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Holographic mesons in Pilch-Warner geometry

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In this paper we study the D7 probe brane scalar fluctuations in global Pilch-Warner background geometry. We chose to work with simple constant solutions that solve the classical embedding equations. This allows one to fix the position of the brane in space and considerably simplify the analysis of its fluctuations. The corresponding meson spectra, obtained by the fluctuations along the transverse directions, admit equidistant structure for the higher modes and a ground state given by the conformal dimension of the operator dual to the fluctuations.

Key words: AdS/CFT correspondence, gauge/gravity correspondence, holographic mesons

INTRODUCTION

The AdS/CFT correspondence is a magnificent duality relating 10 dimensional IIB string theory in the weak coupling regime to a 4 dimensional $SU(N)$ gauge field theory with strong coupling constant, and vice-versa. In this case the gauge field theory lives on the boundary of the spacetime where the strings move. This correspondence gives us the opportunity to study non-perturbative phenomena in Yang-Mills theory with tools available in the classical superstring theory and supergravity.

On the gauge theory side of the original Maldacena setup [1] there is a theory with huge amount of symmetry ($\mathcal{N} = 4$ supersymmetric Yang-Mills theory), and on the string side of the correspondence we have a stack of N $D3$-branes, so that both endpoints of the strings are attached to the same stack of $D3$-branes, therefore the corresponding states transforms in the adjoint representation of the gauge group. Adding flavours in the setup is achieved by introducing a separate stack of N_f $D7$ probe branes [2]. Here, the $SU(N_f)$ is a global flavor symmetry. If we consider the case of $N_f \ll N_c$, where N_c is the number of the $D3$-branes, and further take N_c large, then we have a strongly coupled dual gauge theory, and a stack of $D3$-branes, which is the source of the background geometry, but in the limit of large N_c it can effectively be replaced by $AdS_5 \times S^5$. In this setup we can study the N_f $D7$-branes in the probe limit. The strings stretched between the two stacks of branes have finite length and thus finite energy, so that the quark mass is given by the separation distance and the string tension: $m_q = L/2\pi\alpha'$.

Although variety of meson spectra were found [3, 4] (for a review see [5]), there is still too much supersymmetry we have to deal with in order to achieve string theory description of QCD and the Standard model. To produce more realistic QCD like string theories, deformations of the initial $AdS_5 \times S^5$ geometry [6, 7], or introducing external magnetic or electric fields [8–11], have to be considered. Such configurations will break the supersymmetry and theories with less supersymmetry will emerge. In this context Pilch-Warner geometry [12, 13] is a fine example of such deformed geometry. It is a solution of five-dimensional $\mathcal{N} = 8$ gauged supergravity lifted to ten dimensions, which, in its infrared critical point, preserves 1/4 of the original supersymmetry.

GENERAL SETUP

Pilch-Warner geometry

Pilch-Warner geometry is a solution of five-dimensional $\mathcal{N} = 8$ gauged supergravity lifted to ten dimensions. This geometry interpolates between the maximally supersymmetric $AdS_5 \times S^5$ in the ultraviolet critical point and warped AdS_5 times squashed five-sphere in the infrared point (IR). In this study we will restrict ourself to the IR critical point. An interesting feature on the gravity side is that it preserves 1/8 supersymmetry everywhere while at IR fixed point it is enhanced to 1/4. On the SYM side the IR fixed point corresponds to large N limit of the superconformal $\mathcal{N} = 1$ theory of Leigh-Strassler [15]. The ten-dimensional Pilch-Warner metric is given by

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$$\mathrm{d}s^2 = \mathrm{d}s_{1,4}^2 + \mathrm{d}s_5^2\,, \tag{1}$$

where we have the warped AdS_5 metric

$$\mathrm{d}s_{1,4}^2 = \Omega^2(e^{2A}\,\mathrm{d}s^2(\mathbb{M}^4) + \mathrm{d}r^2)\,, \tag{2a}$$

and the squashed five-sphere metric

$$\begin{aligned}\mathrm{d}s_5^2 =& L_0^2\frac{X^{1/2}\mathrm{sech}\chi}{\overline{\rho}^3}\Big[\mathrm{d}\theta^2 + \frac{\overline{\rho}^6\cos^2\theta}{X}(\sigma_1^2+\sigma_2^2)\\ &+\frac{\overline{\rho}^{12}\sin^2 2\theta}{4X^2}\Big(\sigma_3 + \frac{2-\overline{\rho}^6}{2\overline{\rho}^6}\mathrm{d}\phi\Big)^2 + \frac{\overline{\rho}^6\cosh^2\chi}{16X^2}\\ &\times(3-\cos 2\theta)^2\Big(\mathrm{d}\phi - \frac{4\cos^2\theta}{3-\cos 2\theta}\sigma_3\Big)^2\Big]. \end{aligned}\tag{2b}$$

This is an example of warped geometry where the warp factor Ω is given by: $\Omega^2 = X^{1/2}\cosh\chi/\overline{\rho}$, and the function $X(r,\theta) = \cos^2\theta + \overline{\rho}^6\sin^2\theta$. Our left-invariant one-forms satisfy $\mathrm{d}\sigma_i = \varepsilon_{ijk}\sigma_j\wedge\sigma_k$, so that $\mathrm{d}\widetilde{\Omega}_3^2 = \sigma_i\sigma_i$ is the metric on the unit 3-sphere. In global coordinates they take the following form:

$$\begin{aligned}\sigma_1 &= \frac{1}{2}(\sin\beta\mathrm{d}\alpha - \cos\beta\sin\alpha\,\mathrm{d}\gamma),\\ \sigma_2 &= -\frac{1}{2}(\cos\beta\,\mathrm{d}\alpha + \sin\beta\sin\alpha\,\mathrm{d}\gamma),\\ \sigma_3 &= \frac{1}{2}\,(\mathrm{d}\beta + \cos\alpha\,\mathrm{d}\gamma)\,.\end{aligned}\tag{3}$$

At the IR point $r\to-\infty$, $\chi = \mathrm{arccosh}\left(\frac{2}{\sqrt{3}}\right)$, $\overline{\rho} = 2^{1/6}$, and $A(r) = r/L$. The AdS radius L is given in terms of the AdS radius L_0 of the UV spacetime by $L = (3/2^{5/3})L_0$. One finds that at the IR point the metric takes the form

$$\mathrm{d}s_{1,4}^2 = \Omega^2(e^{\frac{2r}{L}}\,\mathrm{d}s^2(\mathbb{M}^4) + \mathrm{d}r^2)\,, \tag{4a}$$

and the squashed five-sphere metric

$$\begin{aligned}\mathrm{d}s_5^2 =& \frac{2}{3}L^2\Omega^2\Big[\mathrm{d}\theta^2 + \frac{4\cos^2\theta}{3-\cos 2\theta}(\sigma_1^2+\sigma_2^2)\\ &+\frac{4\sin^2 2\theta}{(3-\cos 2\theta)^2}\sigma_3^2 + \frac{2}{3}\Big(\mathrm{d}\phi - \frac{4\cos^2\theta}{3-\cos 2\theta}\sigma_3\Big)^2\Big].\end{aligned}\tag{4b}$$

As shown in [14] there is a natural global $U(1)_\beta$ action $\beta\to\beta + const$ which rotates σ_1 into σ_2 and leaves σ_3 invariant. We adopt the set up where the S^3 Euler angle $\beta\to\beta+2\phi$ is shifted to give a solution with a global $U(1)_R = U(1)_\phi$ symmetry. Performing this coordinate transformation on the solution (4a) and (4b) we arrive at the final result for the Pilch-Warner metric in global coordinates

$$\mathrm{d}s_{1,4}^2(IR) = L^2\Omega^2(-\cosh^2\rho\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\rho\mathrm{d}\Omega_3^2), \tag{5a}$$

$$\begin{aligned}\mathrm{d}s_5^2(IR) =& \frac{2}{3}L^2\Omega^2\Big[\mathrm{d}\theta^2 + \frac{4\cos^2\theta}{3-\cos 2\theta}(\sigma_1^2+\sigma_2^2)\\ &+\frac{4\sin^2 2\theta}{(3-\cos 2\theta)^2}(\sigma_3+\mathrm{d}\phi)^2 + \frac{2}{3}\Big(\frac{1-3\cos 2\theta}{\cos 2\theta - 3}\Big)^2\\ &\times\Big(\mathrm{d}\phi - \frac{4\cos^2\theta}{1-3\cos 2\theta}\sigma_3\Big)^2\Big],\end{aligned}\tag{5b}$$

where $\mathrm{d}\Omega_3^2 = \mathrm{d}\phi_1^2 + \sin^2\phi_1(\mathrm{d}\phi_2^2 + \sin^2\phi_2\mathrm{d}\phi_3^2)$ is the metric on the 3-sphere, and

$$\Omega^2 = \frac{2^{1/3}}{\sqrt{3}}\sqrt{3-\cos(2\theta)} \tag{6}$$

is the warp factor at the IR point.

R-R and NS-NS potentials

The Pilch-Warner background includes non-trivial Ramond-Ramond (R-R) and Neveu-Schwarz (NS-NS) form fields entering the $D7$ probe brane action. The full action is given by two terms – a Dirac-Born-Infeld (DBI) term and a Wess-Zumino (WZ) term [16, 17]

$$\begin{aligned}S_{D_7} &= S_{DBI} + S_{WZ}\\ &= -T_7\int d^8\xi e^{-\Phi}\sqrt{-\det(P[G]+\mathscr{F})}\\ &\quad -T_7\int\Big(P[C_8] - P[C_6]\wedge\mathscr{F} + \tfrac{1}{2}P[C_4]\wedge\mathscr{F}\wedge\mathscr{F} + \cdots\Big),\end{aligned}\tag{7}$$

where $\mathscr{F} = P[B_2] + 2\pi\alpha' F$, T_7 is the $D7$-brane tension, Φ is the dilaton, F is the worldvolume gauge field, B_2 is the Kalb-Ramond 2-form, and P denotes the pullback of the bulk spacetime tensor to the worldvolume of the brane:

$$P[G]_{ab} = G_{AB}\frac{\partial X^A}{\partial\xi^a}\frac{\partial X^B}{\partial\xi^b}\,. \tag{8}$$

The indices $a,b = 0,\ldots,7$ span the world volume of the $D7$-brane, while $A,B = 0,\ldots,9$ span the whole spacetime. We will consider $D7$-brane embeddings in static gauge where θ and ϕ directions are transverse to the brane

$$\begin{aligned}&\xi^a = (\tau,\rho,\phi_1,\phi_2,\phi_3,\alpha,\beta,\gamma),\\ &a = 0,\ldots,7,\quad \phi = \phi(\rho),\quad \theta = \theta(\rho)\,.\end{aligned}\tag{9}$$

In static gauge the pullback is given by

$$P[G]_{ab} = g_{ab} + G_{mn}\frac{\partial X^m}{\partial \xi^a}\frac{\partial X^n}{\partial \xi^b}\,, \qquad (10)$$

where g_{ab} is the induced metric on $D7$, and G_{mn} $(m,n=\theta,\phi)$ are the metric components in front of the transverse coordinates governing the $D7$-brane fluctuations. D–branes carry an R-R charge which due to charge conservation turns them into stable objects. In the IIB string theory the R-R potentials are C_0, C_2, C_4, C_6, C_8 and corresponding field strengths, which satisfy certain Bianchi identities and equations of motion [16]

$$\Phi = C_0 = 0, \quad F_1 = dC_0 = 0\,, \qquad (11a)$$

$$C_2 = \Re e(A_2), \quad B_2 = \Im m(A_2)\,, \qquad (11b)$$

$$H_3 = dB_2, \quad F_3 = dC_2 - C_0\wedge H_3 = dC_2\,, \qquad (11c)$$

$$\mathrm{d}F_3 = \mathrm{d}H_3 = 0, \quad \mathrm{d}F_5 = H_3\wedge F_3\,, \qquad (11d)$$

$$\mathrm{d}(\star F_3) = -H_3\wedge F_5,\ \mathrm{d}(\star H_3) = F_3\wedge F_5,\ F_5 = \star F_5, \qquad (11e)$$

$$dC_4 + d\widetilde{C}_4 = F_5 + C_2\wedge H_3\,, \qquad (11f)$$

$$F_7 = \star F_3 = dC_6 - C_4\wedge H_3\,, \qquad (11g)$$

$$F_9 = \star F_1 = 0 = dC_8 - C_6\wedge H_3 = C_6\wedge H_3,\ \ \chi = C_8 = 0\,. \qquad (11h)$$

Here the field strengths are defined in terms of the corresponding potentials as

$$H_3 \equiv \mathrm{d}B_2, \quad F_{\mathrm{p}} \equiv \mathrm{d}C_{\mathrm{p}-1} - C_{\mathrm{p}-3}\wedge H_3\,. \qquad (12)$$

In this setup the axion/dilaton system of scalars (11a) and (11h) is trivial along the flow. We also have an ansatz for the self-dual five form

$$F_5 = -\frac{2^{5/3}}{3}L^4\cosh\rho \times \sinh^3\rho(1+\star)\mathrm{d}\tau\wedge\mathrm{d}\rho\wedge\varepsilon(S^3_\phi), \qquad (13)$$

where $\varepsilon(S^3_\phi) = \sin^2\phi_1\,\sin\phi_2\,\mathrm{d}\phi_1\wedge\mathrm{d}\phi_2\wedge\mathrm{d}\phi_3$ is the volume element of the unit 3-sphere S^3_ϕ, and $\star$ is the Hodge star operator. The ansatz for the 2-form potential A_2 at the IR point is also known

$$A_2(IR) = C_2 + iB_2 - \frac{i}{2}e^{-2i\phi}L_0^2\cos\theta\Big(\mathrm{d}\theta - \frac{2i\sin 2\theta}{3-\cos 2\theta}(\sigma_3 + \mathrm{d}\phi)\Big)\wedge(\sigma_1 + i\sigma_2)\,. \qquad (14)$$

Two additional constraints are necessary for (11a) to be consistent, namely

$$F_3\wedge\star H_3 = 0, \qquad F_3\wedge\star F_3 = H_3\wedge\star H_3\,. \qquad (15)$$

Equipped with this setup we are ready to begin our study of the classical D7-brane embeddings and its scalar fluctuations in global Pilch-Warner geometry.

D7-BRANE EMBEDDINGS

In order to proceed with the classical D7 embeddings we will set the worldvolume gauge field $F = 0$. There are three relevant cases of classical D7-brane embeddings we will consider here. The first two correspond to fixed values of θ, and the third one corresponds to fixed value of ϕ. The firs case we are going to consider is $\theta = 0$, $\phi = \phi(\rho)$. The classical profile of the $D7$-brane is governed by the following non-linear equation:

$$\phi''(\rho) + (3\coth\rho + \tanh\rho) \times\Big(\phi'(\rho) + \frac{4}{9}(\phi'(\rho))^3\Big) = 0\,. \qquad (16)$$

The simplest solution to this equation is the constant solution $\phi(\rho) = const$. In the second case we take the ansatz $\theta = \pi/2, \phi = \phi(\rho)$. The classical equation of motion is similar to the previous one

$$\phi''(\rho) + (3\coth\rho + \tanh\rho) \Big(\phi'(\rho) + \frac{8}{9}(\phi'(\rho))^3\Big) = 0\,, \qquad (17)$$

solved again by any constant $\phi(\rho) = const$. Note that if we set $u(\rho) = \phi'(\rho)$ for both cases we get first order Bernoulli type equations. The third case we are going to consider is the ansatz $\phi = 0, \theta = \theta(\rho)$. The classical equation of motion takes complicated form

$$\theta''(\rho) + a(\rho)\theta'(\rho) + b(\rho)(\theta'(\rho))^2 + c(\rho)(\theta'(\rho))^3 + d(\rho)\varsigma(\rho) + e(\rho) = 0\,, \qquad (18)$$

where the corresponding coefficients are

$$a(\rho) = 3\coth\rho + \tanh\rho,$$

$$b(\rho) = \frac{1}{9(z_2-3)^2}\Big[2\tan(\theta(\rho)) \times\big(-\varsigma(\rho)\sqrt{6-2z_2}(z_3 - 13z_1) - 90z_2 + 9z_4 + 117\big)\Big],$$

$$c(\rho) = \frac{28}{9}(\tanh\rho + \coth\rho + \mathrm{csch}2\rho),$$

$$d(\rho) = -\frac{2\sqrt{2}\sin(\theta(\rho))\sqrt{3-z_2}(z_2-7)}{7(z_2-3)^2},$$

$$e(\rho) = \frac{18}{7}\,\frac{\sin(2\,\theta(\rho)) - 3\tan(\theta(\rho))}{(z_2-3)},$$
$$\varsigma(\rho) = \sqrt{14(\theta'(\rho))^2+9},$$
$$z_n = \cos(n\,\theta(\rho)), \quad n = 1,2,3,4.$$

This equation has simple solutions, which satisfy the trigonometric equation $\sin\theta = 0$, i.e. $\theta(\rho) = k\pi$, $k \in \mathbb{N}_0$.

SCALAR FLUCTUATIONS OF THE $D7$-BRANE AND THE MESON SPECTRA

Fluctuations along ϕ, $\theta = 0$

Now we proceed with the study of the scalar fluctuations of the $D7$ probe brane along ϕ and θ, around the classical solution $\phi_{cl} = 0$. We consider the following ansatz for the fluctuations:

$$\theta = 0 + 2\pi\alpha'\Theta, \qquad \phi = 0 + 2\pi\alpha'\Phi, \tag{19}$$

where $\delta\phi = 2\pi\alpha'\Phi$, and $\delta\theta = 2\pi\alpha'\Theta$. The full Lagrangian is the sum of the DBI and WZ Lagrangians

$$\mathscr{L} = \mathscr{L}_{DBI} + \mathscr{L}_{WZ}. \tag{20}$$

Let us investigate only the Φ fluctuations. There are no contributions from $\mathscr{L}_{WZ}$, but it gives non-zero contributions for Θ, as we will see in the next section. The DBI Lagrangian, up to quadratic order in the fluctuations, is given by

$$\mathscr{L}_{DBI} = -\mu_7\sqrt{-g}\Big(1 + \frac{1}{2}g^{ab}G_{\theta\theta}\partial_a\Theta\partial_b\Theta + \frac{1}{2}g^{ab}G_{\phi\phi}\partial_a\Phi\partial_b\Phi\Big), \tag{21}$$

where $g = \det(g_{ab})$ is the determinant of the induced metric. The equation of motion for the fluctuation field Φ is given by the Laplace-Beltrami equation

$$\nabla_a\nabla^a\Phi = \frac{1}{\sqrt{-g}}\,\partial_a\left(\sqrt{-g}\,g^{ab}\,\partial_b\Phi\right) = 0. \tag{22}$$

Expanding equation (22) one finds

$$-\partial_\tau^2\Phi + \cosh^2\!\rho\left(\partial_\rho^2\Phi + (3\coth\rho + \tanh\rho)\partial_\rho\Phi + \frac{1}{\sinh^2\!\rho}\Delta_{\phi_i}\Phi + 3\widetilde{\Delta}_{\alpha_i}\Phi\right) = 0, \tag{23}$$

where $\phi_i = (\phi_1, \phi_2, \phi_3)$, $\alpha_i = (\alpha, \beta, \gamma)$, and

$$\Delta_{\phi_i}\Phi = \frac{1}{\sin^2\phi_1}\partial_{\phi_1}(\sin^2\phi_1\partial_{\phi_1}\Phi) + \frac{1}{\sin^2\phi_1\sin\phi_2}\partial_{\phi_2}(\sin\phi_2\partial_{\phi_2}\Phi) + \frac{1}{\sin^2\!\phi_1\sin^2\!\phi_2}\partial_{\phi_3}^2\Phi, \tag{24a}$$

$$\widetilde{\Delta}_{\alpha_i}\Phi = \frac{1}{\sin\alpha}\partial_\alpha(\sin\alpha\partial_\alpha\Phi) + \frac{1}{\sin^2\!\alpha}\Big(\frac{(\cos(2\alpha)+7)}{8}\partial_\beta^2\Phi + \partial_\gamma^2\Phi - 2\cos\alpha\partial_\beta\partial_\gamma\Phi\Big). \tag{24b}$$

Separation of variables leads to the following spectral equations:

$$\ddot{T}(\tau) + \omega^2 T(\tau) = 0, \tag{25a}$$
$$\widetilde{\Delta}_{\alpha_i}Z(\alpha_i) = -\nu\,Z(\alpha_i), \tag{25b}$$
$$\Delta_{\phi_i}Y^l(\phi_i) = -l\,(l+2)\,Y^l(\phi_i), \tag{25c}$$
$$R''(\rho) + (3\coth\rho + \tanh\rho)R'(\rho) + \Big(\frac{\omega^2}{\cosh^2\!\rho} - \frac{l(l+2)}{\sinh^2\!\rho} - 3\nu\Big)R(\rho) = 0, \tag{25d}$$

where $Y^l(\phi_i)$ are the hyperspherical harmonics, $l \in \mathbb{N}_0$. In order to determine the conformal dimension and the spectrum, we will make the following change of variable $r = \sinh\rho$ in the radial equation. After some simple manipulations we find

$$R''(r) + \frac{3+5r^2}{r(r^2+1)}R'(r) + \Big(\frac{\omega^2}{(r^2{+}1)^2} - \frac{l(l{+}2)}{r^2(r^2{+}1)} - \frac{3\nu}{r^2{+}1}\Big)R(r){=}0. \tag{26}$$

There are two independent solutions $R(r) = R_+(r) + R_-(r)$ given in terms of Gaussian hypergeometric functions

$$R_+(r) = c_1 r^{-2-l}(r^2+1)^{-\frac{\omega}{2}}\,{}_2F_1(a,b;c;z), \tag{27a}$$
$$R_-(r) = c_2 r^l(r^2+1)^{-\frac{\omega}{2}} \times {}_2F_1(a-c+1,b-c+1;2-c;z), \tag{27b}$$

where the arguments of the hypergeometric function are as follows:

$$a = (-l - \omega - \sqrt{3\nu+4})/2,$$
$$b = \frac{1}{2}(-l - \omega + \sqrt{3\nu+4})/2,$$
$$c = -l, \qquad z = -r^2 .$$

The second solution $R_-(r)$ is regular at the origin $r = 0$, and at the boundary $r \to \infty$, which makes it our choice for normalizable solution, so that we always have finite fluctuations. The hypergeometric function is a polynomial of degree n if one of the first two arguments is equal to a negative integer $-n, n > 0$. Thus, imposing normalizability for the solution $R_-(r)$ means to chose one of the first two arguments of the hypergeometric function have negative integer values, e.g. $b - c + 1 = -n, n > 0$. This gives us the quantization condition from which one calculates the scalar meson spectrum

$$\omega = \sqrt{4+3\nu} + 2 + l + 2n, \tag{28}$$
$$\omega > 0, \qquad l, n \in \mathbb{N}_0, \qquad -\frac{4}{3} \le \nu \le 0.$$

By the standard AdS/CFT dictionary one can calculate the conformal dimension of the operators corresponding to Φ from the analysis of the radial equation (26) at the boundary $r \to \infty$. For large r we have the following asymptotic equation:

$$R''(r) + \frac{5}{r}R'(r) - \frac{3\nu}{r^2}R(r) = 0, \tag{29}$$

which is solved by

$$R(r) = c_1 r^{-\sqrt{3\nu+4}-2} + c_2 r^{\sqrt{3\nu+4}-2} . \tag{30}$$

This solution contains normalizable and non-normalizable parts that behaves as $r^{k_1} = r^{\Delta-4+p}$, and $r^{k_2} = r^{-\Delta+p}$, for some constant p. Taking the difference of the powers one finds the conformal dimension

$$\Delta = \frac{k_1 - k_2}{2} + 2 = 2 + \sqrt{3\nu+4}, \tag{31}$$

where $k_1 = -2 + \sqrt{3\nu+4}$, $k_2 = -2 - \sqrt{3\nu+4}$. Equation (31) allows us to express the spectrum in terms of the conformal dimension

$$\omega = \Delta + l + 2n . \tag{32}$$

From (32) we see that the energy of the ground state is given by the conformal dimension of the operator dual to the fluctuations. For higher modes the spectrum is equidistant. This is consistent with similar results for the fluctuations in different background geometries [3, 7]. To calculate the spectrum of conformal dimensions we need to quantise the parameter ν from equation (25b). Separation of variables of the kind $Z = A(\alpha)\, e^{im_1\beta}\, e^{im_2\gamma}$ leads to the following quantized values of ν:

$$\nu = (m+m_2)^2 + m + m_2 - \frac{m_1^2}{4}, \tag{33}$$
$$m, m_1 \in \mathbb{N}_0, \quad \frac{m_1}{3} < m_2 \le m_1, \quad -\frac{4}{3} \le \nu \le 0 .$$

Fluctuations along ϕ, $\theta = \pi/2$

This case needs more careful treatment, because at $\theta = \pi/2$ the radius of the 3-sphere spanned by (α, β, γ) shrinks to zero. This means that a direct substitution of $\theta = \pi/2$ into the equation of motion could cause problems. As it turns out the dependence on θ can be factor out. The equation of motion is once again the Laplace-Beltrami equation $\nabla_a \nabla^a \Phi = 0$, which written explicitly gives

$$A(\theta)(I_0(\Phi) + \cos^2\theta I_2(\Phi) + \cos^4\theta I_4(\Phi)) = 0, \tag{34}$$

Here $A(\pi/2) = 0$, so that setting $\theta = \pi/2$ causes the entire equation to become trivial, which is not a case of interest. The relevant equations for the fluctuation Φ comes from setting the coefficients I_0, I_2 and I_4 equal to zero, which leads to the following set of equations:

$$\Delta^{(a)}_{\alpha_i}\Phi = 0, \tag{35a}$$

$$-\partial_\tau^2\Phi + \partial_\rho^2\Phi\cosh^2\rho + \partial_\rho\Phi\,(2\cosh(2\rho)+1)\coth\rho + \coth^2\rho\,\Delta_{\phi_i}\Phi - 5\cosh^2\rho\,\Delta^{(b)}_{\alpha_i}\Phi = 0, \tag{35b}$$

$$-\partial_\tau^2\Phi + \partial_\rho^2\Phi\cosh^2\rho + \partial_\rho\Phi\,(2\cosh(2\rho)+1)\coth\rho + \coth^2\rho\,\Delta_{\phi_i}\Phi - 3\cosh^2\rho\,\Delta^{(c)}_{\alpha_i}\Phi = 0, \tag{35c}$$

where we have the following differential operators:

$$\Delta_{\phi_i}\Phi = \frac{1}{\sin^2\phi_1}\,\partial_{\phi_1}\left(\sin^2\phi_1\,\partial_{\phi_1}\Phi\right) + \frac{1}{\sin^2\phi_1\,\sin\phi_2}\,\partial_{\phi_2}\left(\sin\phi_2\,\partial_{\phi_2}\Phi\right) + \frac{1}{\sin^2\phi_1\sin^2\phi_2}\,\partial^2_{\phi_3}\Phi\,, \tag{36a}$$

$$\Delta^{(a)}_{\alpha_i}\Phi = \frac{1}{\sin\alpha}\,\partial_\alpha\left(\sin\alpha\,\partial_\alpha\Phi\right) + \frac{1}{\sin^2\alpha}\left(\partial^2_\beta\Phi + \partial^2_\gamma\Phi - 2\,\partial^2_{\beta\gamma}\Phi\cos\alpha\right), \tag{36b}$$

$$\Delta^{(b)}_{\alpha_i}\Phi = \frac{1}{\sin\alpha}\left(\partial_\alpha\left(\sin\alpha\,\partial_\alpha\Phi\right) - \frac{1}{10\sin\alpha}\left(\partial^2_\beta\Phi\left(\cos(2\alpha)-11\right) - 10\,\partial^2_\gamma\Phi + 20\,\partial^2_{\beta\gamma}\Phi\cos\alpha\right)\right), \tag{36c}$$

$$\Delta^{(c)}_{\alpha_i}\Phi = \frac{1}{\sin\alpha}\left(\partial_\alpha\left(\sin\alpha\,\partial_\alpha\Phi\right) - \frac{1}{4\sin\alpha}\left(\partial^2_\beta\Phi\,\left(\cos(2\alpha)-5\right) - 4\,\partial^2_\gamma\Phi + 8\,\partial^2_{\beta\gamma}\Phi\cos\alpha\right)\right). \tag{36d}$$

After separation of variables one finds

$$\ddot T(\tau) + \omega^2\,T(\tau) = 0\,,\quad \Delta_{\phi_i}Y^l(\phi_1,\phi_2,\phi_3) = -l\,(l+2)\,Y^l(\phi_1,\phi_2,\phi_3)\,, \tag{37a}$$

$$\Delta^{(a)}_{\alpha_i}Z(\alpha,\beta,\gamma) = 0,\quad \Delta^{(b)}_{\alpha_i}Z = \lambda\,Z,\quad \Delta^{(c)}_{\alpha_i}Z = \mu\,Z\,, \tag{37b}$$

$$R''_{(b)}(\rho) + (3\coth\rho + \tanh\rho)\,R'_{(b)}(\rho) + \left(\frac{\omega^2}{\cosh^2\rho} - \frac{l\,(l+2)}{\sinh^2\rho} - 5\,\lambda\right)R_{(b)}(\rho) = 0\,, \tag{37c}$$

$$R''_{(c)}(\rho) + (3\coth\rho + \tanh\rho)\,R'_{(c)}(\rho) + \left(\frac{\omega^2}{\cosh^2\rho} - \frac{l\,(l+2)}{\sinh^2\rho} - 3\,\mu\right)R_{(c)}(\rho) = 0\,. \tag{37d}$$

If one subtracts (35b) and (35c) a relation between the eigenvalues μ and λ is found, namely $5\,\lambda = 3\,\mu$, which makes equations (37c) and (37d) equivalent to each other. Therefore one needs to study only one of them. The radial equation (37d) has the same form and solutions as equation (26), but the only difference here being the different values for μ. The spectrum also has the same form as (32)

$$\omega = \Delta + l + 2n\,, \tag{38}$$

where the conformal dimension is $\Delta = \sqrt{4+3\,\mu} + 2$. In order to study the eigenvalue μ, we need to make sure that equations $\Delta^{(c)}_{\alpha_i}Z = \mu\,Z$ and $\Delta^{(a)}_{\alpha_i}Z(\alpha,\beta,\gamma) = 0$ are simultaneously satisfied. For that purpose we express $\partial^2_{\beta\gamma}Z\cos\alpha$ from (35a)

$$\partial^2_{\beta\gamma}Z\cos\alpha = \frac{1}{2}\frac{\sin^2\alpha}{\sin\alpha}\partial_\alpha(\sin\alpha\partial_\alpha Z) + \frac{1}{2}\partial^2_\beta\Phi + \frac{1}{2}\partial^2_\gamma\Phi,$$

and substitute this back into $\Delta^{(c)}_{\alpha_i}Z = \mu\,Z$ to find:

$$\partial^2_\beta Z(\alpha,\beta,\gamma) = 2\,\mu\,Z(\alpha,\beta,\gamma)\,. \tag{39}$$

Separation of variables $Z = A(\alpha)\,B(\beta)\,G(\gamma)$ gives an equation for $B(\beta)$

$$B''(\beta) - 2\,\mu\,B(\beta) = 0\,. \tag{40}$$

This equation has a simple solution of the form

$$B(\beta) = B_1\,e^{\sqrt{2\mu}\,\beta} + B_2\,e^{-\sqrt{2\mu}\,\beta}\,. \tag{41}$$

Therefore, if we want to study $\Delta^{(c)}_{\alpha_i}Z = \mu Z$, so that equation $\Delta^{(a)}_{\alpha_i}Z(\alpha,\beta,\gamma) = 0$ to be also satisfied, we must have the following separation of variables $Z = A(\alpha)\,e^{\sqrt{2\mu}\,\beta}\,e^{in_\gamma\gamma}$ in $\Delta^{(c)}_{\alpha_i}Z = \mu\,Z$, which gives an equation for $A(\alpha)$

$$A''(\alpha) + \cot\alpha A'(\alpha) - \frac{4\mu\sin^2\alpha + 2\mu(\cos(2\alpha)-5) + 4n^2_\gamma + 8in_\gamma\sqrt{2\mu}\cos\alpha}{4\sin^2\alpha}A(\alpha) = 0, \tag{42}$$

The solution is a combination of hypergeometric functions. Requiring finite fluctuations everywhere one finds the general form of the eigenvalue μ:

$$\mu = -k^2/2\,, \tag{43}$$

where $k \in \mathbb{N}_0$. Therefore we conclude that $\mu \leq 0$. But real and positive energy requires $\mu \geq -4/3$. which leads to only two possible values for μ, namely 0 and -1/2.

Although $\theta = 0$ and $\theta = \pi/2$ cases look similar due to their spectra, they are physically different cases. The $\theta = \pi/2$ is the massive direction from the point of view of the brane probe. Also in this case we have an enhancement of the supersymmetry from $\mathcal{N} = 1$ to $\mathcal{N} = 2$.

Fluctuations along θ, $\phi = 0$

Next we consider fluctuations of $D7$ along the θ transverse direction. In this case there are contributions from both the DBI and WZ parts of the action. Considering only quadratic lagrangian for the fluctuations one finds the equation of motion

$$-\partial_\tau^2\Theta + \cosh^2\rho\Big(\partial_\rho^2 + (3\coth\rho + 4\tanh\rho)\partial_\rho + \frac{\Delta_{\phi_i}}{\sinh^2\rho} + 3?\vec{\Delta}_{\alpha_i} - 6\Big)\Theta = 0\,, \quad (44)$$

where $\Delta_{\phi_i}\Theta$ and $\widetilde{\Delta}_{\alpha_i}\Theta$ are the same as in (24a) and (24b). Separation of variables leads to the same equations as (25a), (25b) and (25c), but slightly different radial equation

$$R''(\rho) + (3\coth\rho + 4\tanh\rho)R'(\rho) + \Big(\frac{\omega^2}{\cosh^2\rho} - \frac{l(l+2)}{\sinh^2\rho} - 3\nu - 6\Big)R(\rho) = 0\,. \quad (45)$$

Making the change $r = \sinh\rho$ we get the following equation:

$$R''(r) + \frac{(8r^2+3)}{r^3+r}R'(r) + \Big(\frac{\omega^2}{(r^2+1)^2} - \frac{l(l+2)}{r^2(r^2+1)} - \frac{3\nu-6}{r^2+1}\Big)R(r) = 0\,. \quad (46)$$

One can show that a solution regular at the origin $r = 0$, and at the boundary $r \to \infty$, is given by

$$R(r) = c\, r^l\left(r^2+1\right)^{-\frac{1}{4}\sqrt{4\omega^2+9}-\frac{3}{4}}\,{}_2F_1\left(\frac{l}{2}+\frac{\Delta}{2}-\frac{1}{4}\sqrt{4\,\omega^2+9}, \frac{l}{2}-\frac{\Delta}{2}-\frac{1}{4}\sqrt{4\,\omega^2+9}+2; l+2; -r^2\right). \quad (47)$$

Quantizing one of the first two arguments of the hypergeometric function gives us the meson spectrum

$$\omega^2 = (l+2n+\Delta)^2 - \frac{9}{4}\,, \quad (48)$$

where $\Delta = \left(4+\sqrt{25+12\,\nu}\right)/2$ is the conformal dimension, $l, n \in \mathbb{N}_0$, $-2 \geq \nu \geq -10/3$. Once again the spectrum is equidistant in its higher modes, but the ground state is not equal to the conformal dimension of the operators dual to the fluctuations. The additional shift in the ground state $(n, l = 0)$ could be resolved by studying the symmetries of the theory and considering some supersymmetric D-brane embeddings. Other origins of the shift are also not excluded.

CONCLUSION

Quantum chromodynamics is the most successful theory describing the strong nuclear force so far. At low energy QCD is strongly coupled, which means that the force between the quarks grows immensely and they tend to form particles called hadrons – a phenomenon known as confinement. In this low energy regime of the theory the usual perturbative techniques are not applicable, which forces us to look for alternative non-perturbative methods. Such alternative techniques arise in String theory in the context of the AdS/CFT correspondence, where the physics of the supersymmetric Yang-Mills systems can be understood by that of the D-brane dynamics and vice versa. The original form of the conjecture focuses on the $\mathcal{N} = 4$ super Yang-Mills theory, which is a gauge theory with a huge amount of symmetry. On the other hand generically QCD is neither supersymmetric, nor conformal. One way to make the correspondence more applicable to realistic gauge theories, such as QCD, is to reduce the amount of the supersymmetry. This goal can be achieved in several different ways, one of which is deforming the original $AdS_5 \times S^5$ geometry. This was the approach we adopted here by looking at the dynamics of the flavor D7-brane embedded in a deformed background called Pilch-warner geometry.

In this study we obtained the classical embedding equations for the D7 probe brane in global Pilch-Warner background geometry for three relevant cases.

Although highly non-linear the embedding equations have simple constant solutions, which allowed us to fix the probe brane position in space and considerably simplify the study of its fluctuations. In the case when the number of the flavour branes is much smaller than the number of the color branes, the analysis of the scalar fluctuations of the D7 probe brane lead us to analytical results for the meson spectra. All obtained spectra are equidistant in the higher modes, but not all of them have ground states equal to the conformal dimension of the operators dual to the fluctuations.

The fluctuations of the D5-brane as well as the fluctuations of the world volume gauge field can also be studied. Some more complicated cases include turning on external electric or magnetic fields, which further break the supersymmetry. The asymptotic behaviour of the brane embedding equations can be used to extract information about the dual gauge theory, namely the quark condensate, which mixes the left and right degrees of the fundamental matter and leads to a breaking of their chiral symmetry.

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ХОЛОГРАФСКИ МЕЗОНИ В ГЕОМЕТРИЯ НА ПИЛЧ-УОРНЪР

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(Резюме)

AdS/CFT съответствието е удивителна дуалност, свързваща десет-мерна IIB суперструнна теория с малка константа на връзката с четири-мерна суперсиметрична калибровъчна $SU(N)$ теория с голяма константа на връзката, и обратно. В случая калибровъчната теория живее на границата на пространството, в което се движат струните. Това съответствие ни дава възможност да изучаваме непертурбативни проблеми в калибровъчни теории на Янг-Милс чрез методи на класическата Суперструнна теория или Супергравитация.

В оригиналната версия на съответствието от струнна гледна точка имаме стек от N_c на брой паралелни D_3-брани, които генерират ефективната $AdS_5 \times S^5$ геометрия на пространството, а от другата страна имаме $N = 4$ суперсиметрична калибровъчна теория на Янг-Милс. При тази конфигурация краищата на струните се закрепят върху брани от един и същ стек, което прави състоянията да се трансформират в присъединеното представяне на калибровъчната група, а това означава, че липсва фундаментална материя като кварки. Ако към конфигурацията от D_3-брани добавим N_f на брой D_7 пробни брани ($N_f \ll N_c$) ще получим състояния трансформиращи се по фундаменталното представяне на калибровъчната група и следователно ще получим фундаментална материя.

Въвеждането на допълнително външно магнитно или електрично поле деформира първоначалната $AdS_5 \times S^5$ геометрия, което води до нарушаване на суперсиметрията и постигане на теории с по-малко суперсиметрия. Също така се нарушава и киралната симетрия, което води до образуването на кварков кондензат и конфайнмънт, така че струнното описание да се доближава все повече до описание на Квантовата хромодинамика.

В настоящият труд сме изследвали спектъра на скаларните флуктуации на D_7 пробна брана в геометрия на Пилч-Уорнър в глобални координати. Тази геометрия представлява деформирано по определен начин $AdS_5 \times S^5$ пространство, което е решение на 5-мерна $N = 8$ супергравитация вдигната до 10-мерие, като запазва 1/4 от първоначалната суперсиметрия в инфрачервената критична точка, и 1/8 навсякъде другаде.