Fundamental quantum limit in Mach-Zehnder interferometer

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In this article we discuss the concept of standard quantum (shot-noise) limit and fundamental quantum limit (completely quantum mechanical) in Mach-Zehnder Interferometer. With the method of linear invariants the three independent quantum fluctuations are determined in Schrödinger Minimum Uncertainty States. On the base of more general Uncertainty Relation (Schrodinger-1930), we accurately define the notion of fundamental quantum limit. The analytical consideration involves the general term Covariance (compare to Variance) of two quantum variables. Explicit new formula for the fundamental quantum limit is obtained.

Key words: quantum mechanics, uncertainty relations, Heisenberg limit, Schrödinger limit, interferometry

INTRODUCTION

The light interferometry is a one of the primary methods to prove experimentally basic laws in physics. For example a lot of discussions in the scientific literature was done, dedicated to improve gravitational radiation using Quantum Mechanical interferometry [1,2]. Much work has been done on reduction of the quantum noise by using input light prepared in non-classical states [3–9]. Because of the particle nature of the light, there exists some fundamental limitations of its sensitivity, which are the subject of present article: standard Quantum Limit(semiclassical) and the Fundamental Quantum Limit, the latter known as Heisenberg Limit [10–12]. Introducing the more accurate Uncertainty Relation than that of Heisenberg, we will precise the formulation of Quantum Limit. Classical and semiclassical treatments of these measurements is less precise as quantum one. Quantum interferometry is the best tool for phase estimation due to its sensitivity. The goal of quantum interferometry is to estimate phases beyond the standard quantum limit. It was discovered that squeezed vacuum, injected into the normally unused port of an interferometer, provides sensitivity below the shot-noise limit [1]. It is possible to reach even better sensitivity if a nonlinear interaction between photons in the Mach-Zehnder interferometer takes place (for example – parametric down-conversion, [1]).

In this article we consider shortly the method of linear integrals of motion (in second section). In the third section we interpret the two notions *Standard Quantum Limit* and *Fundamental Quantum Limit*.

We examine the resemblances and distinctions between them and we derive a new formula for the Fundamental Quantum Limit using the general uncertainty relation.

METHOD OF LINEAR INTEGRALS OF MOTION

One of the most revolutionary consequences that quantum mechanics bequeathed as a fundamental principle in physics is the refusal of strong determinism. That is why the uncertainty relation (called uncertainty principle in the beginning of quantum mechanics) plays fundamental role. The uncertainty relation for the two canonical variables was introduced for the first time by Heisenberg (1925), and generalized to any two observables by Schrödinger [13,14]:

$$(\Delta A)^2 (\Delta B)^2 \geq Cov^2(A,B) + |\frac{1}{2}\langle [A,B]\rangle|^2, \quad (1)$$

where the covariance for non-commuting operators A and B is defined as

$$Cov(A,B) \equiv \frac{1}{2}\langle AB+BA\rangle - \langle A\rangle\langle B\rangle, \quad (2)$$

and the variance is defined as $(\Delta A)^2 = Cov(A,A)$.

Neglecting the first term on the right side, we receive the Heisenberg Uncertainty Relation:

$$(\Delta A)^2 (\Delta B)^2 \geq |\frac{1}{2}\langle [A,B]\rangle|^2. \quad (3)$$

We will remind the method of linear integral of motions presented in [15–17] Let us consider a classical system with s degree of freedom and let $u = u(q_1,...q_s, \ p_1,...p_s,t)$ be a dynamical variable of this system. Expressed in terms of Poisson brackets $\{,\}$, the full derivative of u with respect to t is

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \{u, H_{class}\}. \quad (4)$$

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By definition u^{inv} will be an integral of motion (invariant) iff $du^{inv}/dt = 0$, i.e.

$$\frac{\partial u^{inv}}{\partial t} + \{u^{inv}, H_{class}\} = 0. \qquad (5)$$

As far as there is a principle of correspondence between classical and quantum mechanics, the analogy requires the existence of $2s$ Hermitean operators - integrals of motion for any quantum system, and the relevant equations of the quantum invariants are [16]

$$\frac{\partial \widehat{I}_\nu^{inv}}{\partial t} + \frac{1}{i\hbar}[\widehat{I}_\nu^{inv}, \widehat{H}] = 0, \quad \nu = 1, ..., 2s. \qquad (6)$$

Note that these equations for the invariants (6) are different from the Heisenberg equations of motion:

$$\frac{d\widehat{A}}{dt} - \frac{1}{i\hbar}[\widehat{A}, \widehat{H}] = 0. \qquad (7)$$

The same difference in the sign exists in classical mechanics between (5) and the Hamilton equations written in terms of Poisson brackets [18]:

$$\frac{du}{dt} - \{u, H_{class}\} = 0, \quad u = q_k, p_k. \qquad (8)$$

The independent solutions of (6) for any quantum system are also $2s$: $\widehat{I}_\nu^{inv}(t) = \widehat{U}(t)I_\nu^{inv}(0)\widehat{U}^\dagger(t)$, where $U(t)$ is the unitary evolution operator. This difference arise in our days with new impact, due to mismatching some times of the equations for invariants with those for motion (see for example wikipedia). Method of Linear Integrals of Motion was developed for quadratic Hamiltonians $\widehat{H}$ in [15] and full-time evolution for all three independent quantum fluctuations $Cov(q, p), (\Delta q)^2$ and $(\Delta p)^2$ were obtain in [16, 17]. Linear invariants, obeying (6), are expressed through solution of equation for non-stationary harmonic oscillator $\ddot{\varepsilon}(t) + \Omega(t)\varepsilon(t)^2 = 0$. Remarkable advantage of this method is, that the solution ψ of Schrödinger equation $\widehat{H}\psi = i\hbar\frac{\partial \psi}{\partial t}$ are found in explicit form, and also minimize the Schrödinger Uncertainty Relation (1). Becouse of that reason the obtained states are often call Schrödinger Minimum Uncertainty States (SMUS) [19]. This class of SMUS includes MUS introduced by C. Caves [20], Lectures 7.5 − 10, p. 3, Coherent and p. 8, Squeezed states, respectively. SMUS includes the sub-class of Covariance States, with non-vanishing covariance $Cov(A, B) \neq 0$ (2).

STANDARD QUANTUM LIMIT AND FUNDAMENTAL QUANTUM LIMIT

The topic of Fundamental Quantum Limit (Heisenberg Limit) and its generalization was discused also in [21], where it is stated that other limit exists (Schrödinger Limit), but the exact formulation is given here as follows.

In this article we don't argue whether Heisenberg Limit can be beaten or not. We discuss only the two limits: Heisenberg and Schrödinger, taking into account the Covariance.

As a particular experimental setup and to provide our consideration we will consider Mach-Zehnder Interferometer, see Fig. 1. In that case Heisenberg uncertainty principle links phase $(A = \widehat{cos(\phi)})$, [22]) of a state to its photon number $(B = \widehat{n})$, (similarly to [23], p. 322):

$$\Delta\phi\Delta n \geq 1/2, \qquad (9)$$

and for low level bound of the product we receive $\Delta\phi\Delta n = 1/2$.

To escape misunderstanding between the two limits (Standard and Fundamental) we will consider Max-Zehnder Interferometer fed by power laser prepared in Coherent States. Due to particle nature of photons, they obey classical Poisson distribution (see for example [24] or [25], chapter 8.4):

$$P[N = n] = \frac{e^{-\lambda}\lambda^n}{n!} \qquad \lambda > 0. \qquad (10)$$

(In this paragraph N is the photon number, classical random variable, and $n = 0, 1, 2, ...,$. From the
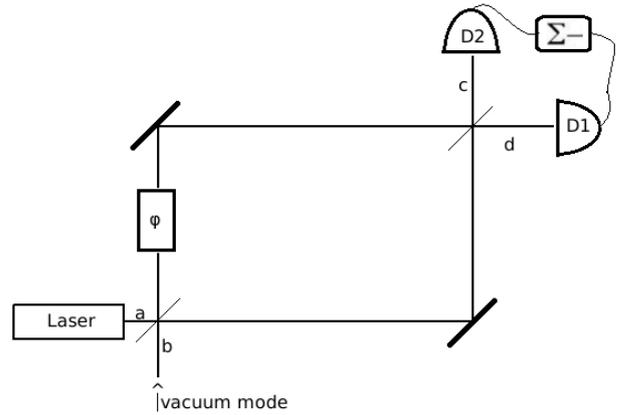
Fig. 1. Schematic representation of the Mach-Zehnder interferometer. The input modes are a coherent and a squeezed-vacuum field, respectively.

central limit theorem in the theory of Probability and Statistics follows $E[N] = \lambda = D[N]$, so in our notation

$N = \lambda = (\Delta N)^2.$

$$(\Delta N) = \sqrt{N}. \tag{11}$$

Minimizing Heisenberg Uncertainty Relation (3) with (11) we get the so called semiclassical Standard Quantum Limit for the phase in Mach-Zehnder interferometer:

$$(\Delta\phi)_{SQL} = \frac{1}{\sqrt{N}}. \tag{12}$$

It is worth to mention that the same dependence can be obtained [9] $(\Delta(N)) = \sqrt{N}$ if we consider N separate single-photon beams, producing the so called "shot noise" and N being the number of photons used. In this case, $cos^2(\phi/2)$ is the probability of the photon exiting at output **c**, and $sin^2(\phi/2)$ is the probability of the photon exiting at output **d** [9] . The Standard Quantum Limit is not fundamental and is only semi-classical: quantum in respect to Heisenberg Uncertainty Relation and classical in respect to state preparation and detection strategy. Standard Quantum Limit for position and momentum observables is discussed in [23, 26, 27]

Let us consider completely Quantum case. The discovery of squeezed states gives a new challenge to quantum interferometry, with the possibility to reach better sensitivity. Injecting non-classical states (such as squeezed states or N00N-states [1, 28]) in unused port **b** will improve the phase sensitivity of Mach-Zehnder interferometer. Quantum Mechanics does not set any restriction on the fluctuation Δn of the photon number operator $\hat{n}$ - the only upper bound is

$$\Delta n \le N \quad (N = \langle\hat{n}\rangle). \tag{13}$$

This limit of photon number uncertainty together with minimized Heisenberg Uncertainty Relation (3) gives the phase limit:

$$(\Delta\phi)_{HL} = \frac{1}{2N}, \tag{14}$$

known in the literature as Heisenberg Limit. Here we use $N = N_{in}/2$, which is the total number of photons in the arm of the interferometer that experiences the phase shift. For Mach-Zehnder Interferometer

$$(\Delta\phi)_{HL} = \frac{1}{N}.$$

And now let us consider the Fundamental Quantum Limit based on General Uncertainty Relation (1).

In terms of photon number and phase operators the Schrödinger Uncertainty Relation is

$$(\Delta\phi)^2(\Delta n)^2 \ge \frac{1}{4} + Cov^2(\phi, n). \tag{15}$$

In similar way as above we define the thorough bound for phase sensitivity. The upper bound for the variance of photon number operator is

$$\Delta n \le N \quad (\text{again} \quad N = \langle\hat{n}\rangle). \tag{16}$$

Taking the root mean square and multiplying (16) by $(\Delta\phi)$ we receive

$$(\Delta\phi)N \ge (\Delta\phi)(\Delta n) \ge \frac{1}{2}\sqrt{1 + 4Cov^2(\phi, n)}. \tag{17}$$

The lower bound of these inequalities (17) gives the rigorous quantum limit (we call it Shrödinger Quantum Limit):

$$(\Delta\phi)_{SL} = \frac{1}{2N}\sqrt{1 + 4Cov^2(\phi, n)}. \tag{18}$$

Similarly to work [21] we present the "uncertainty ellipse" as a cross-section of quasi probability distribution (Wigner function) with horisontal plane. To consider in details the interference pattern in Mach-Zehnder interferometer, it is necessary to take into account the electro-optic coefficients and the orientation of the cristal in arm *g*, leading to presence of Squeezed and Covariant states, which is not subject of this article. For the moment, we will illustrate Squeezed and Covariant states for the two quadratures components, showing the Wigner quasi probability function in presence of non-vanishing covariance. As an example for Covariant states we choose

$$\Psi = \frac{e^{-\frac{1}{2}x^2}e^{\frac{1}{4}ix^2+4}}{\pi^{1/4}e^4},$$

which leads to $Cov(q, p) = \hbar/2$. The Wigner quasi probability function is presented in Fig. 2. The uper plane at $1/e$ from the top presents Heisenberg Limits for q and p respectively. The cross-section is "uncertainty ellipse" with semi-axes equal to Heisenberg Limits for q and p respectively. The actual "uncertainty ellipse" (and more precise vision on this topic) is shown on lower plane with semi-axes equal to Shrödinger Limits for q and p respectively, where the non-vanishing covariance plays essential role. The distance between Heisenberg Limit and Schrödinger
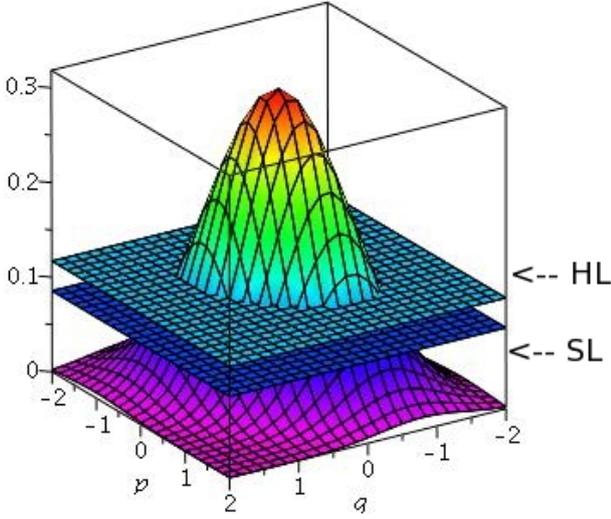
Fig. 2. Quantum fluctuations in Heisenberg limit and Schrödinger limit.

Limit depends on the third independent quantum fluctuation – the covariance.

Going back to Mach-Zehnder interferometer with the same argument ($N = N_{in}/2$) on both ports **c** and **d** we have

$$(\Delta\phi) = \frac{1}{N}\sqrt{1 + 4Cov^2(\phi, n)}. \qquad (19)$$

An argument in favour of general formula (18) (and in particular for Mach-Zehnder interferometer formula (19)) is that in case of power laser beams [30–32] we have strong nonlinearity leading always to covariance term.

Moreover, considering the dark fringes of the interference picture, where the photon number is very small, the states are still mixture of coherent and squeezed state $|\alpha\rangle_a |\zeta\rangle_a$ (even when the time-interval is very large in the case of single-photon registration, and looks like there is no correlation, but it exists).

We would like to emphasize that the precise phase estimation should include covariance, taken from operators, for particular $\widehat{\phi}$. This is still difficult question, because of ambiguously defining phase operator $(\widehat{\phi}, \widehat{exp(i\phi)}, \widehat{cos(i\phi)}, \widehat{sin(i\phi)})$ – which one we should take? [22]. There are a lot of discussions [22, 33, 34] and the problem of defining phase operator still exists. We choose one of the cases described in [22].

CONCLUSIONS

In this article we discuss the concept of standard quantum (shot-noise) limit, the Heisenberg Limit and rigorous Schrödinger Limit (the last two – completely

quantum-mechanically). With the method of linear invariants the three independent quantum fluctuations are determined in Schrödinger Minimum Uncertainty States (SMUS). On the base of more general Uncertainty Relation (Schrödinger-1930), we accurately define the notion of fundamental quantum limit. The newly obtained Schrödinger Limit formula is applied for Mach-Zehnder Interferometer also, fed with coherent light mixed with squeezed vacuum, including the sensitivity of dark fringes in the interferometer. The analytical consideration involves the general term Covariance (compare to Variance) of two quantum variables. This term should be always taken into account, since it increases the fundamental quantum limit, especially in experiments, with strong nonlinearity. Neglecting it, would lead to serious errors in experimental results. Explicit new formulas for arbitrary two non-commuting observables (not only $\widehat{\phi}, \widehat{n}$) for the Schrödinger limit are received in similar way, but this will be a topic of next work.

APPENDIX

Derivation of Schrödinger uncertainty relation

The proof follows Schrödinger derivation, where the substitution in Schwarz inequality is split in two steps, as we will see later. Let ψ denotes the wavefunction of our quantum system, and $\mathcal{H}$ is the corresponding Hilbert space with scalar product $\langle\psi|\varphi\rangle$ and norm $|\psi| = \sqrt{\langle\psi|\psi\rangle}$, where $\psi, \varphi \in \mathcal{H}$. For any two vectors $\psi, \varphi \in \mathcal{H}$ the Schwarz inequality holds

$$|\psi|^2|\varphi|^2 \geq |\langle\psi|\varphi\rangle|^2. \qquad (20)$$

We accept the following notation: $\langle C \rangle \equiv \langle\psi|\widehat{C}|\psi\rangle \equiv \langle\psi|(\widehat{C}\psi)\rangle$, where C is self-adjoin operator, associated with certain physical variable. Let us

substitute $\psi, \varphi \in \mathscr{H}$ into inequality (20) with

$$\psi \to \widehat{C}^{\dagger} \psi^*, \quad \varphi \to \widehat{D}\psi. \qquad (21)$$

The product of two Hermitean operators is in general non-Hermitean, but it could be split into Hermitean part (symmetrical product) and skew-Hermitean part (half of its commutator $[C, D]$):

$$CD = \frac{CD + DC}{2} + \frac{CD - DC}{2}. \qquad (22)$$

This splitting corresponds in many aspects to the splitting of a complex number z into real and imaginary parts - $z = Re(z) + iIm(z) = (z + z^*)/2 + (z - z^*)/2$.

Using inequality.(20), we obtain

$$\langle D^2 \rangle \langle C^2 \rangle \geq |\langle CD \rangle|^2. \qquad (23)$$

If we decompose the right hand side according to (22) and applying $|z|^2 = |Re|^2 + |Im|^2$, where $z = \langle CD \rangle$ and $z^* = \langle D^{\dagger} C^{\dagger} \rangle = \langle DC \rangle$ we get

$$\langle C^2 \rangle \langle D^2 \rangle \geq \left(\frac{\langle CD + DC \rangle}{2} \right)^2 + \left| \frac{\langle CD - DC \rangle}{2} \right|^2. \qquad (24)$$

In order to arrive at the general case instead of the operators A and B we use $C = A - \langle A \rangle \widehat{I}$ and $D = B - \langle B \rangle \widehat{I}$. Than, for the first term on the right we have

$$\left(\frac{\langle CD + DC \rangle}{2} \right)^2 = \left(\frac{\langle (A - \langle A \rangle)(B - \langle B \rangle) + (B - \langle B \rangle)(A - \langle A \rangle) \rangle}{2} \right)^2$$
$$= \left(\frac{\langle AB - A\langle B \rangle - \langle A \rangle B + \langle A \rangle \langle B \rangle + BA - B\langle A \rangle - \langle B \rangle A + \langle B \rangle \langle A \rangle \rangle}{2} \right)^2 = \left(\frac{\langle AB + BA \rangle}{2} - \langle A \rangle \langle B \rangle \right)^2 = Cov^2(A, B).$$

And for the second term on the left

$$\left| \frac{\langle CD - DC \rangle}{2} \right|^2 = \left| \frac{\langle (A - \langle A \rangle)(B - \langle B \rangle) - (B - \langle B \rangle)(A - \langle A \rangle) \rangle}{2} \right|^2 = \left| \frac{\langle AB - BA \rangle}{2} \right|^2 = \left| \frac{1}{2} \langle [A, B] \rangle \right|^2.$$

Finally, we end up Schrödinger derivation with

$$(\Delta A)^2 (\Delta B)^2 \geq Cov^2(A, B) + \left| \frac{1}{2} \langle [A, B] \rangle \right|^2. \qquad (25)$$

If we move the covariance on the left side of the inequality, and use the definition of covariance matrix [24]

$$\Sigma_{\mathbf{A}, \mathbf{B}} = \begin{pmatrix} Var(A, A) & Cov(A, B) \\ Cov(A, B) & Var(B, B) \end{pmatrix},$$

we receive for SUR in very compact, canonical form

$$det(\Sigma_{\mathbf{A}, \mathbf{B}}) \geq \left| \frac{1}{2} \langle [A, B] \rangle \right|^2, \qquad (26)$$

showing some group symmetry in a phase space!

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ФУНДАМЕНТАЛНО КВАНТОВО ОГРАНИЧЕНИЕ В МАХ-ЦЕНДЕР ИНТЕРФЕРОМЕТЪР

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(Резюме)

Статичния шум (на английски: *Shot noise*) съществува понеже феномени като светлината и електрическият ток се състоят от движение на отделни квантови обекти. Светлина идваща от отдалечени звезди например, пристига на малки порции, наречени фотони. Подобни са процесите, когато изследваме тъмните зони на интерференчната картина. При такива малки стойности интензитета вече не е непрекъсната функция. Тогава се регистрират само единични фотони, неравномерно пристигащи по време. Тези явления дават ограничения при реалните физични измервания на много слаби сигнали. В тази връзка е въведено физичното понятие *стандартно квантово ограничение* (на английски - Heisenber limit, shot-noise limit), основаващо се на Хайзенберговото съотношение на неопределеност [1].

В настояшата статия разглеждаме статичния шум в интерферометър на Макс-Цендър (и по-специално в областта на тъмните ивици на интерференчната картина). През 1930 година Шрьодингер обобщава и уточнява Хайзенберговото съотношение на неопределеност [2]. Превод на оригиналната работа на Шрьодингер от немски на английски е направен в [3], където в анотацията към превода са показани нови свойства на съотношението на неопределеност, каквито Хайзенберговото не притежава. Прилагайки по-общото съотношение на неопределеност, ние преразглеждаме това централно понятие *стандартно квантово ограничение*. Аналитичното разглеждане включва понятието ковариация на две квантови променливи. Получена е нова (по-точна) формула за стандартния квантов лимит.

Настоящата публикация е едно продължение на [4], където се дискутира необходимостта от уточняване на *стандартното квантово ограничение,* но точната формула е изведена тук.

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