### Effects of Hall current and rotation on unsteady MHD natural convection flow with heat and mass transfer past an impulsively moving vertical plate in the presence of radiation and chemical reaction

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Effects of Hall current and rotation on unsteady MHD natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an impulsively moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusion is studied. The exact solutions of momentum, energy and concentration equations, under the Boussinesq approximation, are obtained in closed form by the Laplace transform technique. The expressions for skin friction, Nusselt number and Sherwood number are also derived. The variations in fluid velocity, fluid temperature and species concentration are shown graphically whereas numerical values of skin friction, Nusselt number and Sherwood number are presented in tabular form for various values of pertinent flow parameters.

Keywords: Unsteady MHD natural convection flow, radiation, chemical reaction, Nusselt number, Sherwood number.

#### INTRODUCTION

Theoretical and experimental investigations of natural convection flow over vertical surfaces embedded in a porous medium have a wide range of applications in different fields of science and technology. Such configuration exists in several practical systems such as catalytic chemical reactors, thermal insulators, heat exchanger devices, nuclear waste repositories, systems for drying of porous solids, underground energy transport, enhanced recovery of oil and gas, cooling of nuclear reactors, geothermal reservoirs, etc. Considering the importance of such fluid flow problems, large amount of research works have been carried out in this field. Mention should be made of the research studies of Cheng and Minkowycz [1], Nakayama and Koyama [2], Lai and Kulacki [3], Hsieh et al. [4], Nield and Kuznetsov [5] and Gorla and Chamkha [6]. Comprehensive reviews of thermal/species convection in porous media are presented by Pop and Ingham [7], Vafai [8] and Nield and Bejan [9].

The problems of hydromagnetic convective flow in a porous medium have drawn the attention of several researchers in the past due to significant effects of magnetic field on many problems of physical interest, *viz.* boundary layer flow control, plasma studies, geothermal energy extraction, metallurgy, chemical, mineral and petroleum engineering, etc. and on the performance of many engineering devices using electrically conducting fluids, namely, MHD generators, MHD pumps, MHD accelerators, MHD flow-meters, nuclear reactors, etc. Raptis and Kafousias [10] investigated steady hydromagnetic free convection flow through a porous medium bounded by an infinite vertical plate with constant suction velocity. Raptis [11] considered unsteady two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid along an infinite vertical plate embedded in a porous medium. Chamkha [12] investigated unsteady MHD free convection flow through a porous medium supported by a surface. Chamkha [13] also studied MHD natural convection flow near an isothermal inclined surface adjacent to a thermally stratified porous medium. Aldoss et al. [14] investigated combined free and forced convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Kim [15] discussed unsteady MHD free convection flow past a moving semi-infinite vertical porous plate embedded in a porous medium with variable suction. A few representative fields of interest in which combined heat and mass transfer plays an important role are: design of chemical processing equipment; formation and dispersion of fog; distribution of temperature and moisture over agricultural fields

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and groves of trees; damage of crops due to freezing, common industrial sight especially in power plants, etc. In view of these facts, Jha [16] considered hydromagnetic free convection and mass transfer flow past a uniformly accelerated moving vertical plate through a porous medium. Ibrahim et al. [17] investigated the unsteady hydromagnetic free convection flow of a micropolar fluid and the heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Alam and Rahman [18] considered Dufour and Soret effects on MHD free convection heat and mass transfer flow past a vertical porous flat plate embedded in a porous medium. Makinde and Sibanda [19] studied MHD mixed convective flow with heat and mass transfer past a vertical plate embedded in a porous medium with constant wall suction. Makinde [20] analyzed a hydromagnetic mixed convection flow and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium. Eldabe et al. [21] discussed unsteady MHD flow of a viscous and incompressible fluid with heat and mass transfer in a porous medium near a moving vertical plate with time dependent velocity.

The number of investigations of natural convection flow with thermal radiation has increased greatly during the past few decades due to its importance in many practical situations. When natural convection flows occur at high temperature, radiation effects on the fluid flow become significant. Radiation effects on the natural convection flow are important in context of furnace design, electric power generation, thermo-nuclear fusion, glass production, casting and levitation, plasma physics, cosmic flights, propulsion systems, solar power technology, spacecraft re-entry, aerothermodynamics, etc. It is worth noting that unlike convection/conduction the governing equations taking into account the effects of radiation become quite complicated. Hence, many difficulties arise while solving such equations. However, some reasonable approximations are proposed to solve the governing equations with radiative heat transfer. The textbook by Sparrow and Cess [22] describes the essential features of radiative heat transfer. Cess [23] investigated free convection flow past a vertical isothermal plate with thermal radiation using Rosseland diffusion approximation. Hossain and Takhar [24] analyzed the effects of radiation on mixed boundary layer flow near a vertical plate with uniform surface temperature using Rosseland flux model. Bakier and Gorla [25] considered the effects of radiation on mixed convection flow over a horizontal surface embedded in a fluid-saturated porous medium. Takhar et al. [26] analyzed the effects of radiation on MHD free convection flow of a gas past a semiinfinite vertical plate. Chamkha [27] considered solar radiation assisted natural convection in a uniform porous medium supported by a vertical flat plate. Chamkha [28] studied thermal radiation and buoyancy effects on MHD flow over an accelerating permeable surface with heat source or sink. Azzam [29] considered the effects of radiation on MHD free and forced convection flow past a semi-infinite moving vertical plate for high temperature differences. Cookey et al. [30] considered the influence of viscous dissipations and radiation past an infinite heated vertical plate in a porous medium with time-dependent suction. Muthucumaraswamy and Ganesan [31] studied the effects of radiation on a flow past an impulsively started vertical plate with variable temperature. Makinde and Ogulu [32] considered the effects of radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate in the presence of a transverse magnetic field. Mahmoud Mostafa [33] investigated the effects of thermal radiation on unsteady MHD free convection flow past an infinite vertical porous plate taking into account the effects of viscous dissipation. Ogulu and Makinde [34] considered unsteady hydromagnetic free convection flow of a dissipative and radiative fluid past a vertical plate with constant heat flux. Kishore et al. [35] studied effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions. Chamkha et al. [36] investigated radiation effects on mixed convection flow over a wedge embedded in a porous medium filled with nanofluid. Nandkeolyar et al. [37] investigated unsteady hydromagnetic natural convection flow of a dusty fluid past an impulsively moving vertical plate with ramped temperature in the presence of thermal radiation.

In many chemical engineering processes, there occurs chemical reaction between a foreign mass and the fluid in which the plate is moving. Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in numerous industrial applications, *viz.* polymer production, manufacturing of ceramics or glassware, food processing, etc. Afify [38] studied the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface

with chemical reaction in the presence of a transverse magnetic field. Muthucumaraswamy and Chandrakala [39] investigated radiative heat and mass transfer effects on a moving isothermal vertical plate in the presence of chemical reaction. Ibrahim et al. [40] analyzed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Muthucumaraswamy et al. [41] considered thermal radiation effects on hydromagnetic free convection and mass transfer flow past an infinite oscillating isothermal plate in the presence of chemical reaction of first order. Rajesh [42] investigated the effects of thermal radiation and first order chemical reaction on unsteady MHD free convection and mass transfer flow of a dissipative fluid past an infinite vertical porous plate with ramped wall temperature. Sudheer Babu et al. [43] studied radiation and chemical reaction effects on unsteady MHD convection flow past a vertically moving porous plate embedded in a porous medium with viscous dissipation. Nandkeolyar et al. [44] considered unsteady hydromagnetic heat and mass transfer flow of a heat radiating and chemically reactive fluid past a flat porous plate with ramped wall temperature.

Problems of a hydromagnetic natural convection flow in a rotating medium taking into account the effects of radiation are of considerable importance in many areas of geophysics, astrophysics and fluid engineering. In view of this fact, Bestman and Adjepong [45] investigated the unsteady hydromagnetic free convection flow of an incompressible optically thick fluid with radiative heat transfer near a moving flat plate in a rotating medium by imposing time dependent perturbation on a constant plate temperature. Mbeledogu and Ogulu [46] studied the heat and mass transfer of unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer. They applied Rosseland approximation for an optically thick fluid to describe the radiative flux. Bakr [47] discussed the effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Recently, Seth et al. [48] considered the effects of thermal radiation and rotation on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium. It is noticed that when the density of an electrically conducting fluid

is low and/or the applied magnetic field is strong, Hall current plays an important role in determining flow features of the fluid flow problem [49]. Also it is worth noting that both Hall current and rotation induce a secondary flow in the flow-field. In view of this fact, Sarkar et al. [50] studied the effects of Hall current on unsteady MHD free convective flow past an accelerated moving vertical plate with viscous and Joule dissipations. Anjali Devi et al. [51] investigated Hall effects on unsteady MHD free convection flow past an impulsively started porous plate with viscous and Joule dissipations. al. [52] investigated Farhad et unsteady hydromagnetic rotating flow past a moving infinite porous plate in a porous medium with slip condition taking into account the effects of Hall current.

Objective of the present investigation is to study the effects of Hall current and rotation on a hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an impulsively moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusion.

# FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider an unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an impulsively moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusions. Choose the coordinate system in such a way that x'-axis is along the plate in upward direction, y'-axis normal to the plane of the plate and z'-axis perpendicular to the x'y'-plane. The fluid is permeated by uniform transverse magnetic field  $B_0$  applied in a direction parallel to the y'-axis. Both fluid and plate are in rigid body rotation with uniform angular velocity  $\Omega$  about the y'-axis. Initially, i.e. at time  $t' \leq 0$ , both the fluid and plate are at rest and maintained at uniform temperature  $T'_{\infty}$ . Also the level of concentration of fluid is maintained at uniform concentration  $C'_{\infty}$ . At time t' > 0, the plate starts moving with uniform velocity  $u_0$  in x'direction against the gravitational field. At the same time the plate temperature is raised to uniform



Fig. 1. Geometry of the problem

temperature  $T'_w$  and the concentration at the surface of plate is raised to uniform concentration  $C'_w$ . The fluid considered is a gray, emitting-absorbing radiation but non-scattering medium. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate  $K'_2$  between the diffusing species and the fluid. Geometry of the problem is presented in Fig. 1.

Since the plate is of infinite extent along x' and z' directions and is electrically non-conducting, all physical quantities except pressure depend on y' and t' only. The induced magnetic field generated by fluid motion is neglected in comparison to the applied one, i.e., the magnetic field  $\vec{B} \equiv (0, B_0, 0)$ . This assumption is justified because the magnetic Reynolds number is very

small for liquid metals and partially ionized fluids [53]. Also, no external electric field is applied so the effect of polarization of fluid is negligible and we assume the electric field  $\vec{E} \equiv (0,0,0)$ . This corresponds to the case where no energy is added or extracted from the fluid by electrical means [53].

Taking into consideration the assumptions made above, the governing equations for natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid through a porous medium in the presence of thermal and mass diffusions taking Hall current into account, under Boussinesq approximation, in a rotating frame of reference are given by:

$$\frac{\partial u'}{\partial t'} + 2\Omega w' = \upsilon \frac{\partial^2 u'}{\partial {y'}^2} - \frac{\sigma B_0^2}{\rho \left(1 + m^2\right)} \left(u' + mw'\right) - \frac{\upsilon u'}{K_1'} + g\beta' \left(T' - T_{\infty}'\right) + g\beta^* \left(C' - C_{\infty}'\right),\tag{1}$$

$$\frac{\partial w'}{\partial t'} - 2\Omega u' = \upsilon \frac{\partial^2 w'}{\partial {y'}^2} + \frac{\sigma B_0^2}{\rho (1+m^2)} (mu' - w') - \frac{\upsilon w'}{K_1'}, \qquad (2)$$

$$\rho c_{p} \frac{\partial T'}{\partial t'} = k \frac{\partial^{2} T'}{\partial {y'}^{2}} - \frac{\partial q'_{r}}{\partial {y'}}, \tag{3}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial {y'}^2} - K_2' \left( C' - C_{\infty}' \right), \tag{4}$$

Where  $u', w', \upsilon, \rho, \sigma, m = \omega_e \tau_e, \omega_e, \tau_e, g, \beta', \beta^*, T', C', c_p, k, K'_1, q'_r$  and D

are fluid velocity in x'-direction, fluid velocity in z'-direction, kinematic coefficient of viscosity, fluid density, electrical conductivity, Hall current parameter, cyclotron frequency, electron collision

time, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of expansion for species concentration, fluid temperature, species concentration, specific heat at constant pressure, thermal conductivity of

diffusivity, respectively.

the fluid, permeability of the porous medium, radiative flux vector and chemical molecular Initial and boundary conditions to be satisfied are

The local radiant for the case of an optically thin gray gas Raptis [54] is expressed as

$$\frac{\partial q'_r}{\partial y'} = -4a^* \sigma^* \left(T_{\infty'}^{\prime 4} - T^{\prime 4}\right). \tag{6}$$

where  $a^*$  is absorption coefficient and  $\sigma^*$  is the Stefan Boltzmann constant.

It is assumed that the temperature difference within the fluid flow is sufficiently small so that fluid temperature  $T'^4$  may be expressed as a linear function of the temperature. This is accomplished  $T'^4 \cong 4T'^3_{\infty}T' - 3T'^4_{\infty}$ .

by expanding  $T'^4$  in a Taylor series about the free stream temperature  $T'_{\infty}$ . Neglecting second and higher order terms,  $T'^4$  is expressed as

(7)

Making use of eqns. (6) and (7) in eqn. (3), we obtain

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial {y'}^2} + 16a^* \sigma T_{\infty}^{\prime 3} \left( T_{\infty}' - T' \right).$$
(8)

Introducing non-dimensional quantities and parameters

$$y = y'/u_{0}t_{0}, u = u'/u_{0}, w = w'/u_{0}, t = t'/t_{0}, \theta = (T' - T'_{\infty})/(T'_{w} - T'_{\infty}),$$

$$C = (C' - C'_{\infty})/(C'_{w} - C'_{\infty}), K^{2} = \Omega \upsilon/u_{0}^{2}, M = \sigma B_{0}^{2} \upsilon/\rho u_{0}^{2}, G_{r} = \upsilon g \beta'(T'_{w} - T'_{\infty})/u_{0}^{3},$$

$$G_{c} = \upsilon g \beta^{*} (C'_{w} - C'_{\infty})/u_{0}^{3}, P_{r} = \rho \upsilon c_{p}/k, S_{c} = \upsilon/D, R = 16a^{*} \sigma \upsilon^{2} T'_{\infty}^{3}, K_{1} = K'_{1}u_{0}^{2}/\upsilon^{2}$$
and  $K_{2} = \upsilon K'_{2}/u_{0}^{2},$ 

$$(9)$$

eqns. (1), (2), (4) and (8), in non-dimensional form, become

$$\frac{\partial u}{\partial t} + 2K^2 w = \frac{\partial^2 u}{\partial y^2} - \frac{M(u + mw)}{(1 + m^2)} - \frac{u}{K_1} + G_r \theta + G_c C,$$
(10)

$$\frac{\partial w}{\partial t} - 2K^2 u = \frac{\partial^2 w}{\partial y^2} + \frac{M(mu - w)}{(1 + m^2)} - \frac{w}{K_1},\tag{11}$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - R\theta, \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - K_2 C,$$
(13)  
where

$$K^2$$
,  $M$ ,  $G_r$ ,  $G_c$ ,  $P_r$ ,  $S_c$ ,  $R$ ,  $K_1$ , and  $K_2$  are  
rotation parameter, magnetic parameter, thermal  
Grashof number, solutal Grashof number, Prandtl  
number, Schmidt number, radiation parameter,

$$\begin{array}{ll} t \leq 0 & : \ u = 0, \quad w = 0, \quad \theta = 0 \quad C = 0 \\ t > 0 & : \ u = 1, \quad w = 0, \quad \theta = 1, \quad C = 1 \\ & u \rightarrow 0, \quad w \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \end{array}$$

Combining eqns. (10) and (11), we obtain

permeability parameter and chemical reaction parameter, respectively.

Initial and boundary conditions (5), in nondimensional form, become

$$\begin{cases} \text{for all } y, \\ \text{at } y = 0, \\ \text{as } y \to \infty. \end{cases}$$
(14)

G. S. Seth et al.: Effects of Hall current and rotation on a unsteady MHD natural convection flow with heat

$$\frac{\partial F}{\partial t} - 2iK^2F = \frac{\partial^2 F}{\partial y^2} - NF - \frac{F}{K_1} + G_r\theta + G_cC,$$
(15)  
where  $F = u + iw$  and  $N = \frac{M(1 - im)}{(1 + m^2)}.$ 

Initial and boundary conditions (14), in compact form, are given by

$$t \le 0 \quad : F = 0, \quad \theta = 0 \quad C = 0 \quad \text{for all } y,$$
  

$$t > 0 \quad : F = 1, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0,$$
  

$$F \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad y \to \infty.$$
(16)

Eqns. (12), (13) and (15) subject to the initial and boundary conditions (16) are solved analytically with the help of Laplace transform technique. Exact solutions for fluid velocity F(y,t), fluid temperature  $\theta(y,t)$  and species concentration C(y,t) are presented in the following form after simplification

$$F(y,t) = \frac{1}{2} \left( 1 + \frac{a_{1}}{b_{1}} + \frac{a_{2}}{b_{2}} \right) \left\{ e^{y\sqrt{\rho}} \operatorname{erfc} \left( \sqrt{\beta t} + \frac{y}{2\sqrt{t}} \right) + e^{-y\sqrt{\rho}} \operatorname{erfc} \left( -\sqrt{\beta t} + \frac{y}{2\sqrt{t}} \right) \right\} + \frac{a_{1}}{2b_{1}} \left[ e^{b_{t}} \left\{ e^{y\sqrt{\rho}_{t}(\alpha+b_{1})} \operatorname{erfc} \left( \sqrt{(\alpha+b_{1})t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) + e^{-y\sqrt{\rho}_{t}(\alpha+b_{1})} \operatorname{erfc} \left( -\sqrt{(\alpha+b_{1})t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) + \frac{a_{1}}{2b_{1}} \left[ e^{b_{t}} \left\{ e^{y\sqrt{\rho}_{t}(\alpha+b_{1})} \operatorname{erfc} \left( \sqrt{(\alpha+b_{1})t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) + e^{-y\sqrt{\rho}_{t}(\alpha+b_{1})} \operatorname{erfc} \left( -\sqrt{(\alpha+b_{1})t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) \right] + \frac{a_{2}}{2\sqrt{t}} \left[ e^{b_{1}} \left\{ e^{y\sqrt{\rho}_{t}(\alpha+b_{1})t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) - e^{-y\sqrt{b_{1}+\beta}} \operatorname{erfc} \left( -\sqrt{(b_{1}+\beta)t} + \frac{y}{2\sqrt{t}} \right) \right\} - e^{y\sqrt{\rho}_{r}\alpha} \times \operatorname{erfc} \left( \sqrt{\alpha t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) - e^{-y\sqrt{\rho}_{r}\alpha} \operatorname{erfc} \left( -\sqrt{\alpha t} + \frac{y}{2}\sqrt{\frac{P_{r}}{t}} \right) \right] + \frac{a_{2}}{2b_{2}} \left[ e^{b_{2}t} \left\{ e^{y\sqrt{\delta}_{t}(K_{2}+b_{2})} \times \operatorname{erfc} \left( \sqrt{(K_{2}+b_{2})t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) + e^{-y\sqrt{S_{r}(K_{2}+b_{2})}} \operatorname{erfc} \left( -\sqrt{(K_{2}+b_{2})t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] + e^{-y\sqrt{S_{r}(K_{2}+b_{2})}} \operatorname{erfc} \left( -\sqrt{(K_{2}+\beta)t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) + e^{-y\sqrt{S_{r}(K_{2}+b_{2})}} \operatorname{erfc} \left( -\sqrt{(K_{2}+\beta)t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] + e^{-y\sqrt{S_{r}(K_{2}+b_{2})}} \operatorname{erfc} \left( -\sqrt{(K_{2}+\beta)t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] + e^{-y\sqrt{S_{r}(K_{2}+b_{2})}} \operatorname{erfc} \left( -\sqrt{(K_{2}+\beta)t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] + e^{-y\sqrt{S_{r}(K_{2}+b_{2})}} \operatorname{erfc} \left( -\sqrt{(K_{2}+\beta)t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] + e^{-y\sqrt{S_{r}(K_{2}+\beta)t}} \operatorname{erfc} \left( -\sqrt{K_{2}t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] \right] + e^{-y\sqrt{S_{r}(K_{2}+\beta)t}} \operatorname{erfc} \left( -\sqrt{K_{2}t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] + e^{-y\sqrt{S_{r}(K_{2}+\beta)t}} \operatorname{erfc} \left( -\sqrt{\alpha t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] \right] + e^{-y\sqrt{S_{r}(K_{2}+\beta)t}} \operatorname{erfc} \left( -\sqrt{\alpha t} + \frac{y}{2}\sqrt{\frac{S_{r}}{t}} \right) \right] \right] \left[ e^{y\sqrt{\rho}} + e^{y\sqrt{\rho}}$$

$$C(y,t) = \frac{1}{2} \left\{ e^{y\sqrt{S_cK_2}} \operatorname{erfc}\left(\sqrt{K_2t} + \frac{y}{2}\sqrt{\frac{S_c}{t}}\right) + e^{-y\sqrt{S_cK_2}} \operatorname{erfc}\left(-\sqrt{K_2t} + \frac{y}{2}\sqrt{\frac{S_c}{t}}\right) \right\}.$$
(19)

where  $\alpha = R / P_r$ ,  $a_1 = G_r / (1 - P_r)$ ,  $\beta = \left(N + \frac{1}{K_1} - 2iK^2\right)$ ,  $a_2 = G_c / (1 - S_c)$ ,  $b_1 = (P_r \alpha - \beta) / (1 - P_r)$ ,  $b_2 = (S_c K_2 - \beta) / (1 - S_c)$ .

## SOLUTION IN THE CASE OF UNIT PRANDTL NUMBER AND UNIT SCHMIDT NUMBER

The solution (17) for fluid velocity is not valid for fluids with  $P_r = 1$  and  $S_c = 1$ . Since Prandtl number  $P_r$  is a measure of the relative strength of viscosity to thermal conductivity of the fluid and Schmidt number  $S_c$  is a measure of the relative strength of viscosity to chemical molecular diffusivity of the fluid, the case

 $P_r = 1$  and  $S_c = 1$  corresponds to those fluids for which viscous, thermal and concentration boundary layer thicknesses are of same order of magnitude. There are some fluids of practical interest which belong to this category [55]. Setting  $P_r = 1$  and  $S_c = 1$  in eqns. (12) and (13) and following the same procedure as before, exact solutions for fluid velocity F(y,t), fluid temperature  $\theta(y,t)$  and species concentration C(y,t) are obtained and presented in the following form

$$F(y,t) = \frac{\left(1 - \delta_{1} - \delta_{2}\right)}{2} \left\{ e^{y\sqrt{\beta}} \operatorname{erfc}\left(\sqrt{\beta t} + \frac{y}{2\sqrt{t}}\right) + e^{-y\sqrt{\beta}} \operatorname{erfc}\left(-\sqrt{\beta t} + \frac{y}{2\sqrt{t}}\right) \right\} + \frac{\delta_{1}}{2} \left\{ e^{y\sqrt{\alpha}} \operatorname{erfc}\left(\sqrt{\alpha t} + \frac{y}{2\sqrt{t}}\right) + e^{-y\sqrt{\alpha}} \operatorname{erfc}\left(-\sqrt{\alpha t} + \frac{y}{2\sqrt{t}}\right) \right\} + \frac{\delta_{2}}{2} \left\{ e^{y\sqrt{K_{2}}} \operatorname{erfc}\left(\sqrt{K_{2}t} + \frac{y}{2\sqrt{t}}\right) + e^{-y\sqrt{K_{2}}} \operatorname{erfc}\left(-\sqrt{K_{2}t} + \frac{y}{2\sqrt{t}}\right) \right\},$$

$$(20)$$

$$\theta(y,t) = \frac{1}{2} \left\{ e^{y\sqrt{\alpha}} \operatorname{erfc}\left(\sqrt{\alpha t} + \frac{y}{2\sqrt{t}}\right) + e^{-y\sqrt{\alpha}} \operatorname{erfc}\left(-\sqrt{\alpha t} + \frac{y}{2\sqrt{t}}\right) \right\},\tag{21}$$

$$C(y,t) = \frac{1}{2} \left\{ e^{y\sqrt{K_2}} erfc\left(\sqrt{K_2 t} + \frac{y}{2\sqrt{t}}\right) + e^{-y\sqrt{K_2}} erfc\left(-\sqrt{K_2 t} + \frac{y}{2\sqrt{t}}\right) \right\},$$
(22)  
where  $\delta_t = \frac{G_r}{1 + \frac{1}{2}}$  and  $\delta_t = \frac{G_c}{1 + \frac{1}{2}}$ 

where  $\delta_1 = \frac{G_r}{(\beta - \alpha)}$  and  $\delta_2 = \frac{G_c}{(\beta - K_2)}$ .

It is noticed from the solutions (18) and (19) that the solutions (21) and (22) may also be directly obtained by setting  $P_r = 1$  and  $S_c = 1$  in the solutions (18) and (19).

#### SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The expressions for the primary skin friction  $\tau_x$ , secondary skin friction  $\tau_y$ , Nusselt number  $N_u$  and Sherwood number  $S_h$ , which are measures of shear stress at the plate due to primary flow, shear stress at the plate due to secondary flow, rate of heat transfer at the plate and rate of mass transfer at the plate respectively, are presented in the following form

$$\tau_{x} + i\tau_{y} = \left(1 + \frac{a_{1}}{b_{1}} + \frac{a_{2}}{b_{2}}\right) \left\{ \sqrt{\beta} \left( erfc\left(\sqrt{\beta t}\right) - 1 \right) - \frac{1}{\sqrt{\pi t}} e^{-\beta t} \right\} + \frac{a_{1}}{b_{1}} \left[ e^{b_{1}t} \left\{ \sqrt{P_{r}\left(\alpha + b_{1}\right)} \times \left( erfc\left(\sqrt{\alpha + b_{1}}\right)t \right) - 1 \right) - \sqrt{\frac{P_{r}}{\pi t}} e^{-(\alpha + b_{1})t} - \sqrt{(\beta + b_{1})} \left( erfc\left(\sqrt{(\beta + b_{1})t}\right) - 1 \right) + \frac{1}{\sqrt{\pi t}} e^{-(\beta + b_{1})t} \right\} + \left( -\sqrt{P_{r}\alpha} \left( erfc\left(\sqrt{\alpha t}\right) - 1 \right) + \sqrt{\frac{P_{r}}{\pi t}} e^{-\alpha t} \right] + \frac{a_{2}}{b_{2}} \left[ e^{b_{2}t} \left\{ \sqrt{S_{c}\left(K_{2} + b_{2}\right)t} \left( erfc\left(\sqrt{(K_{2} + b_{2})t}\right) - 1 \right) + \left( -\sqrt{\frac{S_{c}}{\pi t}} e^{-(K_{2} + b_{2})t} - \sqrt{(b_{2} + \beta)} \left( erfc\left(\sqrt{(b_{2} + \beta)t}\right) - 1 \right) + \frac{1}{\sqrt{\pi t}} e^{-(b_{2} + \beta)t} \right\} + \left( -\sqrt{S_{c}K_{2}} \left( erfc\left(\sqrt{K_{2}t}\right) - 1 \right) + \sqrt{\frac{S_{c}}{\pi t}} e^{-K_{2}t} \right],$$

$$(23)$$

$$N_{u} = \sqrt{P_{r}\alpha} \left( erfc\left(\sqrt{\alpha t}\right) - 1 \right) - \sqrt{\frac{P_{r}}{\pi t}} e^{-\alpha t},$$
(24)

$$S_{h} = \sqrt{S_{c}K_{2}} \left( erfc\left(\sqrt{K_{2}t}\right) - 1 \right) - \sqrt{\frac{S_{c}}{\pi t}} e^{-K_{2}t}$$

#### **RESULTS AND DISCUSSION**

In order to highlight the influence of rotation, Hall current, thermal buoyancy force, concentration buoyancy force, magnetic field, radiation, chemical reaction and mass diffusion on the flow-field in the boundary layer region, numerical values of primary and secondary fluid velocities, computed from the analytical solution (17), are displayed graphically versus boundary layer co-ordinate y in figures 2 to 9 for various values of  $K^2$ , m, ,  $G_r$ ,  $G_c$ , M, R,  $K_2$  and  $S_c$  taking  $K_1 = 0.2, P_r = 0.71$  and t = 0.2. It is evident from figures 2 to 9 that the primary velocity u and secondary velocity w attain a distinctive maximum

value in the vicinity of the surface of the plate and then decrease properly on increasing boundary layer coordinate y to approach free stream velocity. Fig. 2 shows the effect of rotation on the primary and secondary fluid velocities. It is revealed from Fig. 2 that primary fluid velocity u decreases whereas secondary fluid velocity w increases on increasing  $K^2$ . This implies that rotation tends to retard fluid flow in the primary flow direction whereas it has a reverse effect on fluid flow in the secondary flow direction. This may be attributed to the fact that when the frictional layer near the moving plate is suddenly set into motion, then the Coriolis force acts as a constraint in the main fluid flow, i.e. primary flow to induce cross flow, i.e. secondary flow in the flow-field. Figs. 3 to 5 demonstrate the influence of Hall current, thermal Grashof number and solutal Grashof number on the primary and secondary fluid velocities. It is noticed from Figs. 3 to 5 that both primary and secondary increase fluid velocities on increasing either m or  $G_r$  or  $G_c$ . This implies that Hall current, thermal buoyancy force and concentration buoyancy force tend to accelerate fluid flow in both the primary and secondary flow directions. Figs. 6 to 9 display the effects of magnetic field, radiation, chemical reaction and mass diffusion on the (25)

primary and secondary fluid velocities. It is revealed from Figs. 6 to 9 that both primary and secondary fluid velocities decrease on increasing either M or R or  $K_2$  or  $S_c$ . This implies that magnetic field, radiation and chemical reaction tend to retard fluid flow in both the primary and secondary flow directions. Since Schmidt number  $S_c$  is a measure of relative strength of viscosity to chemical molecular diffusivity,  $S_c$  decreases on increasing chemical molecular diffusivity of the fluid. This implies that mass diffusion tends to accelerate fluid flow in both the primary and secondary flow directions.

The numerical solutions for fluid temperature and species concentration, computed from the analytical solutions (18) and (19), are depicted graphically in Figs. 10 to 13 for various values of radiation parameter R, Prandtl number  $P_r$ , chemical  $K_2$  and reaction parameter Schmidt number  $S_c$  taking time t = 0.2. It is evident from Figs. 10 to 13 that fluid temperature  $\theta$  and fluid concentration C are maximum at the surface of the plate and decrease properly on increasing boundary layer coordinate y to approach the free stream value. Figs. 10 and 11 illustrate the effects of thermal diffusion fluid radiation and on temperature. It is seen from Figs. 10 and 11 that fluid temperature  $\theta$  decreases on increasing either R or  $P_r$  in the boundary layer region. Since  $P_r$  is a measure of the relative strength of viscosity to thermal diffusivity,  $P_r$  decreases on increasing thermal diffusivity. This implies that radiation tends to reduce fluid temperature whereas thermal diffusion has a reverse effect on it. Figs. 12 and 13 demonstrate the effects of chemical reaction and mass diffusion on species concentration. It is noticed from Figs. 12 and 13 that species С concentration decreases on increasing either  $K_2$  or  $S_c$ . This implies that chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it.



Fig. 2: Velocity profiles when m = 1.5,  $G_r = 4$ ,  $G_c = 5$ , M = 10, R = 1,  $K_2 = 0.2$  and  $S_c = 0.22$ 



Fig. 4: Velocity profiles when m = 1.5,  $K^2 = 4$ ,  $G_c = 5$ , M = 10, R = 1,  $K_2 = 0.2$  and  $S_c = 0.22$ 



Fig. 6: Velocity profiles when m = 1.5,  $K^2 = 4$ ,  $G_r = 4$ ,  $G_c = 5$ , R = 1,  $K_2 = 0.2$  and  $S_c = 0.22$ 



Fig. 3: Velocity profiles when  $K^2 = 4$ ,  $G_r = 4$ ,  $G_c = 5$ , M = 10, R = 1,  $K_2 = 0.2$  and  $S_c = 0.22$ .



Fig. 5: Velocity profiles when m = 1.5,  $K^2 = 4$ ,  $G_r = 4$ , M = 10, R = 1,  $K_2 = 0.2$  and  $S_c = 0.22$ 



Fig. 7: Velocity profiles when m = 1.5,  $K^2 = 4$ ,  $G_r = 4$ ,  $G_c = 5$ , M = 10,  $K_2 = 0.2$  and  $S_c = 0.22$ 



Fig. 8: Velocity profiles when m = 1.5,  $K^2 = 4$ ,  $G_r = 4$ ,  $G_c = 5$ , M = 10, R = 1 and  $S_c = 0.22$ 



Fig. 10: Temperature profiles when  $P_r = 0.71$ .



Fig. 12: Concentration profiles when  $S_c = 2$ .

The numerical values of primary skin friction  $\tau_x$  and secondary skin friction  $\tau_z$ , computed from the analytical expression (23), are presented in tabular form in Tables 1 to 5 for various values of  $m, K^2, K_2, S_c, G_r, G_c, M, K_1, R$  and  $P_r$  taking t = 0.2. It is observed from Table 1 that primary skin friction, i.e.  $\tau_x$  decreases on increasing m. With an increase in m, secondary skin friction, i.e.  $\tau_z$  increases, attains a maximum and then decreases when  $K^2 = 2$ ; it increases when



Fig. 9: Velocity profiles when m = 1.5,  $K^2 = 4$ ,  $G_r = 4$ ,  $G_c = 5$ , M = 10, R = 1 and  $K_2 = 0.2$ 



Fig. 13: Concentration profiles when  $K_2 = 0.2$ .

 $K^2 \ge 4$ .  $\tau_x$  and  $\tau_z$  increase on increasing  $K^2$ . This implies that Hall current tends to reduce primary skin friction whereas it has a reverse effect on secondary skin friction when  $K^2 \ge 4$ . Rotation tends to enhance both the primary and secondary skin frictions. It is evident from Table 2 that  $\tau_x$  increases whereas  $\tau_z$  decreases on increasing  $S_c$ .  $\tau_x$  decreases whereas  $\tau_z$  increases on increasing  $K_2$ . This implies that mass diffusion and chemical reaction tend to reduce primary skin friction whereas these have reverse effect on secondary skin G. S. Seth et al.: Effects of Hall current and rotation on a unsteady MHD natural convection flow with heat and mass transfer...

when $K_1 = 0.3, K_2 = 0.2, M = 10, R = 1, P_r = 0.71, G_r = 4, S_c = 0.22$ and $G_c = 5$ .								
L2		$- au_x$			$ au_z$			
$m \downarrow K^2 \rightarrow -$	2	4	6	2	4	6		
0.5	2.4971	2.53666	2.70599	0.890639	1.49983	2.07509		
1.0	1.98013	2.08687	2.35844	1.03103	1.77994	2.4084		
1.5	1.56789	1.70466	2.04754	0.929534	1.8237	2.51996		

**Table 1:** Primary and secondary skin frictions

Table 2: Primary and secondary skin frictions

when $K^2 = 4, m = 1.5, K_1 = 0.3, M = 10, R = 1, P_r = 0.71, G_r = 4$ and $G_c = 1$	5.
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	$- au_x$			$ au_z$			
$K_2 \downarrow S_c \rightarrow -$	0.16	0.22	0.66	0.16	0.22	0.66	
0.2	1.652	1.70466	1.96087	1.9125	1.8237	1.41646	
2.0	1.62962	1.67487	1.87401	1.98757	1.91258	1.56383	
5.0	1.61981	1.65922	1.80761	2.05997	2.00076	1.73388	

friction. It is noticed from Table 3 that  $\tau_z$  decreases on increasing  $G_r$  whereas it increases on increasing  $G_c$ .  $\tau_x$  decreases on increasing either  $G_r$  or  $G_c$ when  $G_c \leq 5$ . This implies that thermal buoyancy force tends to reduce secondary skin friction whereas concentration buoyancy force has a reverse effect on it. Thermal buoyancy force and concentration buoyancy force tend to reduce primary skin friction when  $G_c \leq 5$ . It is worth noting that there exists flow separation at the plate on increasing  $G_c$  when  $G_r = 4$  and 6 and on increasing  $G_r$  when  $G_c = 7$ . It is observed from Table 4 that  $\tau_x$  and  $\tau_z$ increase on increasing *M*.  $\tau_x$  decreases whereas  $\tau_z$ increases on increasing *K*<sub>1</sub>. This implies that magnetic field tends to enhance both primary and secondary skin frictions. Permeability of medium tends to reduce primary skin friction whereas it has a reverse effect on secondary skin friction. It is revealed from Table 5 that  $\tau_x$  and  $\tau_z$  decrease on increasing  $P_r$ .  $\tau_x$  increases on increasing *R* when  $P_r = 0.3$  and it decreases, attains a minimum and then increases on increasing *R* when  $P_r \ge 0.5$ .  $\tau_z$ 

Table 3: Primary and secondary skin frictions
when $K^2 = 4, m = 1.5, K_1 = 0.3, K_2 = 0.2, M = 10, R = 1, P_r = 0.71$ and $S_c = 0.22$ .

~   ~	$- au_x$			$ au_z$			
$G_r \downarrow G_c \rightarrow -$	3	5	7	3	5	7	
2	1.98143	1.07968	0.177923	1.93213	1.99615	2.06018	
4	1.70466	0.802902	-0.09885	1.8237	1.88773	1.95176	
6	1.42788	0.526128	-0.37562	1.71528	1.77931	1.84334	

Table 4: Primary and secondary skin frictions

when $K^2 =$	=4, m=1.	$5, K_2 =$	0.2, R =	$=1, P_r =$	$0.71, G_r$	$=4, S_{c}$	= 0.22 and	$G_c =$	: 5.
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	$- au_x$			$ au_z$		
$M \downarrow K_1 \rightarrow -$	0.1	0.3	0.5	0.1	0.3	0.5
10	2.80552	1.70466	1.42718	1.53869	1.82370	1.92827
15	3.08355	2.14366	1.92434	1.78957	2.12677	2.23223
20	3.35574	2.53327	2.35185	2.00405	2.35463	2.45468

when $K^2 = 4, m = 1.5, M = 10, K_1 = 0.3, K_2 = 0.2, G_r = 4, S_c = 0.22$ and $G_c = 5$ .								
_ 1 _	$ au_z$							
$R \downarrow P_r \rightarrow -$	0.3	0.5	0.71	0.3	0.5	0.71		
1	1.54040	1.6280	1.70466	2.14126	1.96731	1.82370		
5	1.58586	1.59735	1.61619	2.25045	2.17324	2.06695		
9	1.65566	1.65262	1.64707	2.22303	2.2025	2.15664		

 Table 5: Primary and secondary skin frictions

<b>Table 6:</b> Nusselt Number $N_u$ when $t = 0.2$ .								
$N_u$								
$R \downarrow P_r \rightarrow$	0.3	0.5	0.71	7.0				
1	1.10655	1.22687	1.34915	3.43270				
3	1.74676	1.79058	1.85335	3.61987				
5	2.23875	2.25505	2.28743	3.80358				

increases on increasing *R* when  $P_r \ge 0.5$  and it increases, attains a maximum and then decreases on increasing *R* when  $P_r = 0.3$ . This implies that thermal diffusion tends to enhance both the primary and secondary skin frictions. Radiation tends to enhance secondary skin friction when  $P_r \ge 0.5$  and

it tends to enhance primary skin friction when  $P_r = 0.3$ .

The numerical values of Nusselt number  $N_u$ , computed from the analytical expression (24), are presented in Table 6 for various values of R and  $P_r$  whereas that of Sherwood number  $S_h$ , computed from the analytical expression (25), are presented in Table 7 for different values of  $K_2$  and  $S_c$  taking

**Table 7:** Sherwood Number  $S_h$  when t = 0.2

	$S_h$							
$K_2 \downarrow S_c \rightarrow$	2	2.5	3	3.5				
0.2	1.85502	2.07397	2.27192	2.45396				
2	2.45375	2.74337	3.00521	3.246				
5	3.3212	3.71321	4.06762	4.39353				

*t*=0.2. It is noticed from Table 6 that Nusselt number *Nu* increases on increasing either *R* or  $P_r$ . This implies that radiation tends to enhance the rate of heat transfer at the plate whereas thermal diffusion has a reverse effect on it. It is found from Table 7 that Sherwood number  $S_h$  increases on increasing either  $K_2$  or  $S_c$ . This implies that chemical reaction tends to enhance the rate of mass transfer at the plate whereas mass diffusion has a reverse effect on it.

#### CONCLUSIONS

This study presents an investigation of the effects of Hall current and rotation on hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an impulsively moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusion. The significant findings are summarized below:

Rotation tends to retard fluid flow in the primary flow direction whereas it has a reverse effect on the fluid flow in the secondary flow direction. Hall current, thermal buoyancy force, concentration buoyancy force and mass diffusion tend to accelerate fluid flow in both primary and secondary flow directions. Magnetic field, radiation and chemical reaction tend to retard fluid flow in both primary and secondary flow directions. Radiation has a tendency to reduce fluid temperature whereas thermal diffusion has a reverse effect on it. Chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it. Hall current tends to reduce primary skin friction whereas it has a reverse effect on secondary skin friction when  $K^2 \ge 4$ . Rotation tends to enhance both primary and secondary skin frictions. Mass diffusion and chemical reaction tend to reduce primary skin friction whereas these have

reverse effect on secondary skin friction. Thermal buoyancy force tends to reduce secondary skin friction whereas concentration buoyancy force has a reverse effect on it. Thermal buoyancy force and concentration buoyancy force tend to reduce primary skin friction when  $G_c \leq 5$ . Magnetic field tends to enhance both primary and secondary skin frictions. Permeability of medium tends to reduce primary skin friction whereas it has a reverse effect on secondary skin friction. Thermal diffusion tends to enhance both primary and secondary skin frictions. Radiation tends to enhance secondary skin friction when  $P_r \ge 0.5$  and it tends to enhance primary skin friction when  $P_r = 0.3$ . Radiation tends to enhance the rate of heat transfer at the plate whereas thermal diffusion has a reverse effect on it. Chemical reaction tends to enhance the rate of mass transfer at the plate whereas mass diffusion has a reverse effect on it.

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### ЕФЕКТИ НА ТОКОВЕТЕ НА НАLL И НА ВЪРТЕНЕТО В НЕСТАЦИОНАРНО МАГНИТО-ХИДРОДИНАМИЧНО ТЕЧЕНИЕ С ЕСТЕСТВЕНИ КОНВЕКЦИИ И ТОПЛО И МАСОПРЕНАСЯНЕ ЗАД ИМПУЛСИВНО ДВИЖЕЩА СЕ ВЕРТИКАЛНА ПЛОСКОСТ ПРИ ИЗЛЪЧВАНЕ И ХИМИЧНА РЕАКЦИЯ

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#### (Резюме)

Изучени са ефектите на токовете на Hall и на въртенето в нестационарен магнито-хидродинамичен поток с естествени конвекции и топло-масопренасяне зад импулсивно движеща се вертикална пластина. Отчетени са термичната и молекулярната дифузия. Получени са точни решения в затворен вид на хидродинамичните уравнения, уравненията на топлопроводността и на конвективната дифузия с помощта на Лапласова трансформация в прибилжението на Boussinesq. Получени са уравнения за коефициента на триене, числата на Nusselt и на Sherwood. Графично са представени скоротните профили, разпределението на температурата и на веществото, а коефициента на триене и числата на Nusselt и на Sherwood са показани в табличен вид в зависимост от параметрите на течението..