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On the low-temperature critical behaviour of a quantum model of structural phase transitions

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A quantum model describing structural phase transitions in an anharmonic crystal with long-range interaction (decreasing at large distances *r* as $r^{d-\sigma}$, where *d* is the space dimensionality and $0< \sigma \le 2$) is studied at low temperatures. A general expression for the specific heat capacity is derived in the low-temperature region of the *T, λ* - phase diagram, where *T* is the temperature and *λ* is a quantum parameter, associated with the external pressure or the amount of doping. It is shown that in the vicinity of the quantum critical point $(T = 0, \lambda = \lambda_c)$ this expression has a scaling form. The temperature dependence of the critical specific heat capacity *c(T)* is established in the three regions of the phase diagram (renormalized classical region, quantum critical region and quantum disordered region). From the results obtained one can see that $c(T) \sim T^{2d/\sigma}$ in the renormalized classical region and in the quantum critical region, and *c(T)* exponentially tends to zero in the quantum disordered region. The applicability of the results obtained to other models is discussed.

Keywords: bulk critical behaviour, low-temperature effects, specific heat capacity, long–range interaction

INTRODUCTION

Over the past few decades the theory of zero-temperature quantum phase transitions, initiated in 1976 by Hertz, continues to be a subject of great interest [1-4]. These phase transitions, caused by quantum fluctuations rather than thermal ones, appear at zero temperature $(T=0)$ as a function of some non-thermal control parameter (associated with pressure, doping concentration or magnetic fields) or a competition between different parameters describing the basic interaction of the system. Most importantly, the zero-temperature quantum phase transitions can have great impact on the leading T dependence of all observables for a relatively large region of rather low temperatures, compared to characteristic excitation in the system. The low-temperature effects can be explored in the framework of the theory of finite-size scaling (FSS) [5-10]. The most famous model for discussing these properties is the quantum nonlinear $O(n)$ sigma model (QNLσM) [5-7], [11-13]. Its equivalence (in the limit $n \to \infty$) with a quantum version of the spherical model, known as the spherical quantum rotors model (SQRM), has been given in [14]. The SQRM is suitable for a joint description of the quantum and classical fluctuations in dependence on the dimensionality and the geometry of the system [8, 9], [15, 16].

Here we consider a more realistic quantum model with long-range interaction (decreasing at large distances r as $r^{-d-\sigma}$, where d is the space dimensionality and $0<\sigma\le 2$), intended to describe structural phase transitions [17]. The main feature of this model is that the real anharmonic interaction is substituted by its quantum mean spherical approximation reducing the problem to an exactly solvable one. Its exact solvability has been proven in [18]. The relation of this model with the SQRM has been commented in [8]. A rigorous proof of the effect of its quantum fluctuations has been given in [19]. The bulk critical behaviour of the model on the whole (T,λ)-phase diagram, where λ is a quantum parameter, has been studied in [20]. In real systems a phase transition driven by λ can be observed by changing the external pressure or the amount of doping. The finite-size corrections to the free energy for the pure quantum version $(T=0)$ of the model have been derived in [21]. For a more complete discussion of the bulk critical behaviour and the finite-size properties of the model and its generalizations, see Chapter 3 of [16]. In the context of this model it has been shown [22-24] that the Lambert W-function can be applied for a more exact computation of non-universal critical properties with leading logarithmic behaviour at the upper critical dimension of the system.

In this paper we study the low-temperature behaviour of the bulk specific heat capacity in the vicinity of the quantum critical point $(T=0,\lambda=\lambda_c)$ for different space dimensionalities

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($\sigma/2 < d < 3\sigma/2$) of the system and in different regions of the (T,λ)-phase diagram.

Let us note that results for the bulk critical specific heat capacity of classical systems $(\lambda = 0)$ for $d = 2\sigma$ [24] and of quantum systems in the low-temperature region for $d = \sigma$ [25] are available.

THE MODEL

The Hamiltonian of the model is [16, 17]

$$H = \frac{1}{2}\sum_{\mathbf{r}}\left(\frac{P_{\mathbf{r}}^2}{m} - AQ_{\mathbf{r}}^2\right) + \frac{1}{4}\sum_{\mathbf{r},\mathbf{r}'}\varphi(\mathbf{r}-\mathbf{r}')(Q_{\mathbf{r}} - Q_{\mathbf{r}'})^2 + \frac{B}{4N}\left(\sum_{\mathbf{r}} Q_{\mathbf{r}}^2\right)^2, \quad (1)$$

where $P_{\mathbf{r}}$ and $Q_{\mathbf{r}}$ are the operators of the momentum and displacement, respectively, of the particle of mass m at the site $\mathbf{r}$ of a d-dimensional hypercubic lattice. The parameter $A = mv_0^2 > 0$ determines the frequency of a mode which is unstable in the harmonic approximation and the parameter $B > 0$ introduces an anharmonic interaction which is inversely proportional of the particle number N. The harmonic force constants $\varphi(\mathbf{r}-\mathbf{r}')$, which are assumed to decrease at large distances $r = |\mathbf{r}-\mathbf{r}'|$ as $r^{-d-\sigma}$, describe a short-range $(\sigma = 2)$ or a long-range $(0 < \sigma < 2)$ interaction.

The free energy density of the model (1), obtained by using the Approximating Hamiltonian Method, is [18]

$$f = \frac{A^2}{B} f_0 = \frac{A^2}{2B}\left[I_{d,\sigma}\left(\lambda,t,\bar{\Delta}\right) - \frac{1}{2}\left(1+\bar{\Delta}\right)^2 \right], \quad (2)$$

where $\bar{\Delta}$ is the solution of the self-consistent equation

$$\frac{\partial I_{d,\sigma}\left(\lambda,t,\Delta\right)}{\partial \Delta} = 1 + \Delta . \quad (3)$$

In the thermodynamic limit $N \to \infty$ the function $I_{d,\sigma}\left(\lambda,t,\bar{\Delta}\right)$ is defined by

$$I_{d,\sigma}\left(\lambda,t,\Delta\right) = 2k_d t \int_0^{x_D} x^{d-1} \ln\left(2\sinh\left(\frac{\lambda}{2t}\sqrt{\Delta + x^{\sigma}}\right)\right) dx, \quad (4)$$

where $t = T/(4E_0)$ is the dimensionless temperature and $\lambda = \hbar\nu_0/(4E_0)$ is a quantum parameter which is a function of the external pressure or the amount of doping, $E_0 = A^2/(4B)$ is the barrier height of the double well potential in Eq.(1), $x_D = 2\pi(d/S_d)^{1/d}$ is the radius of the effective sphere replacing the Brillouin zone and $k_d = 2(4\pi)^{-d/2}/\Gamma(d/2)$ ($\Gamma(x)$ is the Euler gamma function).

In the disordered phase $(t > t_c)$ $\bar{\Delta}$ is finite and the susceptibility of the system is $\chi = \bar{\Delta}^{-1}$. Setting $\bar{\Delta} = 0$ into Eq. (3) one obtains the critical temperature $t_c(\lambda)$. At the critical value $\lambda_c = (2 - \sigma/d)x_D^{\sigma/2}$ of the quantum parameter the critical temperature is reduced to zero, $t_c(\lambda_c) = 0$.

Eqns. (2) and (3) provide the basis for studying the critical behaviour of the model. For the specific heat capacity $c(T) \equiv -T(\partial^2 f/\partial T^2)$, taking into account Eq. (3), one obtains

$$c(t) = -t\frac{\partial^2 f_0}{\partial t^2} = -\frac{t}{2}\left[\frac{\partial^2 I_{d,\sigma}\left(\lambda,t,\bar{\Delta}\right)}{\partial t^2} + \frac{\partial^2 I_{d,\sigma}\left(\lambda,t,\bar{\Delta}\right)}{\partial\bar{\Delta}\partial t}\left(\frac{\partial\bar{\Delta}}{\partial t}\right)\right]. \quad (5)$$

Let us note that Eq. (3) is similar to the mean-spherical constraint in the SQRM up to the linear Δ-term in the right side of Eq. (3) [8, 9]. This term appears to be essential only above the upper critical dimension.

A GENERAL EXPRESSION FOR THE SPECIFIC HEAT AT LOW TEMPERATURES

At low temperatures the integral, given by Eq. (4), has been studied in the framework of the SQRM for $\sigma = 2$ [8, 9] and $0 < \sigma \le 2$ [15]. In the low-temperature region $(\lambda/t >> 1)$ it can be presented in the form

$$I_{d,\sigma}\left(\lambda,t,\Delta\right)=2\lambda A_{d,\sigma}\left(\Delta\right)-2tB_{d,\sigma}\left(\frac{2t}{\lambda},\frac{\lambda}{2t}\sqrt{\Delta}\right). \quad (6)$$

In Eq. (6) the function $A_{d,\sigma}\left(\Delta\right)$, expressed in terms of the hypergeometric function ${}_2F_1(a,b;c;z)$, has the form

$$A_{d,\sigma}\left(\Delta\right)=\frac{k_d x_D^d}{2d}\sqrt{x_D^\sigma+\Delta}\,{}_2F_1\left(-\frac{1}{2},1;1+\frac{d}{\sigma};\frac{x_D^\sigma}{x_D^\sigma+\Delta}\right), \quad (7)$$

and the function $B_{d,\sigma}\left(x,y\right)$ is

$$B_{d,\sigma}\left(\frac{2t}{\lambda},\frac{\lambda}{2t}\sqrt{\Delta}\right)=\frac{k_d}{\sqrt{\pi}\sigma}\Gamma\left(\frac{d}{\sigma}\right)\left(\frac{2t}{\lambda}\right)^{2\frac{d}{\sigma}}$$

$$\times\tilde{\mathrm{K}}\left(\frac{d}{\sigma}+\frac{1}{2},\frac{\lambda}{2t}\sqrt{\Delta}\right), \quad (8)$$

where the function

$$\tilde{\mathrm{K}}\left(\nu,y\right)\equiv y^{2\nu}\mathrm{K}\left(\nu,y\right) \quad (9)$$

is introduced. The function $\mathrm{K}\left(\nu,y\right)$ in Eq. (9) has been defined and studied in [8, 9]:

$$\mathrm{K}\left(\nu,y\right)\equiv 2\sum_{m=1}^{\infty}\left(my\right)^{-\nu}K_\nu\left(2my\right)$$

$$=\frac{\sqrt{\pi}}{2}\Gamma\left(\frac{1}{2}-\nu\right)y^{-1}+\Gamma\left(\nu\right)\varsigma\left(2\nu\right)y^{-2\nu}-\frac{1}{2}\Gamma\left(-\nu\right)$$

$$+\pi^{2\nu-\frac{1}{2}}\Gamma\left(\frac{1}{2}-\nu\right)y^{-2\nu}\sum_{m=1}^{\infty}\left[\left(m^2+\frac{y^2}{\pi^2}\right)^{\nu-\frac{1}{2}}-m^{2\nu-1}\right], \quad (10)$$

where $K_\nu\left(x\right)$ is the MacDonald function (second modified Bessel function) and $\varsigma\left(x\right)$ is the Riemann zeta function. The asymptotic form of the function $\tilde{\mathrm{K}}\left(\nu,y\right)$ at $y<<1$, obtained by using the asymptotic form of the function $\mathrm{K}\left(\nu,y\right)$ [8, 9], for $1<\nu<3/2$ and $3/2<\nu<2$ is

$$\tilde{\mathrm{K}}\left(\nu,y\right)\approx\frac{\pi^{1/2}}{2}\Gamma\left(\frac{1}{2}-\nu\right)y^{2\nu-1}+\Gamma\left(\nu\right)\zeta\left(2\nu\right)$$

$$+\pi^{2\nu-5/2}\left(\nu-\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-\nu\right)\zeta\left(3-2\nu\right)y^2$$

$$-\frac{1}{2}\Gamma\left(-\nu\right)y^{2\nu}. \quad (11)$$

For the specific heat capacity at low temperatures, from Eqns. (5)-(8), we obtain the following expression

$$c\left(t\right)=\frac{k_d}{\sqrt{\pi}\sigma}\Gamma\left(\frac{d}{\sigma}\right)\left[\Phi_1\left(\frac{d}{\sigma}+\frac{1}{2},\frac{\lambda}{2t}\sqrt{\bar{\Delta}}\right)\right.$$

$$\left.+\frac{1}{2}\Phi_2\left(\frac{d}{\sigma}+\frac{1}{2},\frac{\lambda}{2t}\sqrt{\bar{\Delta}}\right)\left(\frac{t}{\bar{\Delta}}\frac{\partial\bar{\Delta}}{\partial t}\right)\right]\left(\frac{2t}{\lambda}\right)^{2d/\sigma}, \quad (12)$$

in which the functions

$$\Phi_1\left(\nu,y\right)\equiv 2\nu\left(2\nu-1\right)\tilde{\mathrm{K}}\left(\nu,y\right)$$

$$+2\left(4\nu-3\right)y^2\tilde{\mathrm{K}}\left(\nu-1,y\right)+4y^4\tilde{\mathrm{K}}\left(\nu-2,y\right) \quad (13)$$

and

$$\Phi_2\left(\nu,y\right)\equiv-4\left(\nu-1\right)y^2\tilde{\mathrm{K}}\left(\nu-1,y\right)-4y^4\tilde{\mathrm{K}}\left(\nu-2,y\right) \quad (14)$$

are introduced and $\bar{\Delta}=\chi^{-1}$ is the solution of the equation for the inverse susceptibility, Eq. (3), in the low-temperature region.

In the next section we present results for the low-temperature critical inverse susceptibility of the model for systems with different space dimensionalities d in different regions of the $\left(t,\lambda\right)$-phase diagram. These results (see also [16] and [20]) will be used in obtaining the corresponding expressions for the critical specific heat capacity.

THE LOW-TEMPERATURE CRITICAL INVERSE SUSCEPTIBILITY

In the low-temperature region close to the quantum critical point $\left(\Delta<<1\right)$ for $1/2<d/\sigma<3/2$ Eq. (3) has the form

$$\frac{k_d}{2\sqrt{\pi}\sigma}\Gamma\left(\frac{d}{\sigma}\right)\Delta^{\frac{d}{\sigma}-\frac{1}{2}}\left[2\mathrm{K}\left(\frac{d}{\sigma}-\frac{1}{2},\frac{\lambda}{2t}\sqrt{\Delta}\right)\right.$$

$$-\left|\Gamma\left(\frac{1}{2}-\frac{d}{\sigma}\right)\right|\Bigg] == \frac{1}{\lambda} - \frac{1}{\lambda_c}, \qquad (15)$$

where λ_c is the quantum critical point and the function $\mathrm{K}(\nu, y)$ is given by Eq. (10).

On the line $\lambda = \lambda_c$ $(t \to 0^+)$ the solution of Eq. (15) is given by

$$\bar{\Delta} = D_1 t^{\gamma_T^q}, \qquad (16)$$

where the critical exponent $\gamma_T^q = 2$, $D_1 = 4y_0^2/\lambda_c^2$ and y_0 is the solution of the equation $|\Gamma(1/2 - d/\sigma)| = 2\mathrm{K}(d/\sigma - 1/2, y)$. The behaviour of the universal constant y_0 as a function of the dimensionality d of the system in the case of short-range $(\sigma = 2)$ interaction is graphically represented in [8, 9].

By using the critical exponent $\gamma_\lambda = \sigma/(d-\sigma)$, the solution of Eq. (15) can be presented in the form (see [16])

$$\bar{\Delta} \approx D_2 \varepsilon^{\gamma_\lambda} t^{-\gamma_\lambda}, \qquad (17)$$

where $D_2 = \left[\sigma |\sin(\pi d/\sigma)| / (k_d \pi)\right]^{\gamma_\lambda}$, in the two cases: (i) for $1/2 < d/\sigma < 1$ at $\lambda < \lambda_c$, in Eq. (17) $\varepsilon = 1 - \lambda/\lambda_c$, i.e. in the system there is a phase transition driven by t $(t \to 0^+)$ and (ii) for $1 < d/\sigma < 3/2$ at $\lambda_c(t) < \lambda < \lambda_c$, in Eq. (17) $\varepsilon = \lambda/\lambda_c(t) - 1$, i.e. at finite temperatures in the system there is a phase transition driven by λ for very close values of λ to the "shifted" critical value $\lambda_c(t)$.

For $1/2 < d/\sigma < 3/2$ at $\lambda > \lambda_c$ sufficiently close to the quantum critical point by using the critical exponents $\gamma_\lambda^q = 2\sigma/(2d-\sigma)$ and $\phi_T = \sigma/(2d-\sigma)$ the solution of Eq. (15) can be written as (see [16])

$$\bar{\Delta} \approx D_3 \varepsilon^{\gamma_\lambda^q} \left(\frac{t}{\varepsilon^{\phi_T}}\right)^{(\gamma_\lambda^q - \gamma_\lambda)/\phi_T}, \qquad (18)$$

where $D_3 = \left\{2\sigma\sqrt{\pi}/\left[k_d \Gamma(d/\sigma)|\Gamma(1/2 - d/\sigma)|\right]\right\}^{\gamma_\lambda^q}$ and $\varepsilon = 1/\lambda_c - 1/\lambda$.

In the important case $d = \sigma$ Eq. (15) is simplified considerably and has the form

$$2\sinh\left(\frac{\lambda}{2t}\sqrt{\bar{\Delta}}\right) = \exp\left[-\frac{\sigma}{2k_\sigma t}\left(1 - \frac{\lambda}{\lambda_c}\right)\right], \qquad (19)$$

where $k_\sigma = 2(4\pi)^{-\sigma/2}/\Gamma(\sigma/2+1)$. Its solutions are as follows: (i) on the line $\lambda = \lambda_c$ and $t \to 0^+$, i.e. in the quantum critical region,

$$\bar{\Delta} = \Theta^2 t^2/\lambda_c^2, \qquad (20)$$

where $\Theta = 2y_0 = 2\ln\left[(1+\sqrt{5})/2\right] = 0.962424...$ is a universal constant [8, 9]; (ii) in the region where $(1-\lambda/\lambda_c)/t >> 1$ and $\lambda < \lambda_c$, i.e. in the renormalized classical region,

$$\bar{\Delta} \approx \frac{t^2}{\lambda^2}\exp\left[-\frac{\sigma(1-\lambda/\lambda_c)}{k_\sigma t}\right]; \qquad (21)$$

(iii) for $(\lambda/\lambda_c - 1)/t >> 1$ and $\lambda > \lambda_c$, i.e. in the quantum disordered region,

$$\bar{\Delta} \approx \frac{\sigma^2(\delta\lambda)^2}{k_\sigma^2}\left[1 + \frac{4k_\sigma t}{\sigma\lambda\delta\lambda}\exp\left(-\frac{\lambda\delta\lambda}{k_\sigma t}\right)\right], \qquad (22)$$

where $\delta\lambda = 1/\lambda_c - 1/\lambda$. The first term in Eq. (22) is a particular case of Eq. (18) for $d = \sigma$.

Let us note that Eqns. (16)-(18) have been obtained in [20] (see also [16]) in another way. At $\sigma = 2$ Eqns. (16)-(22) coincide with the corresponding ones for the spherical field obtained in [8, 9] for the SQRM with short-range interaction.

THE LOW-TEMPERATURE CRITICAL SPECIFIC HEAT CAPACITY

In the vicinity of the quantum critical point for $1/2 < d/\sigma < 3/2$, taking into account the temperature dependence of $\bar{\Delta}$, it is easy to see that Eq. (12) has a scaling form, where $\lambda\sqrt{\bar{\Delta}}/2t$ and $2t/\lambda$ are the scaling variables.

For $1/2 < d/\sigma < 3/2$ at $\lambda = \lambda_c$ and $t \to 0^+$, i.e. *in the quantum critical region*, substituting Eq. (16) in Eq. (12), for the critical specific heat capacity we have

$$c(t)=\frac{k_d}{\sqrt{\pi}\sigma}\Gamma\left(\frac{d}{\sigma}\right)\left[\Phi_1\left(\frac{d}{\sigma}+\frac{1}{2},y_0\right)\right.$$

$$\left.+\Phi_2\left(\frac{d}{\sigma}+\frac{1}{2},y_0\right)\right]\left(\frac{2t}{\lambda}\right)^{2d/\sigma}. \qquad (23)$$

For $1/2<d/\sigma<1$ at $(1-\lambda/\lambda_c)/t>>1$, i.e. *in the renormalized classical region*, and for $1<d/\sigma<3/2$ in the region where $(\lambda/\lambda_c(t)-1)/t<<1$ and $\lambda_c(t)<\lambda<\lambda_c$, from Eq. (12) and Eq. (17), by using the asymptotic forms of the functions $\Phi_1(\nu,y)$ and $\Phi_2(\nu,y)$ at $y<<1$, we obtain

$$c(t)\approx\left[\frac{4k_d}{\sigma\sqrt{\pi}}\Gamma\left(\frac{d}{\sigma}+1\right)\Gamma\left(\frac{d}{\sigma}+\frac{3}{2}\right)\right.$$

$$\left.\times\varsigma\left(\frac{2d}{\sigma}+1\right)\right]\left(\frac{2t}{\lambda}\right)^{\frac{2d}{\sigma}}. \qquad (24)$$

For $1/2<d/\sigma<3/2$ when $(\lambda/\lambda_c-1)/t>>1$, i.e. *in the quantum disordered region*,

$$c(t)\approx\frac{k_d}{\sqrt{\pi}\sigma}\Gamma\left(\frac{d}{\sigma}\right)\Phi_{1\infty}\left(\frac{d}{\sigma}+\frac{1}{2},\frac{\lambda}{2t}\sqrt{\bar{\Delta}}\right)\left(\frac{2t}{\lambda}\right)^{\frac{2d}{\sigma}}, \qquad (25)$$

where $\bar{\Delta}$ is given by Eq. (18) and by $\Phi_{1\infty}(\nu,y)$ is denoted the asymptotic form of the function $\Phi_1(\nu,y)$, defined by Eq. (13), for $y>>1$.

In the particular case $d=\sigma$ the functions $\Phi_1(3/2,y)$ and $\Phi_2(3/2,y)$ in Eq. (12) are expressed in terms of the polylogarithmic function $Li_s(x)$,

$$\Phi_1\left(\frac{3}{2},y\right)=\sqrt{\pi}\left[3Li_3\left(e^{-2y}\right)+6yLi_2\left(e^{-2y}\right)\right.$$

$$\left.+6y^2Li_1\left(e^{-2y}\right)+4y^3Li_0\left(e^{-2y}\right)\right] \qquad (26)$$

and

$$\Phi_2\left(\frac{3}{2},y\right)=-2\sqrt{\pi}\left[y^2Li_1\left(e^{-2y}\right)+2y^3Li_0\left(e^{-2y}\right)\right]. \qquad (27)$$

By using Eqns. (20)-(22) we obtain the critical specific heat capacity in the tree regions: (i) for $\lambda=\lambda_c$ and $t\to0^+$, i.e. *in the quantum critical region*,

$$c(t)=\frac{k_\sigma}{\sigma}\left[3Li_3\left(e^{-\Theta}\right)+3\Theta Li_2\left(e^{-\Theta}\right)\right.$$

$$\left.+\Theta^2Li_1\left(e^{-\Theta}\right)\right]\left(\frac{2t}{\lambda_c}\right)^2; \qquad (28)$$

(ii) for $(1-\lambda/\lambda_c)/t>>1$, i.e. *in the renormalized classical region*,

$$c(t)\approx\frac{3\varsigma(3)k_\sigma}{\sigma}\left(\frac{2t}{\lambda}\right)^2; \qquad (29)$$

(iii) for $(\lambda/\lambda_c-1)/t>>1$ and $\lambda>\lambda_c$, i.e. *in the quantum disordered region*,

$$c(t)\approx\frac{2\sigma^2\lambda_c(\delta\lambda)^3}{k_\sigma^2t}\exp\left[-\frac{\sigma\lambda_c}{k_\sigma}\left(\frac{\delta\lambda}{t}\right)\right]. \qquad (30)$$

In obtaining Eq. (29) and Eq. (30) the asymptotic forms of the functions $\Phi_1(3/2,y)$ and $\Phi_2(3/2,y)$ for $y<<1$ and $y>>1$ are used, respectively. Let us note that Eq. (30) is a particular case of Eq. (25).

The results for $d=\sigma$ have been obtained in [25].

CONCLUSIONS

In the present study we are interested in the low-temperature behaviour of the critical specific heat capacity of a quantum model with long-range interaction, intended to describe structural phase transitions, Eq. (1).

A general expression for the specific heat capacity in the low-temperature region $(\lambda/t>>1)$ of the (t,λ)-phase diagram, Eq. (12), is derived. Taking into account the temperature dependence of the inverse susceptibility, Eqns. (16)-(18), it is easy to see that in the vicinity of the quantum critical point $(t=0,\lambda=\lambda_c)$ for $1/2<d/\sigma<3/2$ Eq. (12) has a scaling form.

The temperature dependence of the critical specific heat capacity $c(t)$ is obtained in the three regions of phase diagram (renormalized classical region, quantum critical region and quantum disordered region). From Eqns. (23), (24), (28)-(30) one can see that $c(t)$ is going to zero as t raised to the power $2d/\sigma$ in the renormalized classical region and in the quantum critical region, and $c(t)$ exponentially tends to zero as $t \to +0$ in the quantum disordered region.

Finally, the results obtained here for the low-temperature specific heat capacity are directly applicable to the more popular SQRM.

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