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Effect of porosity on the flow and heat transfer between two parallel porous plates with the Hall effect and variable properties under constant pressure gradient

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Keywords: Flow between two parallel plates, temperature-dependent properties, hydromagnetics, porous medium, heat transfer, finite differences.

INTRODUCTION

The flow of an electrically conducting fluid between infinite horizontal parallel plates, known as Hartmann flow, has interesting applications in magnetohydrodynamic (MHD) power generators pumps, etc. Hartmann and Lazarus [1] investigated the effect of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel plates. Exact solutions for the velocity fields were developed [2-5] under different physical effects. Some exact numerical solutions for the heat transfer problem are derived in [6]. Soundalgekar *et al*. [7,8] examined the effect of Hall current on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed constant [7] or varying along the plates in the direction of the flow [8]. Attia [9] examined the effect of Hall current on the velocity and temperature fields of an unsteady Hartmann flow with uniform suction and injection applied perpendicular to the plates.

In these studies the physical properties are assumed to be constant; however, it is known that some physical properties are functions of temperature and assuming constant properties is a good approximation as long as small differences in temperature are involved. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of the physical properties with temperature [10-13]. Klemp *et al*. [14] studied the effect of temperature-dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb [15] solved the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity which has been extended to the transient state by Attia [16]. The influence of the dependence of the physical properties on temperature in the MHD Couette flow between parallel plates was studied [14,15].

In this paper, the transient Hartmann flow through a porous medium of a viscous incompressible electrically conducting fluid is investigated with heat transfer under constant pressure gradient. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature while the Hall current is taken into consideration. The fluid is flowing between two electrically insulating porous plates and is acted upon by an axial constant pressure gradient. A uniform suction and injection and an external uniform magnetic field are applied perpendicular to the surface of the plates. The two plates are kept at two constant but different temperatures and the viscous and Joule dissipations are taken into

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consideration in the energy equation. The flow in the porous medium is described by a differential equation governing the fluid motion based on the Darcy's law which considers the drag exerted by the porous medium [17-21]. The coupled set of the non-linear momentum and energy equations is solved numerically using the method of finite differences to determine the velocity and temperature fields. The effect of porosity of the medium, the Hall current, the suction and injection velocity and the temperature-dependent viscosity and thermal conductivity on both the velocity and temperature distributions is discussed.

FORMULATION OF THE PROBLEM

The fluid flows between two infinite horizontal parallel plates located at the y=±h planes, as shown in Fig. 1. The two plates are porous, insulating and kept at two constant but different temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2>T_1$. A constant pressure gradient is imposed in the axial x-direction and uniform suction from above and injection from below, with velocity vo, are applied impulsively at t=0. A uniform magnetic field Bo, assumed unaltered, is applied perpendicular to the plates in the positive y-direction. The Hall effect is considered and accordingly, a z-component of the velocity is initiated. The viscosity and the thermal conductivity of the fluid depend on temperature exponentially and linearly, respectively, while the viscous and Joule dissipations are not neglected in the energy equation.

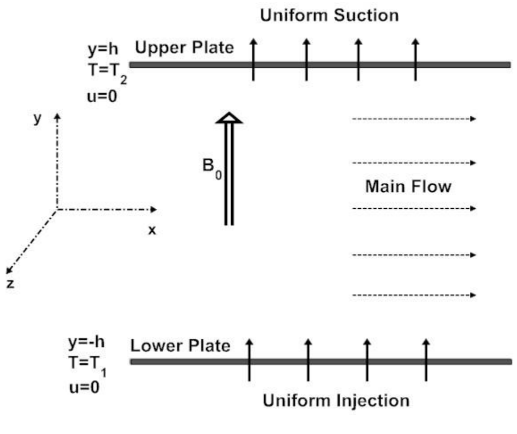

Fig. 1 The geometry of the problem

The flow is through a porous medium where the Darcy model is assumed [19]. The fluid motion starts from rest at t=0, and the no-slip condition at the plates implies that the fluid velocity has neither a z nor an x-component at y=±h. The initial temperature of the fluid is assumed to be equal to T_1 as the temperature of the lower plate. Since the plates are infinite in the x and z-directions, the physical quantities do not change in these directions, which leads to one-dimensional problem.

The flow of the fluid is governed by the Navier-Stokes equation:

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p + \vec{\nabla}.(\mu \vec{\nabla} v) + \vec{J} \wedge \vec{B}_o \tag{1}$$

where, ρ is the density of the fluid, μ is the viscosity of the fluid, $\vec{J}$ is the current density, and $\vec{v}$ is the velocity vector of the fluid, which is given by:

$$\vec{v} = u(y,t)\vec{i} + v_o \vec{j} + w(y,t)\vec{k}$$

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$$\vec{J} = \sigma(\vec{v} \wedge \vec{B}_o - \beta(\vec{J} \wedge \vec{B}_o)). \tag{2}$$

where, σ is the electric conductivity of the fluid and β is the Hall factor [4]. Equation (2) may be solved in $\vec{J}$ to yield:

$$\vec{J} \wedge \vec{B}_o = -\frac{\sigma B_o^2}{1+m^2}((u+mw)\vec{i} + (w-mu)\vec{k}) \qquad \text{... (3)}$$

where, *m* is the Hall parameter and $m = \sigma \beta B_o$. Thus, the two components of the momentum Eq. (1) read:

$$\rho \frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = G + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} - \frac{\sigma B_o^2}{1+m^2}(u+mw) - \frac{\mu}{\overline{K}} u, \tag{4}$$

$$\rho \frac{\partial w}{\partial t} + \rho v_o \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial w}{\partial y} - \frac{\sigma B_o^2}{1+m^2}(w-mu) - \frac{\mu}{\overline{K}} w. \tag{5}$$

where, $\overline{K}$ is the Darcy permeability [19] and the last term in the right side of Eqs. (4) and (5) represents the porosity force in the x- and z-directions respectively. It is assumed that the pressure gradient is applied at t=0 and the fluid starts its motion from rest. Thus

$$t = 0 : u = w = 0. \tag{6a}$$

For t > 0, the no-slip condition at the plates implies that

$$y = -h : u = w = 0, \tag{6b}$$

$$y = h : u = w = 0. \tag{6c}$$

The energy equation describing the temperature distribution for the fluid is given by [18]:

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_o \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \mu((\frac{\partial u}{\partial y})^2 + (\frac{\partial w}{\partial y})^2) + \frac{\sigma B_o^2}{1+m^2}(u^2 + w^2). \tag{7}$$

Where, T is the temperature of the fluid, cp is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The last two terms in the right side of Eq. (7) represent the viscous and Joule dissipations, respectively.

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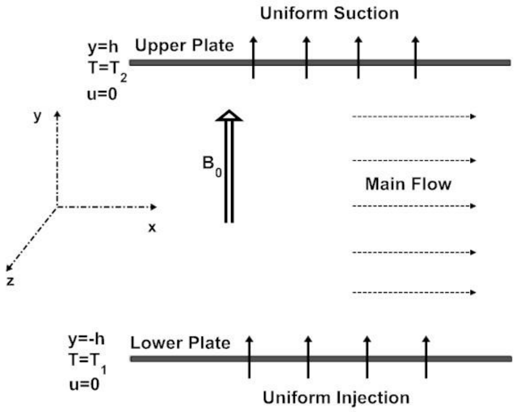

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Introducing the following non-dimensional quantities,

$$(\hat{x},\hat{y},\hat{z}) = \frac{(x,y,z)}{h}, \hat{t} = \frac{t\mu_o}{\rho h^2}, \hat{G} = \frac{\rho G}{h^2 \mu_o^2}, (\hat{u},\hat{w}) = \frac{(u,w)\rho h}{\mu_o}, \theta = \frac{T-T_1}{T_2-T_1},$$

$\hat{f}_1(\theta) = e^{-a_1(T_2-T_1)\theta} = e^{-a\theta}$, a is the viscosity variation parameter,

$\hat{f}_2(\theta) = 1 + b_1(T_2-T_1)\theta = 1 + b\theta$, b is the thermal conductivity variation parameter,

$S = \rho v_o h / \mu_o$ is the suction parameter,

$Ha^2 = \sigma B_o^2 h^2 / \mu_o$, Ha is the Hartmann number,

$M = h^2 / \overline{K}$, is the porosity parameter,

$\Pr = \mu_o c_p / k_o$ is the Prandtl number,

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$Nu_L = (\partial T/\partial \hat{y})_{\hat{y}=-1}$ is the Nusselt number at the lower plate,

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Equations (4) to (8) read (the hats are dropped for simplicity)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = G + f_1(\theta)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f_1(\theta)}{\partial y}\frac{\partial u}{\partial y} - \frac{Ha^2}{1+m^2}(u+mw) - Mu, \tag{9}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = f_1(\theta)\frac{\partial^2 w}{\partial y^2} + \frac{\partial f_1(\theta)}{\partial y}\frac{\partial w}{\partial y} - \frac{Ha^2}{1+m^2}(w-mu) - Mw. \tag{10}$$

$$t = 0 : u = w = 0, \tag{11a}$$

$$t > 0 : y = -1,\ u = w = 0, \tag{11b}$$

$$t > 0 : y = 1,\ u = w = 0. \tag{11c}$$

$$\frac{\partial \theta}{\partial t} + S\frac{\partial \theta}{\partial y} = \frac{1}{\Pr} f_2(\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\Pr}\frac{\partial f_2(\theta)}{\partial y}\frac{\partial \theta}{\partial y} + Ec f_1(\theta)\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) + \frac{EcHa^2}{1+m^2}(u^2+w^2). \tag{12}$$

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Equations (9), (10), and (12) represent a system of coupled non-linear partial differential equations which are solved numerically under the initial and boundary conditions (11) and (13) using the method of finite differences. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method is used at two successive time levels [25]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm [25]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. The computational domain is divided into meshes each of dimension Δt and Δy in time and space, respectively. We define the variables $A = \partial u/\partial y$, $B = \partial w/\partial y$ and $H = \partial \theta/\partial y$ to reduce the second order differential Eqs. (9), (10) and (12) to first order differential equations, and an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations are carried out for the non-dimensional variables and parameters given by, G = 5, Pr = 1, and Ec = 0.2 where G is related to the externally applied pressure gradient and where the chosen given values for Pr and Ec are suitable for steam or water vapor. Grid-independence studies show that the computational domain $0<t<\infty$ and $-1<y<1$ is divided into intervals with step sizes $\Delta t=0.0001$ and $\Delta y=0.005$ for time and space respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u, w, A, B, θ and H for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1<y<1$ at every time step. Less than 7 approximations are required to satisfy

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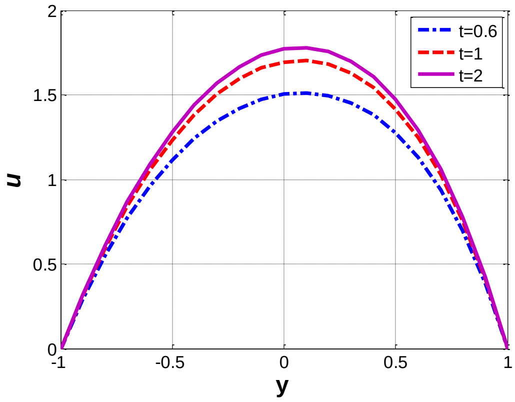
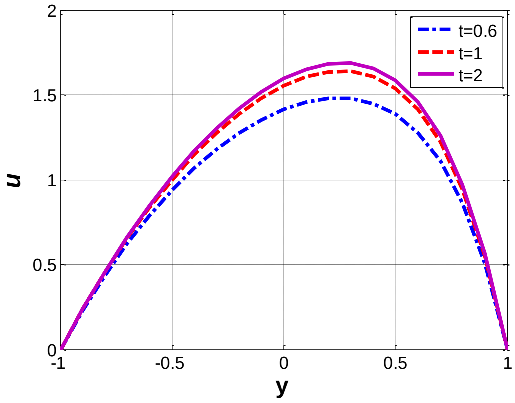
RESULTS AND DISCUSSION

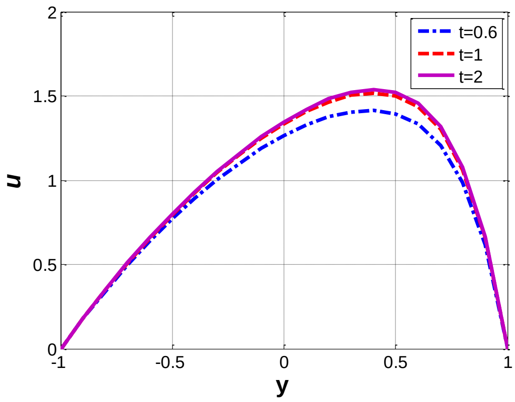
Figures 2-4 show the time development of the profiles of the velocity and temperature for various values of the suction parameter *S* and for *Ha* =1, m=1, *M*=1, a=0.5 and b=0.5.



(a)

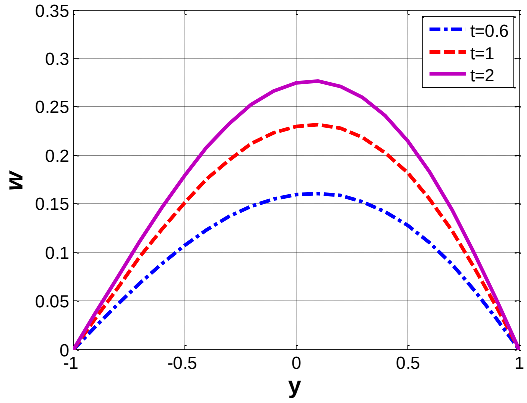


(b)

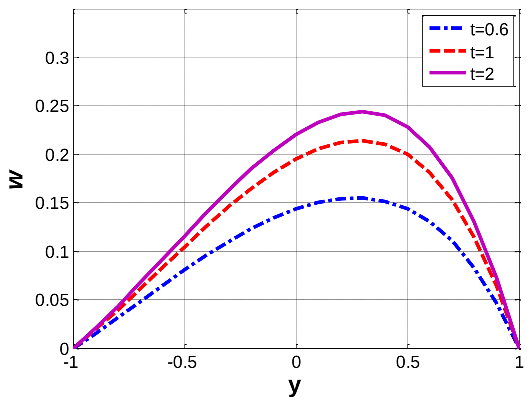


(c)

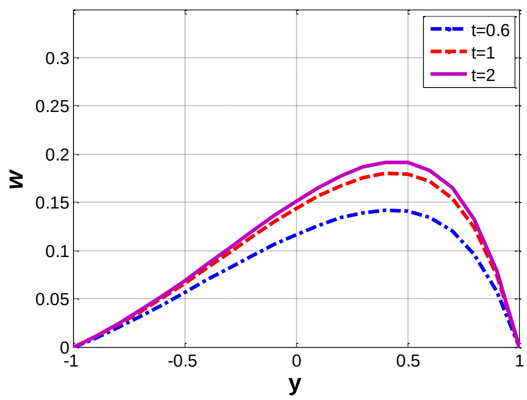
Fig. 2 The evolution of the profile of: *u*; (a) *S*=0; (b) *S*=1; (c) *S*=2. (*Ha*=1, *m*=1, *M*=1, *a*=0.5, *b*=0.5)



(a)



(b)



(c)

Fig. 3 The evolution of the profile of w; (a) S=0; (b) S=1; (c) S=2. (*Ha*=1, *m*=1, *M*=1, a=0.5, b=0.5)

The velocity and temperature distributions reach their steady state monotonically as shown in the figure. The velocity component u reaches steady state faster than *w* which, in turn, reaches steady state faster than θ. This is expected, as u is the source of w, while both u and w are sources of θ. It is also clear from Figs. 2-3 that the velocity components are asymmetric about the centre of the channel because of the effect of the suction.

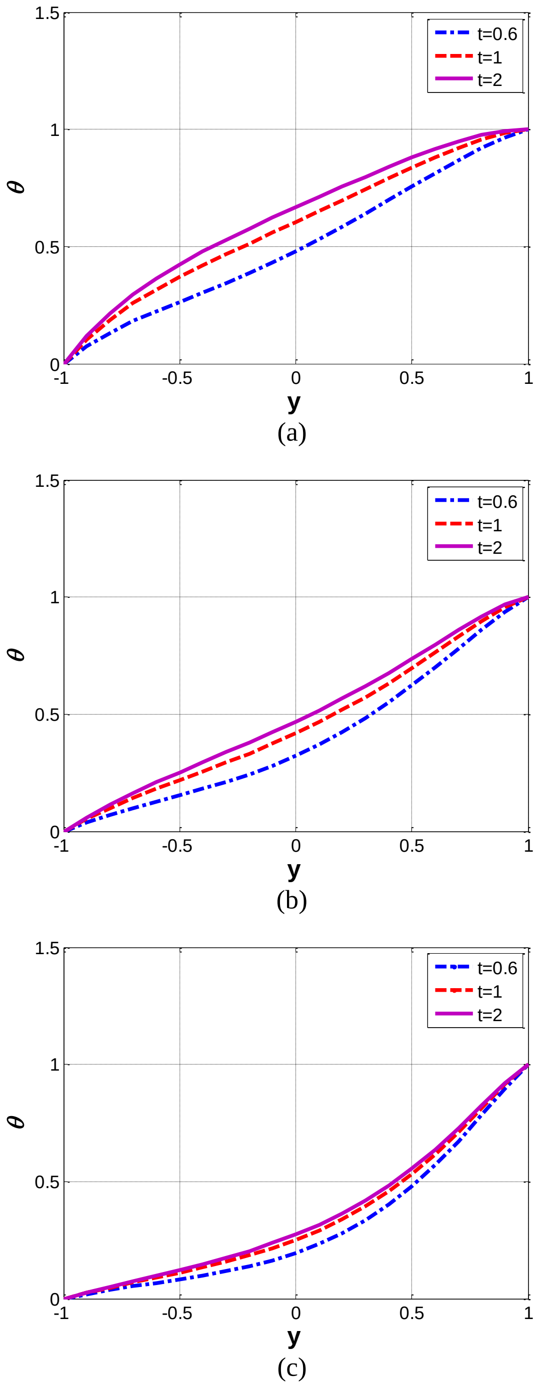


Fig. 4 The evolution of the profile of θ; (a) S=0; (b) S=1; (c) S=2. (*Ha*=1, m=1, *M*=1, a=0.5, b=0.5)

Figures 5-7 present the time progression of the velocity components u and w and the temperature θ at the centre of the channel (y=0) for different values of m and a and for Ha=1, M=1, S=1 and b=0. Increasing the parameter a increases the velocity components u and w and the temperature θ for all values of m as shown in all figures.

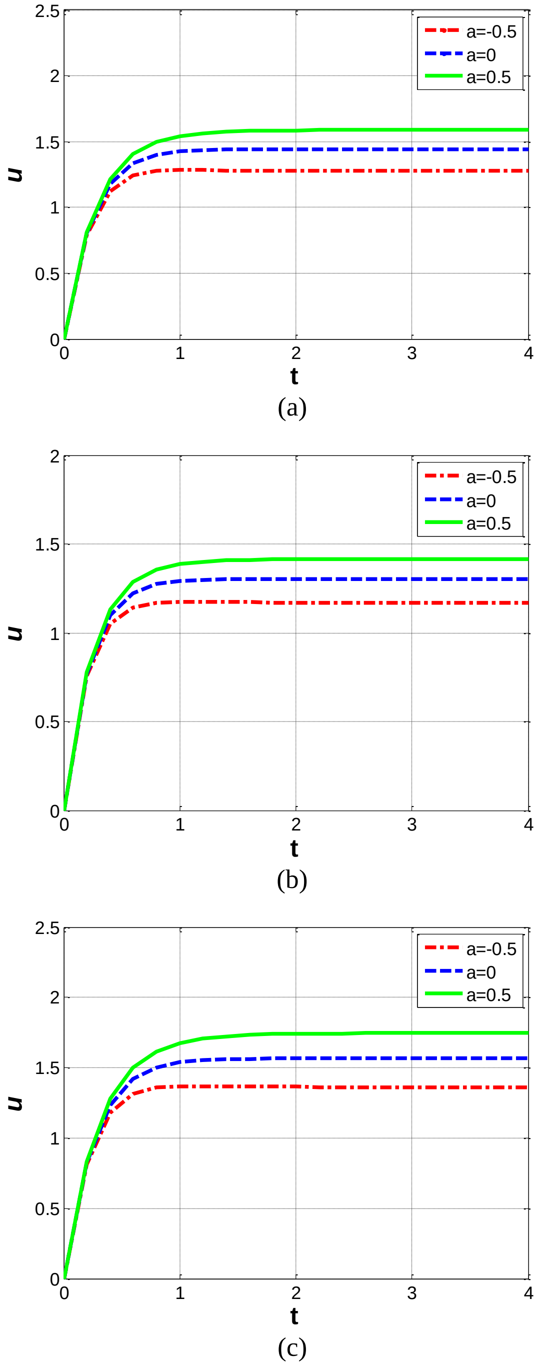


Fig. 5 The evolution of u at y=0 for various values of a and m: (a) m=0; (b) m=1; (c) m=2; . (*Ha=1, M=1,* S=1, b=0)

Figure 5 indicates that u increases with increasing m for all values of a, which can be attributed to the fact that an increment in m decreases the resistive force. Figure 6 shows that w decreases with increasing m for all values of a, which can be attributed to the fact that an increment in m increases the resistive force. Figure 7 shows that θ increases with increasing m for all values of a as a result of increasing the dissipations.

Figures 8-10 present the time progression of the velocity components u and w and the temperature θ at the centre of the channel (y=0) for different values of M and a and for Ha=1, m=1, S=1 and b=0.

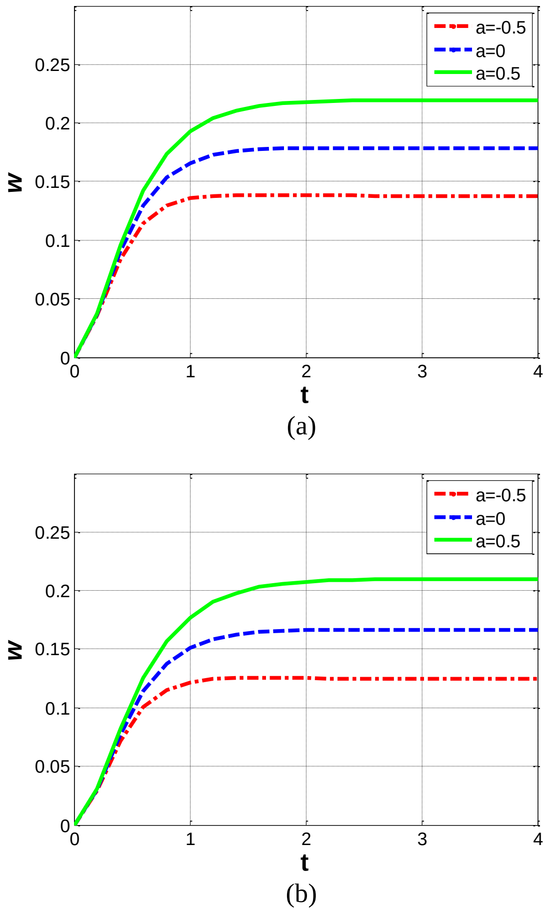


Fig. 6 The evolution of w at y=0 for various values of a and m: (a) m=1; (b) m=2; (Ha=1, M=1, S=1, b=0)

Figures 8-9 indicate that u and w decrease with increasing M for all values of a as a result of the damping effect of the porosity. Figure 10 depicts that the temperature θ decreases with increasing M for all values of a as a result of the damping effect of the porosity which decreases the velocity and velocity gradients and, in turn, deceases the dissipations. Increasing the parameter a increases the velocity components u and w and the temperature θ for all values of M as shown in all figures.

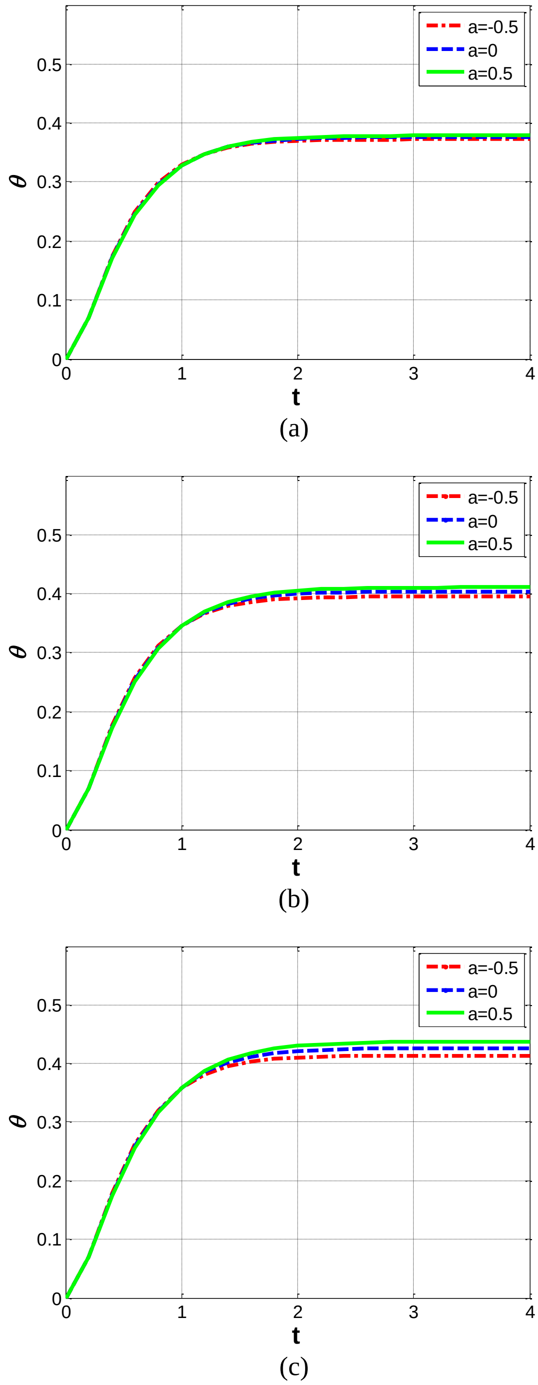


Fig. 7 The evolution of θ at y=0 for various values of a and m: (a) m=0; (b) m=1; (c) m=2 . (Ha=1, M=1, S=1, b=0)

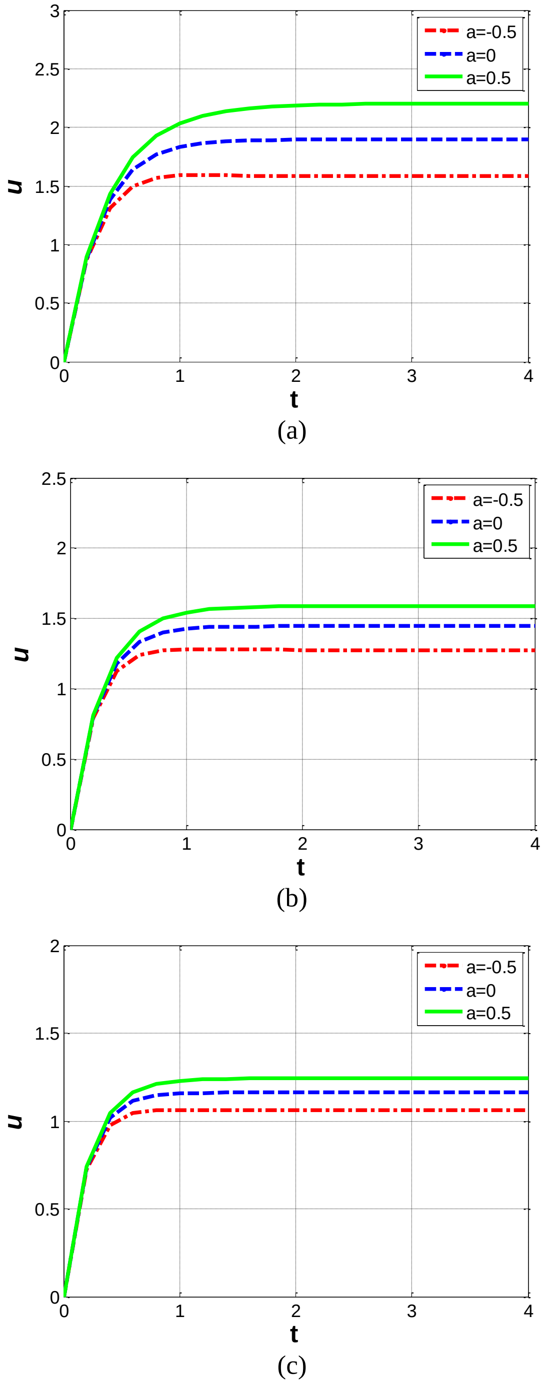


Fig. 8 The evolution of u at y=0 for various values of a and M: (a) M=0; (b) M=1; (c) M=2; . (Ha=1, m=1, S=1, b=0)

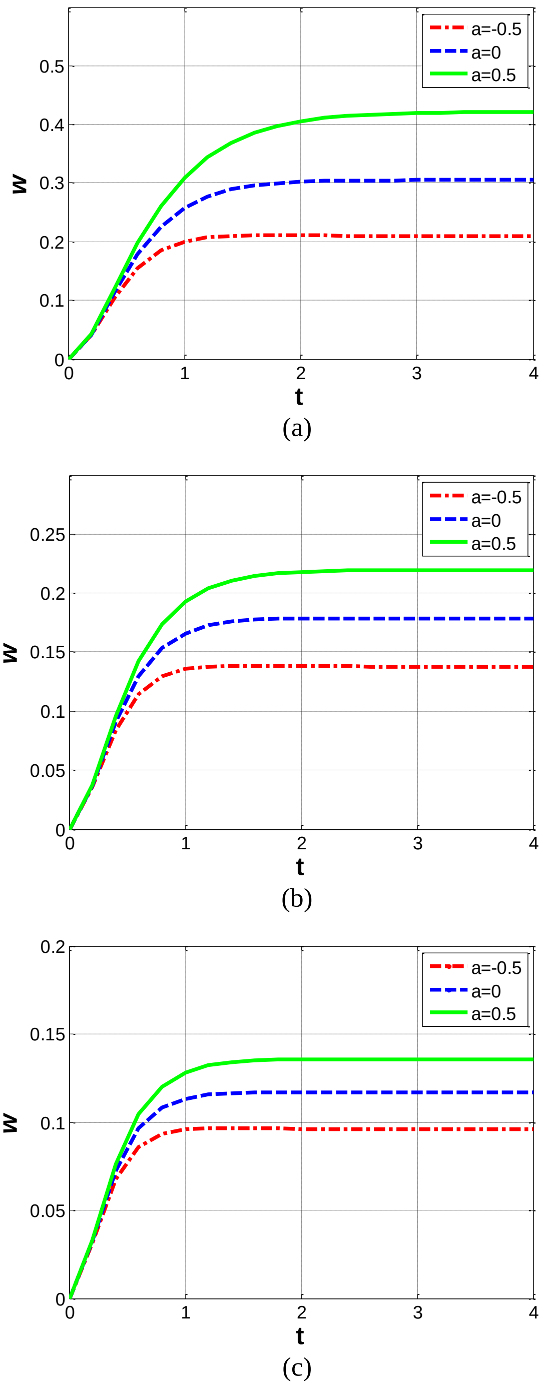


Fig. 9 The evolution of w at y=0 for various values of a and M: (a) M=0; (b) M=1; (c) M=2. (Ha=1, m=1, S=1, b=0)

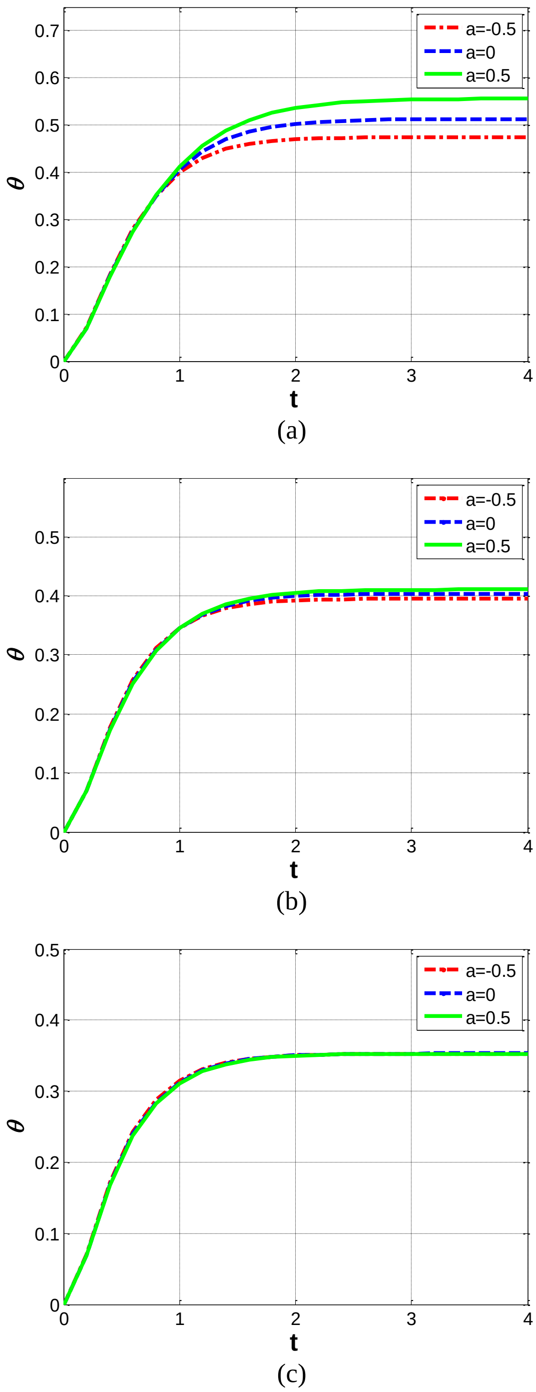


Fig. 10 The evolution of θ at y=0 for various values of a and M: (a) M=0; (b) M=1; (c) M=2;. (Ha=1, m=1, S=1, b=0)

Figure 11 presents the time progression of the temperature θ at the centre of the channel (y=0) for different values of m and b and for *Ha=1, M=1, S=1* and b=0.

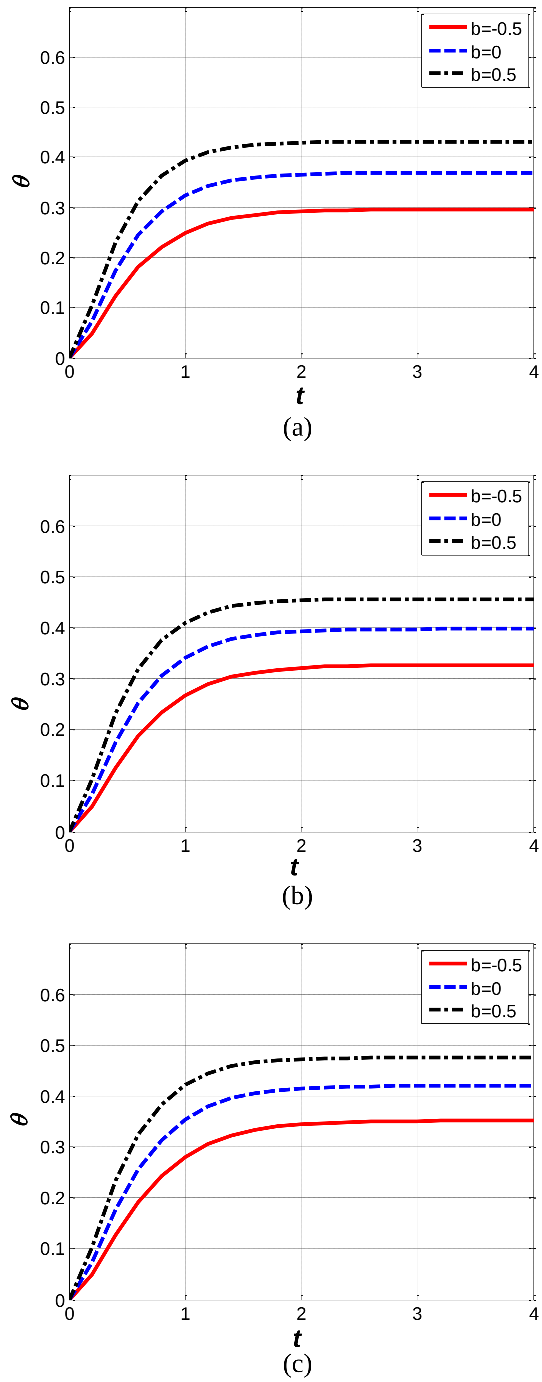
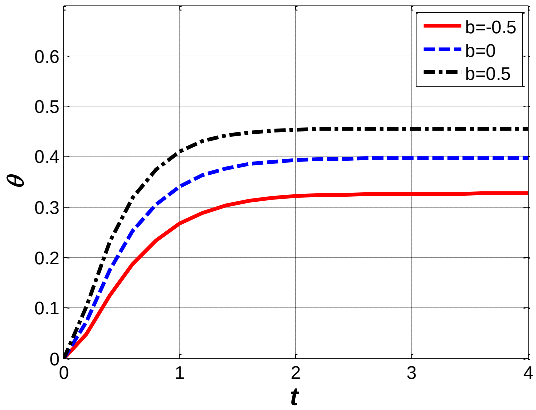
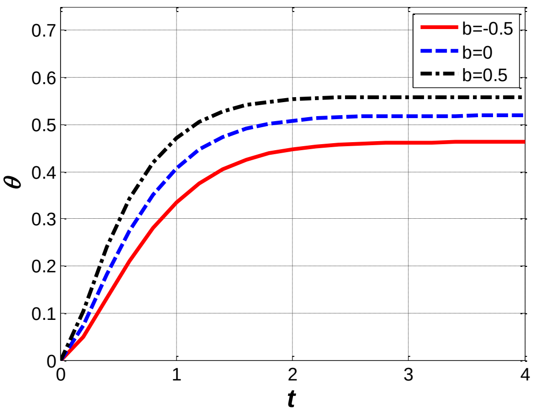


Fig. 11 The evolution of θ at y=0 for various values of b and m: (a) m=0; (b) m=1; (c) m=2. (*Ha=1, M=1, S=1*, a=0)

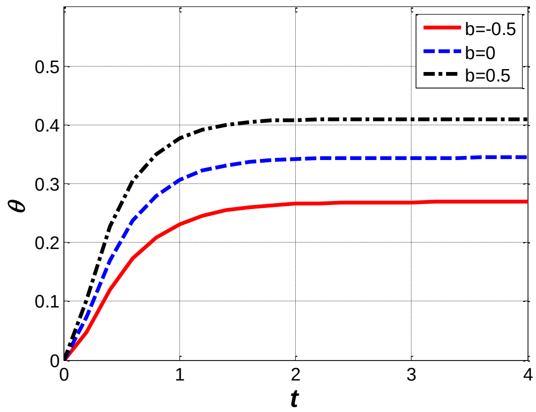
Increasing the parameter b increases the temperature θ for all values of m as shown in all figures. Figure 12 presents the time progression of the temperature θ at the centre of the channel (y=0) for different values of M and b and for Ha=1, M=1, S=1 and a=0.



(a)



(b)



(c)

Fig. 12 The evolution of θ at y=0 for various values of b and M: (a) M=0; (b) M=1; (c) M=2. (Ha=1, m=1, S=1, a=0)

Increasing the parameter b increases the temperature θ for all values of M, as shown in all figures. However, the effect of the parameter b on θ becomes more pronounced for higher values of M.

CONCLUSIONS

The time varying MHD flow through a porous medium between two parallel plates was investigated considering the Hall current under the action of a constant pressure gradient. The viscosity and the thermal conductivity of the fluid are assumed to be temperature dependent. The effect of the porosity parameter M, the Hartmann number Ha, the Hall parameter m, the viscosity variation parameter a and the thermal conductivity variation parameter b on the velocity and temperature fields at the centre of the channel are discussed. Introducing the Hall term gives rise to a velocity component w in the z-direction and affects the main velocity u in the x-direction. It is found that the parameter *a* has a marked effect on the velocity components u and w for all values of M. However, the parameter b has no significant effect on u or w. The porosity parameter M has a marked effect on the velocity and temperature distributions, however, its effect on the velocity and its steady state time is more pronounced than that for the temperature.

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ЕФЕКТ НА ПОРЬОЗНОСТТА ВЪРХУ ТЕЧЕНИЕТО И ТОПЛОПРЕНАСЯНЕТО МЕЖДУ ДВЕ ПОРЬОЗНИ ПЛОЧИ С ЕФЕКТ НА ХОЛ И ПРОМЕНЛИВИ СВОЙСТВА ПРИ ПОСТОЯНЕН ГРАДИЕНТ НА НАЛЯГАНЕТО

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(Резюме)

Изследвано е преходното хидромагнитно течение през порьозна среда между две безкрайни успоредни порьозни плоскости с топлообмен и с отчитането на ефекта на Хол и температурно зависими свойства при постоянен градиент на налгането. Приложено е външно постоянно магнитно поле с равномерно всмукване и инженктиране перпендикулярно на хоризонталните плочи. Получено е числено решение на нелинейните уравнения на движението и енергията с отчитане на дисипация на енергията от вискозитета и ефекта Джаул. Съобщава се за ефекти на порьозността, тока на Хол, температурата зависимост на вискозитета и топлопроводността върху разпределението на скоростта и температурата.