular distribution of particles was demonstrated. This study serves as a reference to future studies of hydrocyclone separation.

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