

Computer modeling and simulations of processes in serial resonance

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The serial resonance is a long known and researched process in the theory of the electrical circuits. The use of modern computer technique and technologies enables quickly, accurately and easily to be designed and analyzed both simple and complex electrical circuits in which, under certain conditions, different resonance phenomena are observed. The article presents two methods for research of the serial resonance in a linear electrical circuit with lumped parameters. The first method is with direct application of the Kirchhoff's and Ohm's laws (the classic method) and the other – by using specialized computer software CircuitMaker. The results that are calculated by both methods are compared and analyzed. The formulated conclusions are presented in the article.

Keywords: Serial Resonance, Quality Factor, Frequency Range.

INTRODUCTION

Over the last few years various computer programs have been created for calculating different mathematical problems. Some of them are highly specialized and related to the analysis of different physical processes, part of which are the analyses of specific electrical circuits. Other programs have a much wider range of options. For example, with programs such as MS Excel large arrays of data can be calculated, and the results can be presented in visual tables and graphics. Usually, such an analysis is related to good knowledge of the theory of the processes under consideration. Very often, it is necessary quickly and accurately to make a research of a certain electrical process without requiring in-depth theoretical knowledge. To make a comparative analysis in the current article the specialized computer software CircuitMaker [1], [2] is used. CircuitMaker allows users to create principal electrical circuits quickly and easily and to model their work.

The accent in this article is on comparing the opportunities for research of a simple electrical process – serial resonance, or as it is also called voltage resonance. It should be emphasized that linear electrical circuits with lumped parameters are analyzed.

For the particular comparative approach, on the one hand, an analysis based on the theory of electrical circuits is used, such as the obtained results are processed and visualized in MS Excel. On the other hand, the same processes are studied by the specialized computer software CircuitMaker. The results that are calculated by both methods are compared and analyzed. The formulated conclusions are presented in the article.

THE WORK OF A RLC CIRCUIT IN A VOLTAGE RESONANCE REGIME

A linear electrical circuit with lumped parameters R , L and C which are serially connected to an AC voltage source $u(t) = u_m \sin(\omega t)$ is analyzed [3]. Fig. 1 presents the corresponding electrical circuit. Due to the fact that the circuit is linear, a complex presentation of its elements can be used [4].

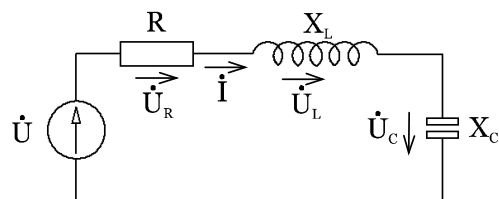


Fig. 1. RLC Serial loop.

The impedance of the circuit Z is determined by (1) [5]:

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = ze^{j\varphi} \quad (1)$$

The element R is the active resistance of the circuit, while $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$ are the reactive resistances. According to Ohm's law, the voltage drops on these resistances are \dot{U}_R , \dot{U}_L and \dot{U}_C , respectively.

According to Kirchhoff's second law and Ohm's law, the complex expression for the current in the circuit is:

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{\dot{U}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{U}{ze^{j\varphi}} = Ie^{-j\varphi}, \quad (2)$$

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where the module and the argument of the impedance Z are expressed by the following dependencies:

$$\begin{cases} z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\ \varphi = \arctg\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \end{cases} \quad (3)$$

In the particular case, when $\omega = \omega_0$ and $\omega_0 L = \frac{1}{\omega_0 C} = \rho$, the circuit is in resonance with a frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, and ρ is the characteristic resistance.

Analysis of the RLC circuit

A specific RLC circuit with the following parameters is considered: $U = 10V$, $R_1 = 1\Omega$, $R_2 = 0,2\Omega$, $L = 1mH$, $C = 10nF$.

The resonance frequency $\omega_0(f_0)$ and the limit frequencies $\omega_1(f_1)$ and $\omega_2(f_2)$, that restrict the amplitude of the current to 3dB, must be determined analytically. Also, the quality factor $Q_1 = Q_1(R_1)$ and $Q_2 = Q_2(R_2)$ must be determined and the corresponding graphics should be drawn.

Fig. 2 and Fig. 3 present the graphics of the normalized values of the module of the impedance $\left(\frac{z}{z_0} = \frac{z}{z_0}(\omega)\right)$ and the module of the current $\left(\frac{I}{I_0} = \frac{I}{I_0}(\omega)\right)$ as a function of the frequency, respectively.

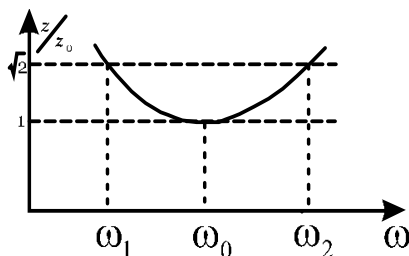


Fig. 2. Frequency band $\frac{z}{z_0} = \frac{z}{z_0}(\omega)$.

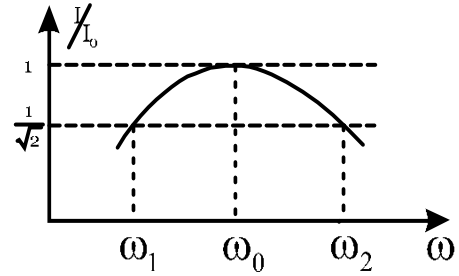


Fig. 3. Frequency band $\frac{I}{I_0} = \frac{I}{I_0}(\omega)$.

The resonance frequency ω_0 is determined by the dependence $\omega_0 = \frac{1}{\sqrt{LC}}$, and the characteristic resistance: $\rho = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$. The quality factor of the circuit is $Q = \frac{\rho}{R}$.

The limit frequencies ω_1 and ω_2 are determined from Fig. 2 and Fig. 3 [5]. At the resonance frequency, the function takes a value $\frac{Z_0}{Z_0} = 1$. For all other values of ω , the function $\frac{Z}{Z_0} > 1$. The frequency band, in its range from ω_1 to ω_2 , is at the level $\sqrt{2}$ (3dB). This allows the module z of the impedance from (3) to be converted to:

$$z_1 = z_2 = R\sqrt{2} = \sqrt{R^2 + \left(\omega_x L - \frac{1}{\omega_x C}\right)^2}, \quad (4)$$

where ω_x are the possible real frequency values which satisfy the condition.

As a result of this, the following system of equations (5) and possible solutions for ω_x (6), are obtained:

$$\begin{cases} \omega_x^2 + \frac{R}{L}\omega_x - \frac{1}{LC} = 0 \\ \omega_x^2 - \frac{R}{L}\omega_x - \frac{1}{LC} = 0 \end{cases} \quad (5)$$

$$\begin{cases} \omega_{1/3} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_{2/4} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{cases} \quad (6)$$

The real solutions for ω_x are (7) and (8):

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \omega_0^2}, \quad (7)$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \omega_0^2}. \quad (8)$$

As a result of the calculations made, the following results are obtained: $\omega_0 = 3.1623E + 0.5$ rad/s ($f_0 \approx 50.33$ kHz), $\rho \approx 316\Omega$. The quality factor is $Q_1 = 316$ and $Q_2 = 1581$. The limit frequencies for the used values of the resistance R are the following: $f'_1 \approx 50.25$ kHz, $f'_2 \approx 50.41$ kHz, $f''_1 \approx 50.31$ kHz, $f''_2 \approx 50.35$ kHz.

After estimating 51 points, which are normally distributed on the two sides of the resonance frequency, the graphic of the normalized values of the current $\frac{I}{I_0} = \frac{I}{I_0} \left(\frac{f}{f_0} \right)$ is obtained (Fig. 4).

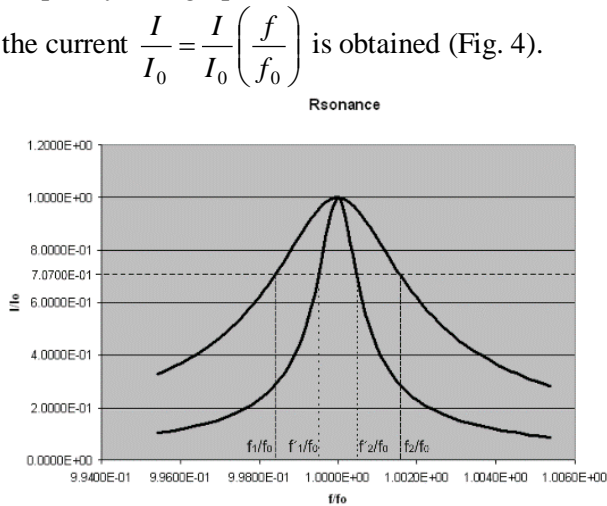


Fig. 4. $\frac{I}{I_0} = \frac{I}{I_0} \left(\frac{f}{f_0} \right)$.

Numerical modeling by using CircuitMaker

Analogous to the approach discussed in [6], Fig. 5 and Fig. 6 present the circuit of the model and the Spice Netlist when $R_1 = 1\Omega$.

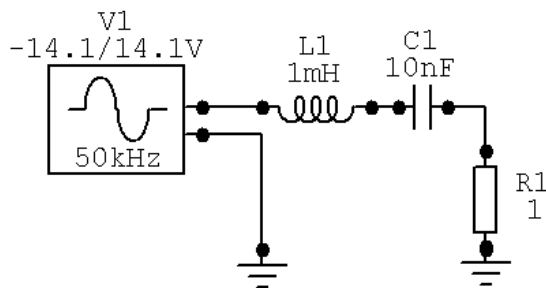


Fig. 5. A circuit of the model.

```
*Spice netlist for Circuit: D:\Rezonans.ckt
L1 4 5 1mH
C1 3 4 10nF
V1 5 0 DC 0 SIN(0 10 50k 0 0) AC 10k 0
R1 0 3 1
.SAVE V(3) V(4) V(5) @!1[p] @!1[i] @c1[p]
@c1[i] @v1[p] v1#branch @v1[z] @r1[p]
.SAVE @r1[i]
*BKGD=RGB 0 0 0
*BINARY RAW FILE
* Selected Circuit Analyses :
.AC LIN 30000 49k 51k
.END
```

Fig. 6. Spice Netlist.

The following parameter of the model is chosen: a source of sinusoidal signal $u_{pp} = [-14.1;14.1]$ V, which simulates power supply of the circuit with an effective voltage value $U = 10V$. In addition, the passive elements are also selected: $R_1 = 1\Omega$, $R_2 = 0.2\Omega$, $L = 1mH$ and $C = 10nF$.

A process which simulates a signal with variable frequency in the range $f = [49 - 51]$ kHz is started. As a result of the process, the graphic of the amplitude-frequency characteristic $I = I(f)$ of the current in the circuit is drawn (Fig. 7).

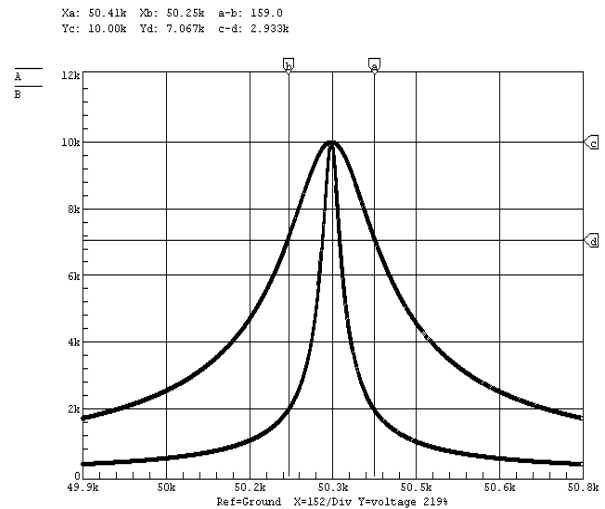


Fig. 7. Characteristic $I = I(f)$.

Fig. 7 presents the characteristics when the values of the resistances are the following: $R_1 = 1\Omega$ and $R_2 = 0.2\Omega$. Fig. 7 shows how the quality factor influences the width of the frequency band Δf .

From the graphic the values of: the resonance frequency $f_0 \approx 50.33$ kHz, the limit frequencies $f_1 \approx 50.26$ kHz, $f_2 \approx 50.40$ kHz and the quality factor $Q = 360$ (for $R_1 = 1\Omega$) are reported.

The same process is repeated in the case when the resistance $R_2 = 0.2\Omega$. In this case, the

characteristic should be graphed in a different scale because the value of the quality factor Q is much higher.

CONCLUSIONS

The advantage of the analytical method is that it provides complete and accurate results for the processes studied in the electrical circuits.

The disadvantage of the analytical method is that it requires in-depth theoretical knowledge in the field that is researched, as well as a large amount of computational work.

Some advantages of the computer simulations by the specialized computer software CircuitMaker are as follows:

- the analysis of the electrical circuit is limited to standard and simple rules for its modeling;
- the results are displayed in an understandable graphical form;
- the parameters of the circuits can be changed easily, that allows the researcher to conduct different numerical experiments.

The disadvantage of the second method is its inaccuracy. The obtained results show a relative error in regard to the analytical method, which is 0.02 % for the limit frequencies f_1 and f_2 and 13.8 % for the quality factor Q . The size of the error depends on the selected scale.

REFERENCES

1. J. Cathey, Theory and Problems of Electronic Devices and Circuits, Second edn., McGraw-Hill, New York, 2002.
2. <http://www.circuitmaker.com>.
3. S. Sharma, Basics of Electrical Engineering, Second edn., I. K. International Publishing House Pvt. Ltd, New Delhi, 2008.
4. J. Bird, Electrical and Electronic Principles and Technology, Second edn., Newnes, GB, 2003.
5. C. Alexander, M. Sadiku, Fundamentals of Electric Circuits, Fourth edn., McGraw-Hill, New York, 2009.
6. V. Milovanski, N. Georgieva, G. Kalpachka, Numerical Modeling and Computer Simulations of Voltage Resonance, in: Mathematics and Natural Sciences (Proc. 6th Int. Sci. Conf.), vol. 2, Blagoevgrad, 2015, p. 105.