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Scattering of solitons from point defects in two coupled Ablowitz-Ladik chains

R. S. Kamburova*, M. T. Primatarowa

Institute of Solid State Physics, Bulgarian Academy of Sciences,
72 Tzarigradsko Chaussee Blvd., 1784 Sofia, Bulgaria

The interaction of solitons with point defects in a system of coupled Ablowitz-Ladik (AL) chains is studied numerically. The system is a discrete analog of coupled nonlinear Schrödinger equations. The interchain coupling which couples opposite sites of the AL chains includes linear and nonlinear interactions. The soliton dynamics depends on the soliton parameters (width, velocity), interchain coupling constant and defect strength. It is obtained that solitons which are exited in one of the two chains can be perfectly switched and at the same time transmitted, trapped or reflected by the attractive impurities. The point defects do not influence the period of energy transfer and it is close to the period for the homogeneous case.

Key words: solitons, Ablowitz-Ladik equation, soliton-impurity interaction

INTRODUCTION

The study of nonlinear waves is receiving much attention due to the potential application within different branches of physics from nonlinear optics to Bose-Einstein condensate. The interplay between discrete diffraction and nonlinearity leads to the formation of discrete solitons. They promise an efficient way to control switching of optical signals in a system of coupled waveguides. So waveguide-based devices have received considerable attention in literature and this field has been extensively explored theoretically and experimentally [1,2]. A discrete coupler involving two waveguides which exchange power as a result of weak overlap of their evanescent fields is the basic realization of a waveguide switch. The rate of power swapped back and forth between waveguides depends on the strength of the coupling, the degree of similarity of the waveguides and the initial pulse energy [3-7]. Waveguide arrays are particularly interesting because of their possible applications in signal processing [8-12]. All considerations regard discrete soliton switching mainly in homogenous waveguides with constant or smoothly varying coupling between them. However in practical applications the properties of inhomogeneous waveguides are more interesting and rather inevitable for switching [13-17].

Widely investigated are the standard discrete nonlinear Schrödinger (NLS) equation, as well as the completely integrable discrete Ablowitz-Ladik (AL) equation [18-20]. Although the two equations have the same linear properties and yield the same NLS equation in the continuum limit, their nonlinear properties are different. This leads to differences in the dynamics of narrow solitons (bright or dark) for the two models. Soliton solutions in two coupled discrete nonlinear chains were found and their stability was investigated in [21-24].

In the present paper we study the interaction of propagating solitons with impurities in two Ablowitz-Ladik chains with a complicated coupling that includes linear and nonlinear interactions between the chains.

THE MODEL

We shall consider two parallel chains of particles described by the following system of coupled Ablowitz-Ladik equations:

$$\begin{aligned} i\frac{\partial \alpha_n}{\partial t} =& M(\alpha_{n+1}+\alpha_{n-1})(1+\gamma|\alpha_n|^2) \\ &+2d\beta_n(1+\gamma|\alpha_n|^2)+\varepsilon\delta_{n,n_0}\alpha_n \\ i\frac{\partial \beta_n}{\partial t} =& M(\beta_{n+1}+\beta_{n-1})(1+\gamma|\beta_n|^2) \\ &+2d\alpha_n(1+\gamma|\beta_n|^2)+\varepsilon\delta_{n,n_0}\beta_n \end{aligned} \tag{1}$$

$\alpha_n(t)$ [$\beta_n(t)$] is the amplitude of an excitation at site n of the first (second) chain, interacting with an impurity of the strength ε localized at the point n_0. M is the coupling interaction between neighboring particles in one and the same chain. The two chains are coupled to each other through the real parameter d which governs the interchain coupling between opposite sites (nondispersive) and includes linear and nonlinear terms. The parameter γ determines the type of the soliton solution (bright for $\gamma > 0$ and dark for $\gamma < 0$) of the AL equation. In what follows we con-

* To whom all correspondence should be sent:
krad@issp.bas.bg

sider only bright solitons and set $\gamma = 1$ due to the scaling property of the AL system.

First we shall consider the homogeneous case ($\varepsilon = 0$). We briefly outline the influence of the coupling between opposite sites on the soliton properties [24]. Equation (1) can be derived from the Hamiltonian

$$H = \sum_n \Big[M(\alpha_n\alpha^*_{n-1} + \alpha^*_n\alpha_{n-1} + \beta_n\beta^*_{n-1} + \beta^*_n\beta_{n-1}) + 2d(\alpha_n\beta^*_n + \alpha^*_n\beta_n) \Big] \quad (2)$$

using the deformed Poisson brackets [19,20]

$$\begin{aligned} \{\alpha_n, \alpha^*_m\} &= i(1+|\alpha_n|^2)\delta_{n,m}\,, \\ \{\alpha_n, \alpha_m\} &= \{\alpha^*_n, \alpha^*_m\} = 0\,, \\ \{\beta_n, \beta^*_m\} &= i(1+|\beta_n|^2)\delta_{n,m}\,, \\ \{\beta_n, \beta_m\} &= \{\beta^*_n, \beta^*_m\} = 0 \end{aligned} \quad (3)$$

and the equations of motion

$$\frac{\partial \alpha_n}{\partial t} = \{H, \alpha_n\}\,, \quad \frac{\partial \beta_n}{\partial t} = \{H, \beta_n\}\,. \quad (4)$$

The system (1) is nonintegrable but has two integrals of motion, the Hamiltonian H and the total number of particles

$$N = \sum_n [\ln(1+|\alpha_n|^2) + \ln(1+|\beta_n|^2)]\,. \quad (5)$$

For $d = 0$ and the symmetric reduction $\alpha_n(t) \equiv \beta_n(t)$ the system (1) turns in an AL equation with the well known bright soliton solution:

$$\alpha_n(t) = \beta_n(t) = \sinh\frac{1}{L}\,\mathrm{sech}\frac{n - vt}{L}\,e^{i(kn - \omega t)} \quad (6)$$
$$v = -2ML\sinh\frac{1}{L}\sin k, \qquad \omega = 2M\cosh\frac{1}{L}\cos k$$

The parameters k (wavenumber) and L (width) determine the velocity v and frequency ω of the soliton. In this case the conserved quantities have the form

$$H = 8M\sinh\frac{1}{L}\cos k, \quad N = 4/L \quad (7)$$

and it holds $\omega = \partial H/\partial N$.

In the continuum limit $\alpha_n(t) \to \alpha(x,t)$, $\beta_n(t) \to \beta(x,t)$ which holds for wide solitons ($L \gg 1$) and for $\alpha(x,t) \equiv \beta(x,t)$ the system (1) reduces to the standard NLS equation of the form

$$i\frac{\partial \alpha}{\partial t} = 2(M+d)\alpha + M\frac{\partial^2 \alpha}{\partial x^2} + 2(M+d)|\alpha|^2\alpha \quad (8)$$

with the bright soliton solution

$$\alpha(x,t) = \varphi_0\mathrm{sech}\frac{x - vt}{L}\,e^{i(kx - \omega t)} \quad (9)$$
$$\varphi_0 = \frac{1}{L}\sqrt{\frac{M}{M+d}}, \qquad v = -2Mk,$$
$$\omega = 2(M+d) - Mk^2 + \frac{M}{L^2}\,.$$

An attempt to include the discreteness effects which become important for narrow solitons ($L \sim 1$) will lead to a correction of the velocity of the form $\Delta v \sim \varphi_0^2$.

Fig. 1 shows the propagation of two narrow solitons with equal amplitudes [$\alpha_n(t) \equiv \beta_n(t)$] excited simultaneously in the two chains for different coupling constants. As can be expected the coupling between opposite sites in the two chains d which besides the linear term has also a nonlinear term changes significantly the soliton's amplitude through the factor $\sqrt{M/(M+d)}$ as well as the velocity with the amount Δv. For positive values of d the amplitude φ_0 and the velocity become larger [figures 1(a)] while for negative values of d they become smaller [figure 1(a$'$)].

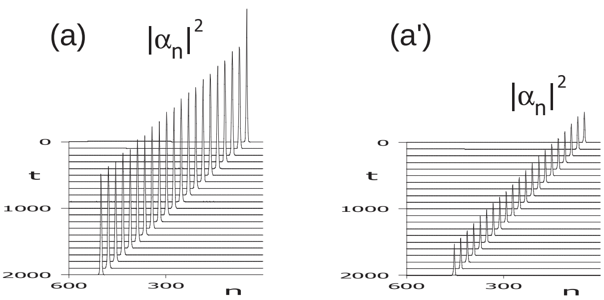

Fig. 1. Propagation of equal narrow ($L = 2$) solitons in the two chains with $M = -1$, $k = 0.1$ and (a): $d = 0.628$; (a$'$): $d = -0.628$. The time is in units of $1/|M|$.

SCATTERING OF BRIGHT SOLITONS FROM POINT DEFECTS

The inhomogeneous static case $\varepsilon \neq 0$, $k = v = 0$ is investigated in detail in [25]. Now we shall study the propagation of an AL soliton which at the initial time is launched in one of the chains

$$\alpha_n(0) = \sinh\frac{1}{L}\,\mathrm{sech}\frac{|n - n_s|}{L}\,e^{ikn}\,, \quad \beta_n(0) = 0 \quad (10)$$

solving numerically the system (1). The simulations are carried out for 250 sites of each chain and periodic boundary conditions. n_s is the place where the soliton is launched initially and is far enough from the defect.

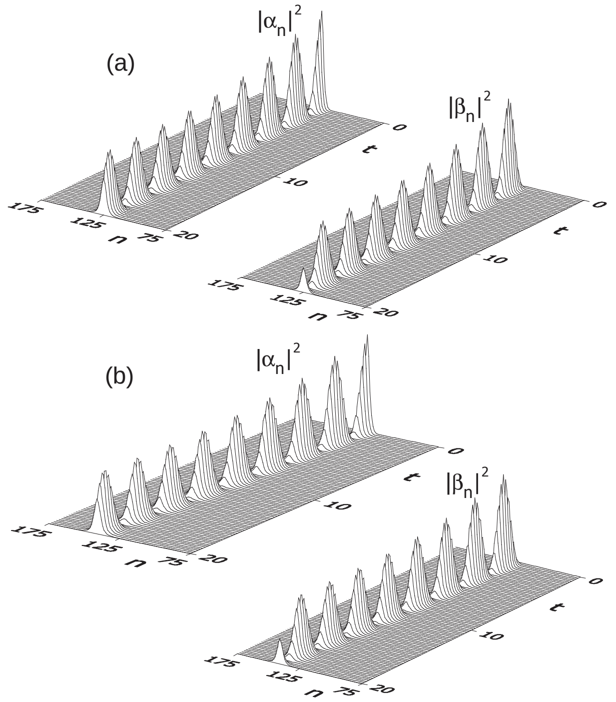

Fig. 2. Perfect switching of a narrow soliton ($L = 2$) in the homogenous case with $d = 0.628$ for (a): $k = 0$ and (b): $k = 0.4$. The time is in units of $1/|M|$.

The width of the solitons is $L = 2$, which underline the discreteness of the system. For wide solitons we can use the continuum approximation and (1) turns in a system of coupled NLS equations. We have chosen $M = -1$ which determines that impurities with $\varepsilon > 0$ are repulsive while impurities with $\varepsilon < 0$ are attractive.

In the homogeneous linear case ($\varepsilon = 0$, $\gamma = 0$) the excitation will transfer from one chain to the other and back with a period

$$t_0 = \pi/2|d|, \qquad (11)$$

where $2d$ is the linear coupling. For our complicated homogeneous model ($\varepsilon = 0$, $\gamma = 1$), which is not linear we observe that an energy exchange between the two chains take place with nearly the same period t_0 and the energy exchange rate depends on the strength of the coupling. For small values of the coupling constant the soliton is only partially transferred. When the coupling increases the transferred rate grows. This behavior is due to the nonlinear coupling terms. We obtained that a soliton can transfer (perfect soliton switching) when the simple condition

$$4|d|L^2 \gg 1 \qquad (12)$$

is fulfilled. Fig. 2 shows perfect soliton switching for the homogenous chains ($\varepsilon = 0$) and different soliton velocities. The process does not depend on the sign of the coupling d. The period from the numerical simulations for the static case [$k = v = 0$, figure 2(a)] as well as for the propagating soliton [$k = 0.4$, $v = 0.81$, figure 2(a)] is approximately 2.5 and is in a good agreement with the value calculated from (11).

For large values of the coupling constant between opposite sites of the chains d the soliton switching is preserve.

In the case of perfect soliton switching ($|d| = 0.628$) the scattering pattern depends strongly on the initial soliton velocity and the strength of the defect. For repulsive defects ($\varepsilon > 0$) the soliton can by only transmitted or reflected. The evolution is more complex in the case of attractive defects ($\varepsilon < 0$). Fig. 3 shows the dynamics of a soliton with a given small velocity ($v = 0.146$) when it interacts with an attractive impurity. Depending on the strength of the impurity

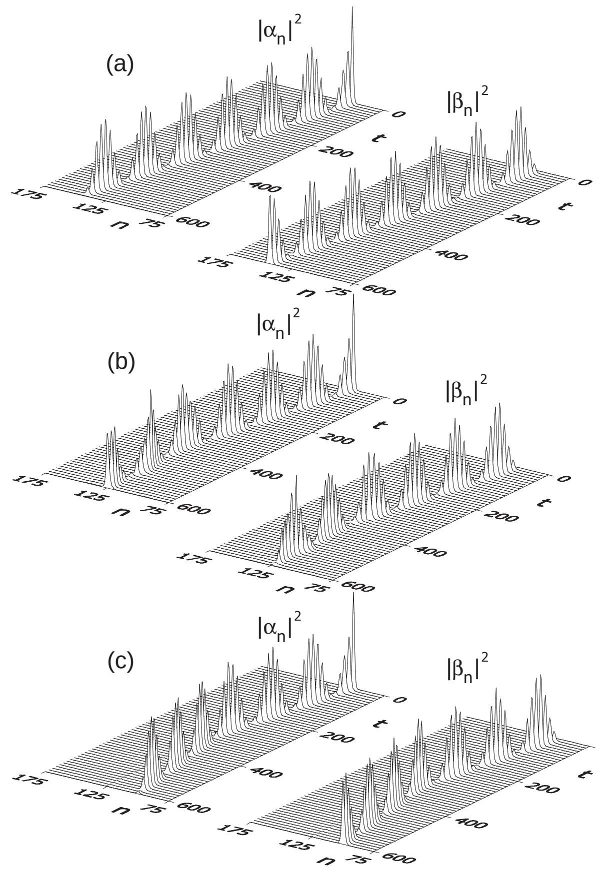

Fig. 3. Scattering of a soliton launched initially in one of the chains at $n_s = 100$ with $k = 0.07$ from an attractive defect placed at $n_0 = 125$ for (a): $\varepsilon = -0.01$; (b): $\varepsilon = -0.1$; (c): $\varepsilon = -1$.

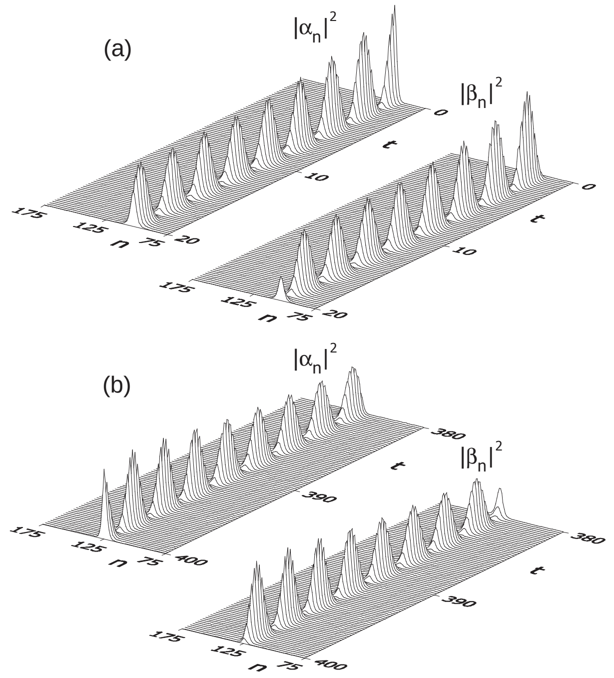

Fig. 4. Evolution of the trapped soliton shown in figure 3(b) for the initial time interval (a) and the time interval after the soliton interaction with the impurity (b).

the soliton can be transmitted [Fig. 3(a), $\varepsilon = -0.01$], trapped [Fig. 3(b), $\varepsilon = -0.1$] or reflected [Figu. 3(c), $\varepsilon = -1$]. We have obtained that in all three cases the soliton transfers from one chain to the other and back periodically. The period of the energy transfer is the same as for homogeneous chains and has a value close to 2.5. Fig. 4 demonstrates in detail the period of the soliton transfer for the parameters when the soliton is trapped. The soliton period before the interaction with the defect [figure 4(a)] and after the interaction with the defect [figure 4(b)] remains the same.

CONCLUSION

We have studied the interaction of discrete bright solitons with impurities in a system of two coupled Ablowitz-Ladik chains. Perfect soliton switching can be obtained for large enough values of the coupling constant. We have investigated the interaction of propagating solitons with point defects. The numerical simulations have shown that solitons which are excited in one of the two inhomogeneous chains with a given velocity can be perfectly switched and at the same time transmitted, trapped or reflected by the attractive impurities depending on the impurity strength.

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РАЗСЕЙВАНЕ НА СОЛИТОНИ ОТ ТОЧКОВИ ДЕФЕКТИ В СИСТЕМА ОТ ДВЕ СВЪРЗАНИ ВЕРИЖКИ НА АБЛОВИЦ–ЛАДИК

Р. Камбурова, М. Приматарова

Институт по физика на твърдото тяло, Българска академия на науките,
бул. "Цариградско шосе" №72, 1784 София, България

(Резюме)

Изследвана е еволюцията на дискретни солитони в система от две свързани чрез параметъра d нехомогенни ($\varepsilon \neq 0$) верижки на Абловиц-Ладик. Тази система представлява дискретен аналог на системата от две свързани нелинейни уравнения на Шрьодингер. В хомогенния случай ($\varepsilon = 0$) тя е неинтегрируема, но има два интеграла на движение.

$$i\frac{\partial \alpha_n}{\partial t} = M(\alpha_{n+1} + \alpha_{n-1})(1 + \gamma|\alpha_n|^2) + 2d\beta_n(1 + \gamma|\alpha_n|^2) + \varepsilon\delta_{n,n_0}\alpha_n$$
$$i\frac{\partial \beta_n}{\partial t} = M(\beta_{n+1} + \beta_{n-1})(1 + \gamma|\beta_n|^2) + 2d\alpha_n(1 + \gamma|\beta_n|^2) + \varepsilon\delta_{n,n_0}\beta_n\,.$$

Разгледано е разпространението на светъл солитон ($\gamma > 0$), който първоначално е формиран само в едната от хомогенните верижките. Определени са периодът на прехвърляне на възбуждението и условието за пълно солитонно превключване. За линейния случай прехвърлянето е с период $t_0 = \pi/|d|$. Числените изследвания показват, че този период е почти същият за изследвания модел, който не е линеен и пълно прехвърляне на енергията на солитона от едната верижка в другата и обратно има при условие $4|d|L^2 \gg 1$, където L е ширината на солитона.

Процесът на превключване се запазва при големи стойности на d. Той не зависи от знака на свързващия коефициент и скоростта на солитона остава непроменена.

Солитонната динамика в нехомогенната система зависи от параметрите на солитона (ширина и скорост), свързващата константа и стойността на дефекта. Линейните точкови дефекти не влияят върху периода на енергиен обмен и той е близък по стойност на този за хомогенния случай. Числените изследвания показват, че когато в едната верижка е формиран солитон с определена скорост той запазва периодичната си динамика на преминаване от едната верижка в другата и обратно, като в същото време в зависимост от стойността на дефекта на привличане може да премине, да се захване или отрази от него.