Mixed convection flow of a viscoelastic fluid through a vertical porous channel influenced by a moving magnetic field with Hall and ion-slip currents, rotation, heat radiation and chemical reaction

J. K. Singh[1*], G. S. Seth[2], Naveen Joshi[1] and C. T. Srinivasa[1]

[1]*Dept. of Mathematics, V. S. K. University, Ballari-583105, INDIA*
[2]*Dept. of Applied Mathematics, IIT (ISM), Dhanbad-826004, INDIA*

Received September 28, 2017; Accepted July 23, 2019

A mathematical analysis is presented for mixed convection hydromagnetic flow of an incompressible, electrically and thermally conducting and chemically reacting viscoelastic fluid through a vertical porous channel filled with porous medium. The effects of Hall and ion-slip currents, rotation, heat radiation and chemical reaction are also considered. The flow system is influenced by an applied magnetic field which is moving with same velocity as that of the moving right wall of the channel. The MHD flow in the rotating fluid system is developed due to periodic pressure gradient, movement of the right wall and buoyancy forces. The governing coupled partial differential equations solved analytically by separating the variables and the solutions for velocity, temperature and concentration are presented in closed form. In order to discuss the effects of flow governing parameters, the numerical results for velocity, fluid temperature and concentration are computed and demonstrated through graphs while the numerical results for skin friction, rate of heat and mass transfer in terms of Nusselt and Sherwood numbers are presented through tables. It is observed that moving magnetic field produces comparatively less drag force in comparison to stationary magnetic field. In case of moving magnetic field, the magnetic field raises the fluid velocity in the primary flow direction in the left half of the channel while it reduces in the right half of the channel because magnetic field is fixed relative to the moving right wall of the channel.

Keywords: Moving magnetic field, viscoelastic fluid, mixed convection, rotation, Hall and ion-slip currents.

INTRODUCTION

Investigation of combined heat and mass transfer characteristics of natural convection flows are significant due to their simultaneous occurrence in many natural systems, transport procedures and industrial applications. It is widely accepted that natural convection arises due to density variations in the field of gravity. For incompressible fluids density variation due to pressure are negligible. However, the density changes due to non-uniform heating and non-uniform species distribution cannot be neglected because they are responsible for natural convection in incompressible fluids. It is praiseworthy to note that, in a rotating fluid system natural convection can be also set up by the action of centrifugal force which is proportional to the density of the fluid. Flow and heat transfer in gas turbine is an example of such situation. Natural convection of heat and mass transfer may find some significant applications in chemical industries and in designing of control systems of heat exchangers. Many research scientists investigated heat and mass transfer natural convection flow of incompressible fluids under different considerations. Some relevant contributions on the topic are due to Angirasa *et al.* [1], Alagoa and Tay [2], Osalusi *et al.* [3], Jha and Ajibade [4], Singh *et al.* [5], Khan *et al.* [6], Narahari and Debnath [7], Kar *et al.* [8], Barik *et al.* [9], Manglesh and Gorla [10], Chamkha and Al-Rashidi [11], Ibrahim *et al.* [12], Seth and Sarkar [13], Yabo *et al.* [14], Butt and Ali [15], Singh *et al.* [16], Iqbal *et al.* [17], Misra and Adhikary [18] and Falade *et al.* [19]. Study of transport of heat and mass through a porous medium become important because similar to magnetic force, permeability of the porous medium also produces a fluid controlling force in the flow-field. A considerable attention has been given by the researchers to the study of hydromagnetic natural convection flow of conducting fluids through porous medium in the recent years owing to its numerous applications in petrochemical industries, geothermal energy extraction, heat exchangers etc. Stimulated from the industrial applications, researchers [2, 5, 6, 8-10, 12-13, 16-19] investigated hydromagntic natural convective flow of conducting fluids through a uniform porous medium under different situations using various computational and analytical methods. In an ionized fluid, the electrons and ions are drifting with different velocities and the diffusion velocity of electrons is much larger than that of ions (Cramer and Pai [20]). Therefore, the diffusion velocities of ions are usually neglected in Ohm's law. However, in industrial

* To whom all correspondence should be sent:
E-mail: s.jitendrak@yahoo.com

process where electromagnetic forces are large, we may not neglect the diffusion velocity of the ions. In such a case both the diffusion velocities of electrons and ions are included in Ohm's law and phenomenon of Hall and ion-slip effects comes into the picture. For such flows, the combined effects of Hall and ion-slip currents play a prominent role in the determination of flow behaviour. Combined effects of Hall and ion-slip currents on hydromagnetic flows are investigated by the researchers [21-26]. Ojjela and Kumar [27] studied the influence of Hall and ion-slip currents on unsteady MHD two-dimensional heat and mass transfer flow of a chemically reacting micropolar fluid through a porous channel. Subsequently, Hossain *et al.* [28] discussed the effects of Hall and ion-slip currents on MHD natural convection and mass transfer flow past an oscillating vertical porous plate with heat source and rotation. Moreover, Singh *et al.* [29] analysed the combined influences of Hall and ion-slip currents on heat and mass transfer natural convection flow past an oscillating vertical plate with ramped wall temperature and time dependent concentration. Combined effects of Hall and ion-slip currents on unsteady MHD natural convection flow of a rotating fluid over a vertical plate due to moving free stream is recently presented by Singh *et al.* [30]. Research studies made on non-Newtonian fluid dynamics have been drawn the attention of many researchers during last few decades due to its widespread applications in polymer sciences in manufacturing process of plastics, food processing, petrochemical engineering and biological fluids. Non-Newtonian fluids are subdivided into several categories. Viscoelastic fluids are one of the subcategories in which fluids have both the viscous and elastic attributes. Walters [31, 32] has presented a theoretical model in his classical research papers known as Walters'-B fluid model and this model opened the new scope for the engineers and scientists to study the dynamics of viscoelastic fluids. Stimulated from the widespread applications, researchers [33-41] studied the dynamics of viscoelastic fluid under different considerations. Garg *et al.* [42] analysed the heat transfer characteristic of viscoelastic MHD oscillatory natural convection flows through a vertical porous channel with Hall effects. Subsequently, Singh *et al.* [43] presented MHD oscillatory convective flow of viscoelastic fluid through a channel filled with porous medium with heat radiation and Hall current. Moreover, Ramesh and Devakar [44] discussed effects of external magnetic field and heat transfer on the peristaltic flow of Walters B fluid in a vertical channel. Combined influences of Hall and ion-slip currents on MHD free convection boundary layer flow of non-Newtonian tangent hyperbolic fluid past a vertical surface in a porous medium with Ohmic dissipation is discussed by Gaffar *et al.* [45]. Recently, Singh *et al.* [46] considered the effects of Hall and ion-slip currents on unsteady MHD boundary layer flow of a rotating Walters-B' fluid over an infinite vertical porous plate in a uniform porous medium with fluctuating wall temperature and concentration.

In most of the research investigations, applied magnetic field is considered to be fixed relative to the fluid. However, the moving magnetic field plays a prominent role in determination of flow characteristics of the hydromagnetic flows. In this intended research study we analysed unsteady mixed convection hydromagnetic flow of a viscoelastic fluid through a vertical porous channel filled with porous medium in the presence of a moving magnetic field with Hall and ion-slip currents, rotation, heat radiation and chemical reaction. The hydromagnetic flow in the rotating fluid system is developed due to periodic pressure gradient, movement of the right wall and buoyancy forces. The governing coupled partial differential equations solved analytically by separating the variables. It is observed that suction reduces concentration in the left half of the channel while it enhances concentration in the right half of the channel because suction and injection are simultaneously taking place through left and right walls of the channel.

MATHEMATICAL MODEL OF THE PROBLEM

In the present problem we considered fully developed MHD laminar flow of an electrically and thermally conducting and chemically reacting viscoelastic fluid through a porous medium bounded by two infinite vertical porous walls $(-\infty \le x' \le \infty, -\infty \le y' \le \infty)$ placed at $z = -z_0/2$ and $z = z_0/2$. The flow system is influenced by a uniform magnetic field $\vec{B}\ (0, 0, B_0)$ applied along a direction normal to the plane of the walls. Initially, the walls of the channel and fluid are consider being at rest and the whole flow system is rigidly rotating with a uniform angular velocity $\vec{\Omega}\ (0, 0, \Omega)$ about a direction normal to the plane of the walls. Since the walls of the channel are assumed to be porous, consider simultaneous suction and injection take place through the opposite walls of the channel respectively with a uniform transpiration velocity w_0. The left wall of the channel

$z = -z_0 / 2$ is kept fixed while the right wall of the channel $z = z_0 / 2$ is moving with velocity $w_0 f'(t')$. The applied magnetic field is also considered to be moving with same velocity as that of right wall of the channel in x'-direction i.e. the magnetic field is fixed relative to the right wall of the channel. The temperature and concentration of the left wall is kept fixed while the temperature and concentration of the right wall varying periodically with the time. The MHD flow through the vertical channel is developed due to a periodic pressure gradient applied along x' -direction, oscillatory movement of the right wall of the channel and the buoyancy forces arise from temperature and concentration differences in the field of gravity. All the physical variables in this problem will be function of z' and t' because flow is fully developed and laminar. Geometry of the physical problem is presented in Fig. 1.

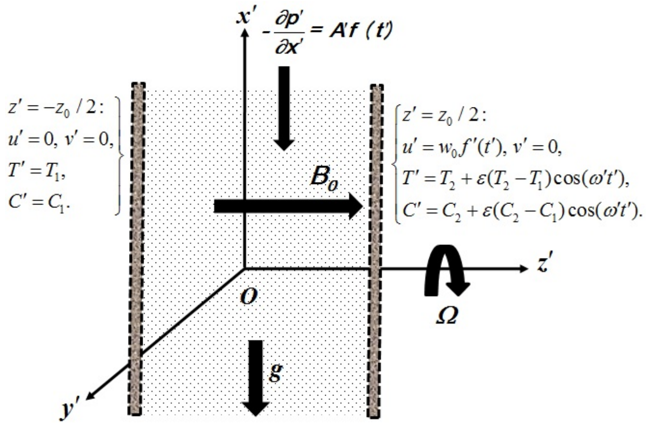


Fig. 1 Geometry of the physical problem.

Fluid is considered to be such that magnetic diffusivity of the fluid is very large, so the induced magnetic field produced by fluid motion is assumed to be negligible in comparison to applied one (Sutton and Sherman [47]). It is also assumed that there does not exist any applied and polarization voltages, so no energy is being added or extracted from the fluid by electrical means (Meyer [48]) i.e. electric field $\vec{E} = (0,0,0)$. In view of the above made assumptions and compatibility with the continuity equation for velocity field $(\nabla.\vec{q} = 0)$, solenoidal relation $(\nabla.\vec{B} = 0)$ and continuity equation for electric field $(\nabla.\vec{J} = 0)$; the fluid velocity $\vec{q}$, magnetic field $\vec{B}$ and current density $\vec{J}$ may assume as $\vec{q} = (u', v', w_0)$, $\vec{B} = (0,0,B_0)$ and $\vec{J} = (J'_x, J'_y, 0)$ respectively.

The equations governing the motion of mixed convection flow of a viscoelastic fluid through a vertical porous channel filled with porous medium in the presence of a moving magnetic field with Hall and ion-slip currents, rotation and thermal and concentration buoyancy forces under Boussinesq approximation are described by

$$\begin{aligned}&\frac{\partial u'}{\partial t'} + w_0 \frac{\partial u'}{\partial z'} - 2\Omega v' \\ &= -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \upsilon \frac{\partial^2 u'}{\partial z'^2} + \frac{\beta'}{\rho}\frac{\partial^3 u'}{\partial z'^2 \partial t'} \\ &-\frac{\sigma B_0^2}{\rho(\alpha_e^2 + \beta_e^2)}\left[\alpha_e u' - \beta_e v' - K_1 w_0 f'(t')\right] \\ &-\frac{\upsilon u'}{k'} + g\beta_T (T' - T'_\infty) + g\beta_C (C' - C'_\infty),\end{aligned} \tag{1}$$

$$\begin{aligned}&\frac{\partial v'}{\partial t'} + w_0 \frac{\partial v'}{\partial z'} + 2\Omega u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} + \frac{\beta'}{\rho}\frac{\partial^3 v'}{\partial z'^2 \partial t'} \\ &-\frac{\sigma B_0^2}{\rho(\alpha_e^2 + \beta_e^2)}\left[\beta_e u' - \alpha_e v'\right] - \frac{\upsilon v'}{k'},\end{aligned} \tag{2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p'}{\partial z'}, \tag{3}$$

where $\alpha_e = 1 + \beta_e \beta_i$ and

$$K_1 = \begin{cases} 1 & \text{when } \mathrm{B}_0 \text{ is moving and} \\ & \text{fixed relative to the moving} \\ & \text{porous wall,} \\ 0 & \text{when } \mathrm{B}_0 \text{ is stationary} \\ & \text{and fixed relative to the fluid.} \end{cases}$$

Equation (3) shows the constancy of pressure in a direction normal to the plane of the channel walls. The pressure gradient term $-(1/\rho)(\partial p' / \partial y')$ is not present in equation (2) because there is a net cross flow in y'-direction.

The fluid considered being optically thin with a relatively low density, then the energy equation with heat radiation is

$$\frac{\partial T'}{\partial t'} + w_0 \frac{\partial T'}{\partial z'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial z'^2} - \frac{4\alpha^2 (T' - T_1)}{\rho C_p}. \tag{4}$$

The concentration equation with first order chemical reaction is given by

$$\frac{\partial C'}{\partial t'} + w_0 \frac{\partial C'}{\partial z'} = D\frac{\partial^2 C'}{\partial z'^2} - K'(C' - C_1). \tag{5}$$

The conditions to be satisfied at the boundary walls of the channel are

$$\left.\begin{array}{l}\text{At } z'=\dfrac{z_0}{2}:\begin{cases}u'=w_0 f'(t'),\ v'=0,\\ T'=T_2+\varepsilon(T_2-T_1)\cos(\omega' t'),\\ C'=C_2+\varepsilon(C_2-C_1)\cos(\omega' t').\end{cases}\\ \text{At } z'=-\dfrac{z_0}{2}:\{u'=v'=0,\ T'=T_1,\ C'=C_1.\end{array}\right\} \quad (6)$$

To non-dimensionalize the equations governing the fluid motion i.e. equations (1)-(5) and boundary conditions (6), we define the following non-dimensional quantities:

$$\left.\begin{array}{l} z=z'/z_0,\ x=x'/z_0,\ u=u'/U_0,\\ v=v'/U_0,\ t=t'w_0/z_0,\ \omega=\omega' z_0/w_0,\\ f=f'/w_0,\ T=(T'-T_1)/(T_2-T_1),\\ C=(C'-C_1)/(C_2-C_1).\end{array}\right\} \quad (7)$$

Using the non-dimensional quantities defined in equation (7), to the equations (1) and (2) and then combining both, we get the following non-dimensional equation

$$S\left(\frac{\partial q}{\partial t}+\frac{\partial q}{\partial z}+\frac{\partial p}{\partial x}\right)=\frac{\partial^2 q}{\partial z^2}+\beta\frac{\partial^3 q}{\partial z^2\partial t}$$
$$-X_3 q+G_T T+G_C C-K_1^* f(t), \quad (8)$$

where

$$\left.\begin{array}{l} q=u+iv,\ S=z_0 w_0/\upsilon,\ K^2=\Omega z_0^2/\upsilon,\\ M^2=\sigma B_0^2 z_0^2/\rho\upsilon,\ k_1=k'/z_0^2,\\ \beta=\beta' w_0/z_0\upsilon,\ G_T=g\beta_T z_0^2(T_2-T_1)/\upsilon w_0,\\ G_C=g\beta_C z_0^2(C_2-C_1)/\upsilon w_0.\end{array}\right\}$$

The energy equation (4) and concentration equation (5) in non-dimensional form become

$$S\,\mathrm{Pr}\left(\frac{\partial T}{\partial t}+\frac{\partial T}{\partial z}\right)=\frac{\partial^2 T}{\partial z^2}-N^2 T, \quad (9)$$

$$SSc\left(\frac{\partial C}{\partial t}+\frac{\partial C}{\partial z}\right)=\frac{\partial^2 C}{\partial z^2}-K_2 C, \quad (10)$$

where $\mathrm{Pr}=\upsilon\rho C_p/k$, $N^2=Q_0 z_0^2/k$, $Sc=\upsilon/D$, $K_2=K' z_0^2/D$.

The conditions at the boundary walls of the channel in non-dimensional form become

$$\left.\begin{array}{l}\text{At } z=1/2: q=T=C=f(t)=1+\varepsilon\cos(\omega t).\\ \text{At } z=-1/2: q=T=C=0.\end{array}\right\} \quad (11)$$

The coupled partial differential equations (8)-(10) together with the boundary conditions (11) describe the mathematical model of the present physical problem which is required to be solved.

SOLUTION OF THE PROBLEM

Since the MHD flow through the vertical channel is developed due to a periodic pressure gradient applied along x'-direction, oscillatory movement of the right wall of the channel and the buoyancy forces arise from temperature and concentration differences. Thus, the temperature, concentration, fluid velocity and pressure gradient along x'-direction are assumed as

$$T(z,t)=T_0(z)+\frac{\varepsilon}{2}\left(T_1(z)e^{i\omega t}+T_2(z)e^{-i\omega t}\right), \quad (12)$$

$$C(z,t)=C_0(z)+\frac{\varepsilon}{2}\left(C_1(z)e^{i\omega t}+C_2(z)e^{-i\omega t}\right), \quad (13)$$

$$q(z,t)=q_0(z)+\frac{\varepsilon}{2}\left(q_1(z)e^{i\omega t}+q_2(z)e^{-i\omega t}\right), \quad (14)$$

$$-\frac{\partial p}{\partial x}=A\left(1+\frac{\varepsilon}{2}\left(e^{i\omega t}+e^{-i\omega t}\right)\right). \quad (15)$$

Using equations (12)-(15) to the PDE's system given by equations (8)-(10), these reduces to the following system of ODE's

$$T_0''-S\,\mathrm{Pr}\,T_0'-N^2 T_0=0, \quad (16)$$
$$C_0''-SScC_0'-K_2 C_0=0, \quad (17)$$
$$q_0''-Sq_0'-X_3 q_0=-K_1^*-G_T T_0-G_C C_0, \quad (18)$$
$$T_1''-S\,\mathrm{Pr}\,T_1'-(N^2+i\omega S\,\mathrm{Pr})T_1=0, \quad (19)$$
$$C_1''-SScC_1'-(K_2+i\omega Sc)C_1=0, \quad (20)$$
$$(1+i\omega\beta)q_1''-Sq_1'-(X_3+i\omega S)q_1$$
$$=-K_1^*-G_T T_1-G_C C_1, \quad (21)$$
$$T_2''-S\,\mathrm{Pr}\,T_2'-(N^2-i\omega S\,\mathrm{Pr})T_2=0, \quad (22)$$
$$C_2''-SScC_2'-(K_2-i\omega Sc)C_2=0, \quad (23)$$
$$(1-i\omega\beta)q_2''-Sq_2'-(X_3-i\omega S)q_2$$
$$=-K_1^*-G_T T_2-G_C C_2. \quad (24)$$

The conditions at the boundary walls of the channel after use of equations (12) to (14) yield

$$\left.\begin{array}{l}\text{At } z=1/2: T_0=T_1=T_2=1,\\ C_0=C_1=C_2=1,\ q_0=q_1=q_2=1.\\ \text{At } z=-1/2: T_0=T_1=T_2=0,\\ C_0=C_1=C_2=0,\ q_0=q_1=q_2=0.\end{array}\right\} \quad (25)$$

Solving equations (16)-(24) subject to the boundary conditions (25), the solution for temperature, concentration and fluid velocity are given by

$$T_i=\frac{e^{r_i z-\frac{s_i}{2}}-e^{s_i z-\frac{r_i}{2}}}{2\sinh\left(\dfrac{r_i-s_i}{2}\right)}, \qquad i=0,1,2, \quad (26)$$

$$C_i = \frac{e^{u_i z - \frac{v_i}{2}} - e^{v_i z - \frac{u_i}{2}}}{2\sinh\left(\frac{u_i - v_i}{2}\right)}, \qquad i = 0, 1, 2, \tag{27}$$

$$\begin{aligned}
q_i &= \frac{e^{x_i z - \frac{y_i}{2}} - e^{y_i z - \frac{x_i}{2}}}{2\sinh\left((x_i - y_i)/2\right)} \\
&+ \frac{K_1^{**}}{X_3}\left[1 + \frac{e^{x_i z}\sinh(y_i/2) - e^{y_i z}\sinh(x_i/2)}{2\sinh\left((x_i - y_i)/2\right)}\right] \\
&+ \frac{G_T}{2\sinh\left((r_i - s_i)/2\right)\sinh\left((x_i - y_i)/2\right)} \\
&\times \left[\begin{array}{l}
E_{r_i} e^{-\frac{s_i}{2}}\left\{e^{x_i z}\sinh\left(\frac{r_i - y_i}{2}\right) - e^{y_i z}\sinh\left(\frac{r_i - x_i}{2}\right)\right. \\
\left. - e^{r_i z}\sinh\left(\frac{x_i - y_i}{2}\right)\right\} - E_{s_i} e^{-\frac{r_i}{2}}\left\{e^{x_i z}\sinh\left(\frac{s_i - y_i}{2}\right)\right. \\
\left. - e^{y_i z}\sinh\left(\frac{s_i - x_i}{2}\right) - e^{s_i z}\sinh\left(\frac{x_i - y_i}{2}\right)\right\}
\end{array}\right] \\
&+ \frac{G_C}{2\sinh\left((u_i - v_i)/2\right)\sinh\left((x_i - y_i)/2\right)} \\
&\times \left[\begin{array}{l}
E_{u_i} e^{-\frac{v_i}{2}}\left\{e^{x_i z}\sinh\left(\frac{u_i - y_i}{2}\right) - e^{y_i z}\sinh\left(\frac{u_i - x_i}{2}\right)\right. \\
\left. - e^{u_i z}\sinh\left(\frac{x_i - y_i}{2}\right)\right\} - E_{v_i} e^{-\frac{u_i}{2}}\left\{e^{x_i z}\sinh\left(\frac{v_i - y_i}{2}\right)\right. \\
\left. - e^{y_i z}\sinh\left(\frac{v_i - x_i}{2}\right) - e^{v_i z}\sinh\left(\frac{x_i - y_i}{2}\right)\right\}
\end{array}\right],
\end{aligned} \tag{28}$$

where $i = 0, 1, 2$.

Substituting the expressions for T_i, C_i and $q_i, i = 0, 1, 2$ from equations (26), (27) and (28) to the equations (12), (13) and (14) respectively, the solutions for temperature, concentration and fluid velocity are, respectively, obtained.

Skin friction at the left and right walls of the channel in the primary and secondary flow directions can be obtained by using equations (28) in the following expressions

$$\left.\begin{aligned}
\tau_L &= \tau_{xL} + i\tau_{yL} \\
&= \left[\frac{\partial q_0}{\partial z} + \frac{\varepsilon}{2}\left(\frac{\partial q_1}{\partial z}e^{i\omega t} + \frac{\partial q_2}{\partial z}e^{-i\omega t}\right)\right]_{z=-1/2}, \\
\tau_R &= \tau_{xR} + i\tau_{yR} \\
&= \left[\frac{\partial q_0}{\partial z} + \frac{\varepsilon}{2}\left(\frac{\partial q_1}{\partial z}e^{i\omega t} + \frac{\partial q_2}{\partial z}e^{-i\omega t}\right)\right]_{z=1/2}.
\end{aligned}\right\} \tag{29}$$

The rate of heat transfer at the left and right walls of the channel in terms of Nusselt number are presented in the following form

$$\left.\begin{aligned}
Nu_L &= Nu_{L0} + \frac{\varepsilon}{2}\left(Nu_{L1}e^{i\omega t} + Nu_{L2}e^{-i\omega t}\right), \\
Nu_R &= Nu_{R0} + \frac{\varepsilon}{2}\left(Nu_{R1}e^{i\omega t} + Nu_{R2}e^{-i\omega t}\right),
\end{aligned}\right\} \tag{30}$$

where

$$\left.\begin{aligned}
Nu_{Li} &= \frac{(r_i - s_i)e^{-(r_i + s_i)/2}}{2\sinh((r_i - s_i)/2)},\ i = 0, 1, 2, \\
Nu_{Ri} &= \frac{r_i e^{(r_i - s_i)/2} - s_i e^{-(r_i - s_i)/2}}{2\sinh((r_i - s_i)/2)},\ i = 0, 1, 2.
\end{aligned}\right\} \tag{31}$$

The rate of mass transfer at the left and right walls of the channel in terms of Sherwood number are presented in the following form

$$\left.\begin{aligned}
Sh_L &= Sh_{L0} + \frac{\varepsilon}{2}\left(Sh_{L1}e^{i\omega t} + Sh_{L2}e^{-i\omega t}\right), \\
Sh_R &= Sh_{R0} + \frac{\varepsilon}{2}\left(Sh_{R1}e^{i\omega t} + Sh_{R2}e^{-i\omega t}\right),
\end{aligned}\right\} \tag{32}$$

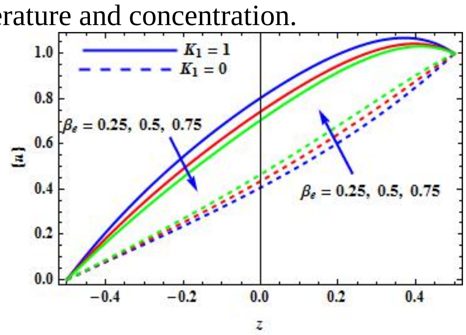
where

$$\left.\begin{aligned}
Sh_{Li} &= \frac{(u_i - v_i)e^{-(u_i + v_i)/2}}{2\sinh((u_i - v_i)/2)},\ i = 0, 1, 2, \\
Sh_{Ri} &= \frac{u_i e^{(u_i - v_i)/2} - v_i e^{-(u_i - v_i)/2}}{2\sinh((u_i - v_i)/2)},\ i = 0, 1, 2.
\end{aligned}\right\} \tag{33}$$

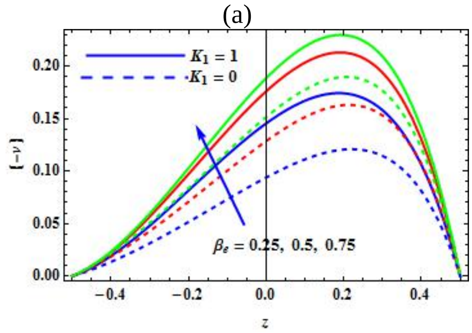
NUMERICAL RESULTS AND DISCUSSION

The analytical solutions for fluid velocity, temperature, species concentration, skin friction, Nusselt number and Sherwood number have been derived in the previous section in terms of various flow governing parameters. The influences of these flow governing parameters on fluid velocity, temperature, species concentration, skin friction, heat and mass transfer at the walls of the channel are computed numerically and presented in graphical and tabular forms by taking $A = 1,\ G_T = 4,\ G_C = 5,\ \Pr = 0.71,\ N = 2,\ Sc = 0.22,\ K_2 = 0.2$ and $\omega t = \pi/2$. The velocity profiles are displayed in figures 2-9 and it can be easily observed from figures 2-9 that moving magnetic field $(K_1 = 1)$ produces comparatively less drag force in comparison to stationary magnetic field $(K_1 = 0)$. Figures 10-11 and 12-13 depict the temperature and concentration profiles respectively.

Figures 2 and 3, respectively, demonstrate the variations in the velocity profiles for various values of Hall current parameter β_e and ion-slip current parameter β_i. It is observed that both the Hall and ion-slip currents raise the fluid velocity in the primary flow direction when magnetic field is fixed relative to the fluid (magnetic field is stationary) i.e. when $K_1 = 0$ while this effect of Hall and ion-slip currents is reversed when magnetic field is fixed relative to the moving wall of the channel (magnetic field is moving) i.e. when $K_1 = 1$. In both the cases when $K_1 = 1$ and $K_1 = 0$, Hall current accelerates the fluid flow in the secondary flow direction while ion-slip current decelerates it in the secondary flow direction. Our result upholds the well known result that Hall current develops the secondary motion in the flow-field. Influence of rotation parameter K^2 on the velocity profiles are depicted in figures 4. In both the cases when $K_1 = 1$ and $K_1 = 0$, rotation (Coriolis force) has tendency to reduce the fluid flow in the primary flow direction while this tendency of rotation is reversed on the fluid flow in the secondary flow direction. It is well accepted that rotation induces secondary motion in the flow-field, our result also agrees with it. Velocity profiles for distinct values of magnetic parameter M^2 are displayed in figures 5. It is noticed that magnetic field suppress the fluid velocity in both the primary and secondary flow directions when $K_1 = 0$, which is due to the fact that magnetic field produces a drag force in the flow-field whose nature is to resist the fluid motion. But when $K_1 = 1$, the magnetic field enhances the fluid velocity in the primary flow direction in the left half of the channel and the fluid velocity in the secondary flow direction throughout the channel while this effect of magnetic field on the fluid velocity in the primary flow direction in the right half of the channel is upturned because magnetic field is fixed relative to the moving right wall of the channel. Deviations in the velocity profiles due to variations in permeability of the porous medium k_1 are presented in figures 6. Figures 6 illustrate that velocity profiles in both the primary and secondary flow directions rise on raising the permeability of the porous medium which implies that the Darcian drag force tends to decelerate the fluid velocity in both the primary and secondary flow directions. Figures 7 demonstrate the effects of the porosity of the walls i.e. suction/injection S on the fluid velocity. When $K_1 = 0$, suction tends to increase the fluid velocity in the primary flow direction and secondary flow direction in the right half of the channel while this trend of velocity in the secondary flow direction is upturned in the left half of the channel. When $K_1 = 1$, suction tends to decrease the fluid velocity in the secondary flow direction and primary flow direction in the left half of the channel while this trend of velocity in the primary flow direction is upturned in the right half of the channel. This may be due to the fact that suction is taking place through the left wall of the channel while the injection is taking place through the right wall of the channel. Influences of oscillations ω on the velocity are shown in the figures 8. When $K_1 = 0$, rise in frequency of oscillations gives rise in fluid velocity in both the primary and secondary flow directions. When $K_1 = 1$, increase in frequency of oscillations gives rise in fluid velocity in the primary flow direction and secondary flow direction in the right half of the channel while this effect is reversed on the fluid flow in the secondary flow direction in left half of the channel. Figures 9 exhibit the variations in the velocity profiles due to change in viscoelastic nature of the fluid i.e. β. It can be seen from figures 9 that, when $K_1 = 0$, the fluid velocity in the primary flow direction falls whereas the fluid velocity in the secondary flow direction rise on raising the viscoelastic parameter. On raising the viscoelastic parameter fluid velocity is rising in both the primary and secondary flow directions when $K_1 = 1$. Figures 10 and 12 depict the influence of suction on fluid temperature and concentration respectively. Suction has tendency to reduce fluid temperature. Suction reduces concentration in the left half of the channel while it enhances concentration in the right half of the channel because suction and injection are simultaneously occuring through left and right walls of the channel. Figures 11 and 13 demonstrate that oscillations tend to rise the fluid temperature and concentration.

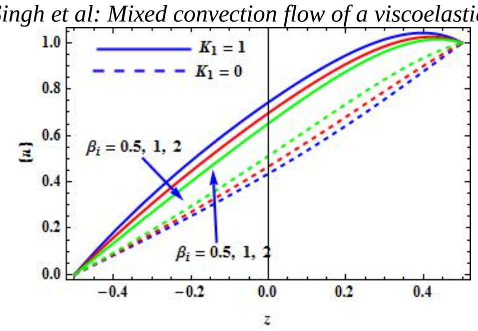


(a)

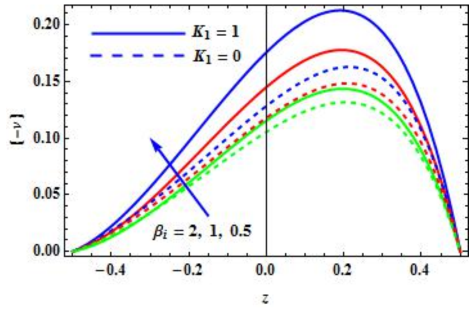


(b)

Fig. 2 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_i = 0.5$, $K^2 = 1$, $M^2 = 9$, $k_1 = 0.3$, $S = 2$, $\omega = 3$ and $\beta = 0.25$.

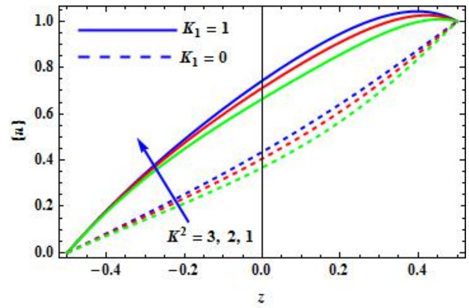


(a)

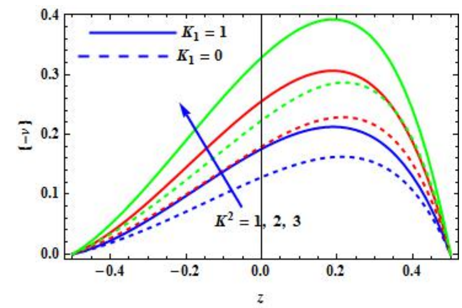


(b)

Fig. 3 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ K^2 = 1,\ M^2 = 9,\ k_1 = 0.3,\ S = 2,\ \omega = 3$ and $\beta = 0.25$.

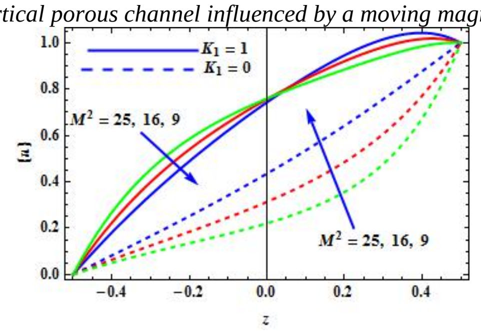


(a)

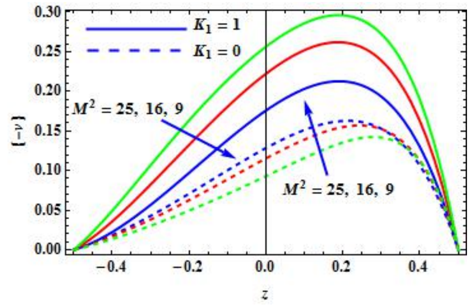


(b)

Fig. 4 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ \beta_i = 0.5,\ M^2 = 9,\ k_1 = 0.3,\ S = 2,\ \omega = 3$ and $\beta = 0.25$.

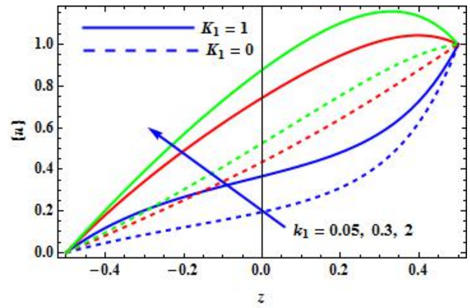


(a)

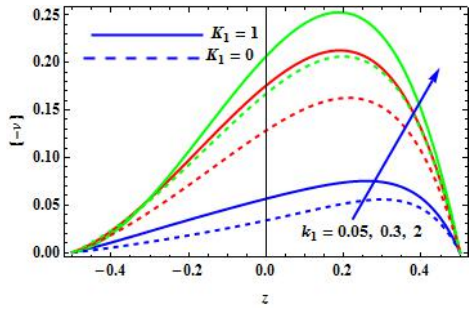


(b)

Fig. 5 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ \beta_i = 0.5,\ K^2 = 1,\ k_1 = 0.3,\ S = 2,\ \omega = 3$ and $\beta = 0.25$.

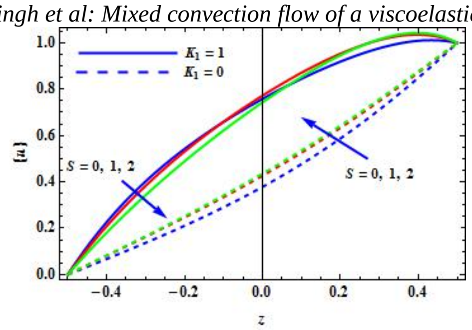


(a)

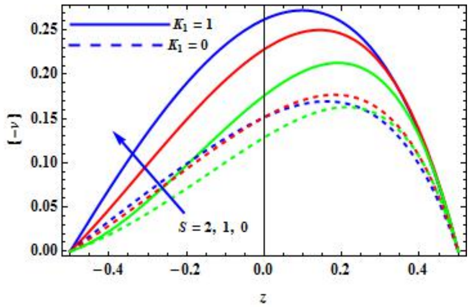


(b)

Fig. 6 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ \beta_i = 0.5,\ K^2 = 1,\ M^2 = 9,\ S = 2,\ \omega = 3$ and $\beta = 0.25$.

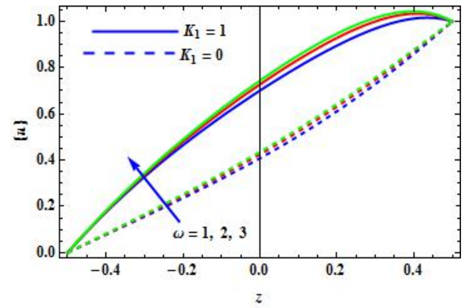


(a)

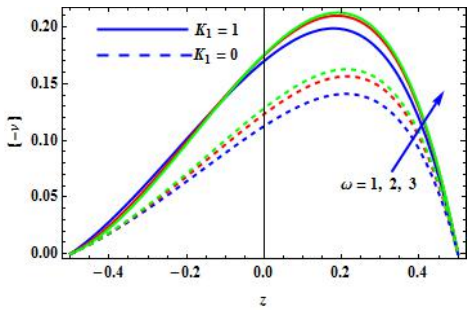


(b)

Fig. 7 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ \ \beta_i = 0.5,\ \ K^2 = 1,\ \ M^2 = 9,\ \ k_1 = 0.3,\ \ \omega = 3$ and $\beta = 0.25$.

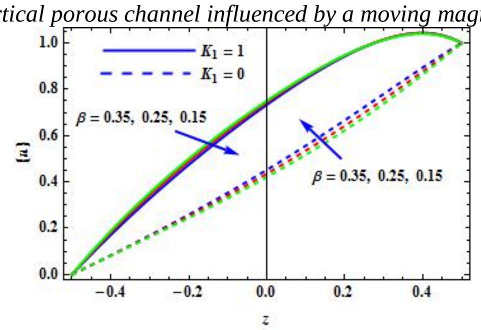


(a)

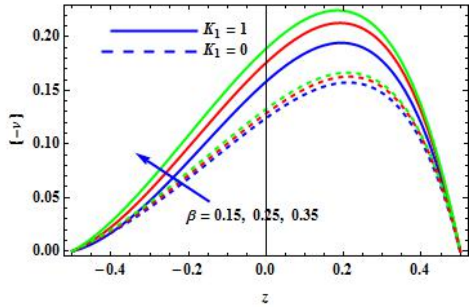


(b)

Fig. 8 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ \ \beta_i = 0.5,\ \ K^2 = 1,\ \ M^2 = 9,\ \ k_1 = 0.3,\ \ S = 2$ and $\beta = 0.25$.



(a)



(b)

Fig. 9 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ \ \beta_i = 0.5,\ \ K^2 = 1,\ \ M^2 = 9,\ \ k_1 = 0.3,\ \ S = 2$ and $\omega = 3$.

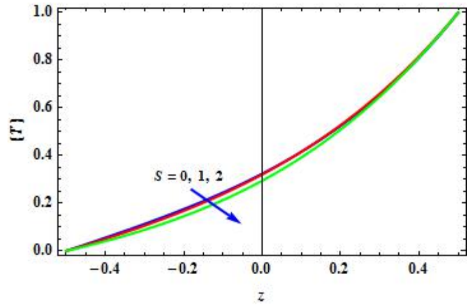


Fig. 10 Temperature profiles when $\omega = 3$.

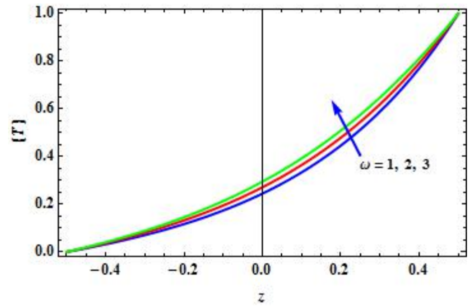


Fig. 11 Temperature profiles when $S = 2$.

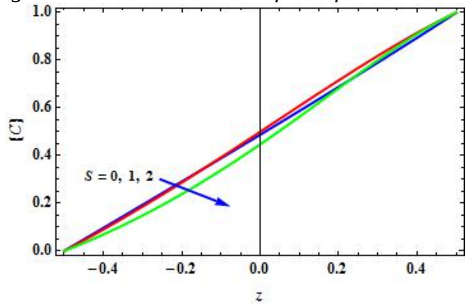


Fig. 12 Concentration profiles when $\omega = 3$.

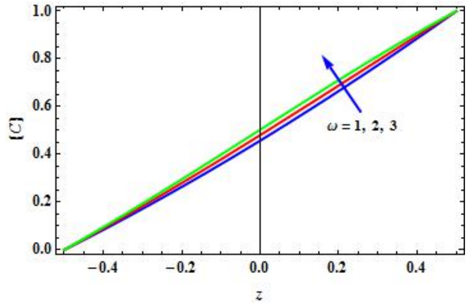


Fig. 13 Concentration profiles when $S = 2$.

Table 1 depicts the influences of flow governing parameters on the skin friction at the moving right wall of the channel in case of both the moving and stationary magnetic field. In case of both the moving and stationary magnetic field, skin friction at the moving wall in the primary flow direction fall down on raising Hall and ion-slip currents while the skin friction at the moving wall in the secondary flow direction raise on rising Hall current, rotation, strength of applied magnetic field, permeability of the porous medium, suction and frequency of oscillations. Ion-slip current has tendency to reduce skin friction at the moving wall in the secondary flow direction. When magnetic field is moving, skin friction at the moving wall in the primary flow direction is reduced by magnetic field and viscoelastic property of the fluid while it is enhanced by suction and frequency of oscillations. When magnetic field is stationary, skin friction at the moving wall in the primary flow direction is raised by rotation, magnetic field and viscoelastic property of the fluid while it is reduced by permeability of the porous medium, suction and frequency of oscillations. Table 2 demonstrates the influences of flow governing parameters on heat and mass transfer at both the walls of the channel. Suction tends to decrease both the heat and mass transfer at the left wall and mass transfer at the right wall of the channel. Oscillations raise both the heat and mass transfer at the left wall while it reduces these at the right wall of the channel.

Table 1 Skin friction at the moving right wall of the channel.

β_e	β_i	K^2	M^2	k_1	S	ω	β	$K_1 = 1$		$K_1 = 0$	
								$-\tau_{xR}$	τ_{yR}	τ_{xR}	τ_{yR}
0.5	**0.5**	**1**	**9**	**0.3**	**2**	**3**	**0.25**	**0.9293**	**1.7302**	**1.2124**	**1.4781**
0.25	0.5	1	9	0.3	2	3	0.25	1.2022	1.4109	1.6016	1.1385
0.75	0.5	1	9	0.3	2	3	0.25	0.7939	1.8636	0.8507	1.6641
0.5	**1**	1	9	0.3	2	3	0.25	0.6951	1.4592	0.8958	1.3118
0.5	**2**	1	9	0.3	2	3	0.25	0.5277	1.1833	0.4365	1.1251
0.5	0.5	**2**	9	0.3	2	3	0.25	0.7261	2.4984	1.4001	2.1039
0.5	0.5	**3**	9	0.3	2	3	0.25	0.4304	3.2281	1.6489	2.6869
0.5	0.5	1	**16**	0.3	2	3	0.25	0.5632	2.2158	2.7083	1.6595
0.5	0.5	1	**25**	0.3	2	3	0.25	0.1130	2.6637	4.1999	1.8005
0.5	0.5	1	9	**0.05**	2	3	0.25	-3.629	0.9086	5.0610	0.7859
0.5	0.5	1	9	**2**	2	3	0.25	2.1401	1.9506	0.2239	1.7246
0.5	0.5	1	9	0.3	**1**	3	0.25	0.7556	1.7313	1.2761	1.4282
0.5	0.5	1	9	0.3	**0**	3	0.25	0.3752	1.6410	1.5523	1.2701
0.5	0.5	1	9	0.3	2	**2**	0.25	0.8140	1.7024	1.2788	1.4312
0.5	0.5	1	9	0.3	2	**1**	0.25	0.5241	1.5749	1.4369	1.2782
0.5	0.5	1	9	0.3	2	3	**0.35**	0.8706	1.7912	1.3868	1.4920
0.5	0.5	1	9	0.3	2	3	**0.15**	0.9537	1.5787	0.9644	1.4144

Table 2 Heat and mass transfers at the channel walls.

S	ω	Nu_L	Nu_R	Sh_L	Sh_R
2	**3**	**0.3801**	**2.0217**	**0.9602**	**0.8190**
1	3	0.4851	1.9957	0.9849	0.9094
0	3	0.5514	2.0746	0.9934	1.0133
2	**2**	0.3434	2.3001	0.9051	0.9608
2	**1**	0.3003	2.5931	0.8483	1.1045

CONCLUSIONS

A mathematical analysis has been presented for mixed convection MHD flow of a rotating viscoelastic fluid through a vertical porous channel filled with porous medium in the presence of a moving magnetic field. The influences of some significant flow governing parameters on these flow variables have been thoroughly discussed in the previous section. Some significant observations are précised below:

- Hall and ion-slip currents raise the fluid velocity in the primary flow direction when magnetic field is stationary. This effect is upturned when magnetic field is moving.
- In case of moving magnetic field, the magnetic field raises the fluid velocity in the primary flow direction in the left half of the channel while this effect is upturned in the right half of the channel because magnetic field is fixed relative to the moving right wall of the channel. Suction shows reverse behaviour as that of magnetic field on the fluid velocity in the primary flow direction.
- Suction reduces concentration in the left half of the channel while it enhances concentration in the right half of the channel because suction and injection are simultaneously taking place through left and right walls of the channel.
- When magnetic field is stationary, magnetic field has tendency to enhance skin friction at the moving wall in the primary flow direction. This tendency is upturned when magnetic field is moving because magnetic field is fixed relative to the moving wall.

NOMENCLATURE

B_0	applied magnetic field (T)
C	non-dimensional species concentration
C'	species concentration (mol/m^3)
C_1	concentration at left wall (mol/m^3)
C_2	a constant concentration (mol/m^3)
C_p	specific heat at constant pressure $(J/kg.K)$
D	chemical molecular diffusivity (m^2/s)
K	rotation parameter
g	acceleration due to gravity $\left(m/s^2\right)$
G_C	solutal Grashof number
G_T	thermal Grashof number
k	thermal conductivity of the fluid $(W/m.K)$
k'	permeability (m^2)
k_1	permeability parameter
K'	chemical reaction constant
K_2	chemical reaction parameter
M	magnetic parameter
N	heat radiation parameter
$\Pr$	Prandtl number
τ_L	non-dimensionless skin friction at the left wall
τ_R	non-dimensionless skin friction at the right wall
S	suction/injection parameter
Sc	Schmdit number
t	non-dimensional time
t'	time (s)
T	non-dimensional fluid temperature
T'	fluid temperature (K)
T_1	Temperature at left wall (K)
T_2	a constant temperature (K)
u	non-dimensional velocity in x'-direction
v	non-dimensional velocity in y'-direction
x	non-dimensional coordinate along the channel wall
z	non-dimensional coordinate normal to channel wall

Greek symbols

α	mean radiation absorption coefficient
β	viscoelastic parameter
β_C	volumetric coefficient of concentration expansion
β_e	Hall current parameter
β_i	ion-slip current parameter
β_T	volumetric coefficient of thermal expansion (K^{-1})
υ	kinematic viscosity (m^2/s)
ρ	fluid density (kg/m^3)
σ	electrical conductivity (S/m)
ω	frequency parameter

APPENDIX

$$X_1 = (1/k_1) + (M^2\alpha_e/(\alpha_e^2+\beta_e^2)),$$
$$X_2 = (2K^2) + (M^2\beta_e/(\alpha_e^2+\beta_e^2)),$$
$$X_3 = X_1 + iX_2,\quad K_1^* = K_1M^2/(\alpha_e^2+\beta_e^2),$$
$$r_0 = \left(S\,\mathrm{Pr} + \sqrt{(S\,\mathrm{Pr})^2+4N^2}\right)/2,$$
$$r_{1,2} = \left(S\,\mathrm{Pr} + \sqrt{(S\,\mathrm{Pr})^2+4(N^2\pm i\omega S\,\mathrm{Pr})}\right)/2,$$
$$s_0 = \left(S\,\mathrm{Pr} - \sqrt{(S\,\mathrm{Pr})^2+4N^2}\right)/2,$$
$$s_{1,2} = \left(S\,\mathrm{Pr} - \sqrt{(S\,\mathrm{Pr})^2+4(N^2\pm i\omega S\,\mathrm{Pr})}\right),$$
$$u_0 = \left(SSc + \sqrt{(SSc)^2+4K_2}\right)/2,$$
$$u_{1,2} = \left(SSc + \sqrt{(SSc)^2+4(K_2\pm i\omega SSc)}\right)/2,$$
$$v_0 = \left(SSc - \sqrt{(SSc)^2+4K_2}\right)/2,$$
$$v_{1,2} = \left(SSc - \sqrt{(SSc)^2+4(K_2\pm i\omega SSc)}\right)/2,$$
$$E_{r_0} = 1/(r_0^2 - Sr_0 - X_3),\ K_1^{**} = K_1^* + SA,$$
$$E_{r_{1,2}} = 1/((1\pm i\omega\beta)r_{1,2}^2 - Sr_{1,2} - (X_3\pm i\omega S)),$$
$$E_{s_0} = 1/(s_0^2 - Ss_0 - X_3),$$
$$E_{s_{1,2}} = 1/((1\pm i\omega\beta)s_{1,2}^2 - Ss_{1,2} - (X_3\pm i\omega S)),$$
$$E_{u_0} = 1/(u_0^2 - Su_0 - X_3),$$
$$E_{u_{1,2}} = 1/((1\pm i\omega\beta)u_{1,2}^2 - Su_{1,2} - (X_3\pm i\omega S)),$$
$$E_{v_0} = 1/(v_0^2 - Sv_0 - X_3),$$
$$E_{v_{1,2}} = 1/((1\pm i\omega\beta)v_{1,2}^2 - Sv_{1,2} - (X_3\pm i\omega S)).$$

REFERENCES

1. D. Angirasa, G.P. Peterson, I. Pop, *Numerical Heat Transfer, Part A: Applications: An International Journal of Computation and Methodology*, **31,** 255 (1997).
2. K. D. Alagoa, G. Tay, *Astr. Phys. Space Sci.*, **260,** 455 (1999).
3. E. Osalusi, J. Side, R. Harris, *Int. Comm. Heat Mass Transfer*, **35,** 908 (2008).
4. B K. Jha, A. O. Ajibade, *Proc. IMechE: Part E: J. Process Mechanical Engineering*, **224,** 91 (2010).
5. K. D. Singh K. Chand, S. Sharma, *Int. J. Math. Sci. Engg. Appls. (IJMSEA)*, **6,** 317 (2012).
6. I. Khan, K. Fakhar, S. Shafie, *J. Phys. Soc. Japan*, 80, 104401 (2011).
7. M. Narahari, L. Debnath, *ZAMM*, **93**, 38 (2013).
8. M. Kar, G. C. Dash, S. N. Sahoo and P. K. Rath, a porous medium with heat source and *J. Eng. Thermophysics*, **22,** 203 (2013).
9. R. N. Barik G. C. Dash, M. Kar, *J. Fluids*, **2013** (2013), 297493.
10. A. Manglesh, M.G. Gorla, *Indian J. Pure Appl. Math.*, **44,** 743 (2013).
11. A. J. Chamkha, S. S. Al-Rashidi, *Heat Transfer Research*, **44,** 215 (2013).
12. S. Md. Ibrahim K. Gangadhar, N.B. Reddy, *J. Appl. Fluid Mech.*, **8,** 529 (2015).
13. G. S. Seth, S. Sarkar, *Bulg. Chem. Comm.*, **47** 66 (2015).
14. I. B. Yabo, B. K. Jha, J-E. Lin, *Appl. Mathematics*, **7,** 2354 (2016).
15. A. S. Butt, A. Ali, *J. Appl. Mech. Tech. Phys.*, **57,** 27 (2016).
16. J. K. Singh, N. Joshi, S. G. Begum, C. T. Srinivasa, *Frontiers in Heat and Mass Transfer*, **7,** 24 (2016).
17. Md. F. Iqbal and S. Ahmad, K. Ali and Md. Ashraf, *J. Porous Media*, **19,** 821 (2016).
18. J. C. Misra and S. D. Adhikary, *Alexandria Engineering Journal*, **55,** 287 (2016).
19. J. A. Falade, J.C. Ukaegbu, A.C. Egere, S. O. Adesanya, MHD oscillatory flow through a porous channel saturated with porous medium, *Alexandria Engineering Journal*, **56,** 147 (2017).
20. K.R. Cramer, S.I. Pai, Magneto-fluid-dynamics for Engineers and Applied Physicists, *McGraw-Hill*, New York (1973).
21. V. Javeri, *Heat Mass Transfer*, **8,** 193 (1975).
22. V. M. Soundalgekar, N. V. Vighnesam and H. S. Takhar, Hall and ion-slip effects in the MHD Couette flow with heat transfer, *IEEE Trans. Plasma Sci.*, **7,** 178 (1979).
23. P.C. Ram, H.S. Takhar, *Fluid Dyn. Res.*, **11,** 99 (1993).
24. H.S. Takhar, B.K. Jha, *J. MHD Plasma Res.*, **8,** 61 (1998).
25. E.M. Abo-Eldahab, M.A.E. Aziz, *Physica Scripta*, **61,** 344 (2000).
26. E.F. Elshehawey, N.T. Eldabe, E.M. Elbarbary, N.S. Elgazery, *Canad. J. Phys.* **82,** 701 (2004).
27. O. Ojjela,d N. N. Kumar, *International Scholarly Research Notice*s, **2014** 646957 (2014).
28. M.D. Hossain, M.A. Samad, M.M. Alam, *Procedia Eng.*, **105,** 56 (2015).
29. J. K. Singh, P. Rohidas, N. Joshi, S. Ghousia Begum, *Int. J. Heat Technol.*, **35,** 37 (2017).
30. J. K. Singh, G. S. Seth, S. Ghousia Begum, *Diffusion Foundations*, **11**, 146 (2017).
31. K. Walters, The motion of a elastico-viscous liquid contained between coaxial cylinders, *Quarterly J. Mech. Appl. Math.*, **13,** 444 (1960).
32. K. Walters, *Quarterly J. Mech. Appl. Math*, **15,** 63 (1962).
33. G. Chakraborty, P. R. Sengupta, *Proc. Nat. Acad. Sci.*, **65,** 75 (1994).
34. V. Sharma, G.C. Rana, *Int. J. Appl. Mech. Eng.*, **6,** 843 (2001).
35. R.C. Sharma, P. Kumar, S. Sharma, *Int. J. Appl. Mech. Eng.*, **7,** 433 (2002).
36. M.M. Nandeppanavar, M.S. Abel, J. Tawade, *Commun. Nonlinear Sci. Numer. Simul.*, **15,** 1791 (2010).
37. T.B. Chang, A. Mehmood, O.A. Beg, M. Narahari, M.N. Islam, F. Ameen, *Commun. Nonlinear Sci. Numer. Simul.*, **16,** 216 (2011).
38. P. G. Siddheshwar, U. S. Mahabaleshwar, *Int. J. Nonlinear Mech.* **40,** 807 (2005).

39. I.J. Uwanta, B.Y. Isah, M.O. Ibrahim, *Int. J. Comput. Appl.*, **36,** 17 (2011).
40. O. Prakash, D. Kumar, Y.K. Dwivedi, *J. Phys.*, **79,** 1457 (2012).
41. T. Hayat, S. Asad, M. Mustafa, H.H. Alsulami, *Chinese Physics B*, **23,** 084701 (2014).
42. B. Garg, K.D. Singh, A. Bansal, *Kragujevac J. Sci.*, **36,** 19 (2014).
43. K.D. Singh, B. Garg, A. Bansal *Proc. Indian Nat. Sci. Acad.*, **80,** 333 (2014).
44. K. Ramesh, M. Devakar, *J. Aerosp. Eng.*, **29** 04015050 (2016).
45. S.A. Gaffar, V.R. Prasad, E.K. Reddy, *Int. J. Appl. Comput. Math.*, **3**, 859 (2017).
46. J. K. Singh, N. Joshi, P. Rohidas, *J. Mechanics* (2017). DOI: 10.1017/jmech.2017.25
47. G. Sutton, A. Sherman, Engineering Magnetohydrodynamics, *McGraw-Hill*, New York, 1965.
48. R.C. Meyer, *J. Aerosp. Sci.*, **25,** 564 (1958).