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Application of finite-difference method for numerical investigation of eigenmodes of anisotropic optical waveguides with an arbitrary tensor

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In this paper 3D finite-difference methods is developed for analyzing anisotropic optical waveguides. An eigenvalue matrix equation is derived through considering simultaneously four transverse field components. The numerical results show that the proposed scheme is highly efficient and yields complex effective indices while requiring match less computer memory and calculation time than the commonly used methods. Algorithm is used to study modes on a electrooptic polymer waveguide and a liquid-crystal optical waveguide with arbitrary director orientation, clearly demonstrated in the numerical examples.

Key words: 3D finite-difference optical waveguides, complex effective indices, fullvector modesolver

INTRODUCTION

In fiber and integrated optics, a fundamental problem is to compute the eigenmodes of optical waveguides. Several techniques are commonly used to compute the electromagnetic modes of waveguides, including finite element methods, mode-matching techniques, method of lines, and finite difference methods. Many methods completely neglect the anisotropy of the constituent materials. Of those that do account for material anisotropy, most require that the permittivity tensor be diagonal when expressed in the coordinate system of the waveguide. In this paper, we present an finite-difference (FD) method for solving full-vector modes of optical waveguides with arbitrary permittivity tensor, i.e. with general three-dimensional (3-D) anisotropy. Only fewer FD works deal with structures including anisotropic materials [1-5] and they at most considered anisotropy in the transverse plane, i.e. in the waveguide cross-section. In [5] was developed a more rigorous vector FD mode solver, based on the traditional approach for anisotropic waveguides, but again assuming non-diagonal anisotropy only in the transverse plane (in permittivity tensor all $\varepsilon_{xz}, \varepsilon_{yz}, \varepsilon_{zx}, \varepsilon_{zy} = 0$). Now, we derive a matrix standard eigenvalue problem from the FD method, which is easier to solve for the all modes. For absorbing boundary condition is used method of perfectly matched layers (PMLs) for anisotropic media [6] at the outer boundaries of the computational domain. In so doing, leaky waveguide modes having complex propagation constants can also be analyzed.

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For optimize numerical realization we proposed scheme with only one complex array similar complex potential of Riemann-Silberstein [7], instead four field components (two for transverse electric field components E_x, E_y and two for transverse magnetic field components H_x and H_y - total four arrays).

The eigenmodes and eigenvalues of the waveguide were calculated using an iterative shift-inverse power method with Rayleigh criteria.

FORMULATION

For analysis of a waveguide is used computational domain (Fig. 1), where the waveguide cross-section in the transverse $x-y$ plane is truncated and surrounded by PML regions of thickness d. The incorporation of PML regions allows the analysis of leaky modes.

We consider anisotropic material, which permittivity tensor $[\varepsilon]$ is formulated as:

$$[\varepsilon] = \varepsilon_0 \left\{ \begin{matrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{matrix} \right\}, \qquad (1)$$

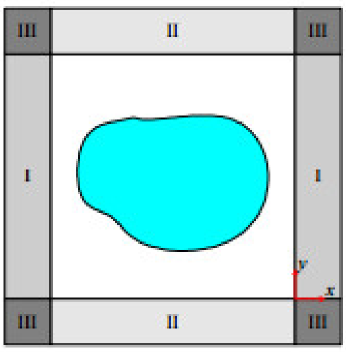


Fig. 1. The cross-section of arbitrary waveguide with the PML, placed at the edges of the computing domain.

where ε_0 is the permittivity of free space, and $\varepsilon_{i,j}$ may be a complex value.

For permeability tensor $[\mu]$ is assumed diagonal structure:

$$[\mu] = \mu_0 \left\{ \begin{array}{ccc} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{array} \right\}, \tag{2}$$

where μ_0 is the permeability of free space, and $\mu_{i,j}$ probably are equal to 1 for non-magnetic waveguide structures.

Assuming a z dependence of $e^{-i\beta z}$ for all fields and a time dependence of $e^{i\omega t}$, with β being the modal propagation constant Maxwell's equations can be written as:

$$\nabla \cdot ([\varepsilon]\vec{E}) = 0 \tag{3}$$

$$\nabla \times \vec{E} = -i\omega([\mu]\vec{H}) \tag{4}$$

$$\nabla \cdot ([\mu]\vec{H}) = 0 \tag{5}$$

$$\nabla \times \vec{H} = i\omega([\varepsilon]\vec{E}) \tag{6}$$

In the matrix form assuming $\dfrac{\partial}{\partial z} = -i\beta$ curl equations can be written as:

$$\left\{ \begin{array}{ccc} 0 & i\beta & \dfrac{\partial}{\partial y} \\ -i\beta & 0 & -\dfrac{\partial}{\partial x} \\ -\dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} & 0 \end{array} \right\} \left\{ \begin{array}{c} E_x \\ E_y \\ E_z \end{array} \right\} = -\omega\mu_0 \left\{ \begin{array}{ccc} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{array} \right\} \left\{ \begin{array}{c} H_x \\ H_y \\ H_z \end{array} \right\} \tag{7}$$

$$\left\{ \begin{array}{ccc} 0 & i\beta & \dfrac{\partial}{\partial y} \\ -i\beta & 0 & -\dfrac{\partial}{\partial x} \\ -\dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} & 0 \end{array} \right\} \left\{ \begin{array}{c} H_x \\ H_y \\ H_z \end{array} \right\} = \omega\varepsilon_0 \left\{ \begin{array}{ccc} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{array} \right\} \left\{ \begin{array}{c} E_x \\ E_y \\ E_z \end{array} \right\} \tag{8}$$

For eigenmode form equations we need to eliminate E_z and H_z form (7) and (8).

Expression for H_z can be obtained from equation (5) or from third of equations (7).

$$H_z = \frac{1}{i\beta\mu_{zz}} \left(\mu_{xx}\frac{\partial H_x}{\partial x} + \mu_{yy}\frac{\partial H_y}{\partial y} \right) \quad \text{or}$$
$$H_z = -\frac{1}{i\omega\mu_0\mu_{zz}} \left(-\frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial x} \right). \tag{9}$$

For E_z in general case we have from third of equation (8)

$$E_z = -\frac{\varepsilon_{zx}}{\varepsilon_{zz}E_x} - \frac{\varepsilon_{zy}}{\varepsilon_{zz}E_y} + \frac{1}{i\omega\varepsilon_0\varepsilon_{zz}} \left(-\frac{\partial H_x}{\partial y} + \frac{\partial H_y}{\partial x} \right). \tag{10}$$

By substituting the E_z and H_z expressions into (7) and (8) can be derived eigenvalue matrix equation for the four transverse field components.

Our new original proposal is to replace these four field components with only one complex filed component $\vec{F}$ and its conjugate $\vec{F}^*$

$$\vec{F} = \frac{1}{2}\vec{E} + i\frac{1}{2}\vec{H}. \tag{11}$$

Evidently we have

$$\vec{E} = \vec{F} + \vec{F}^*, \qquad i\vec{H} = \vec{F} - \vec{F}^* \tag{12}$$

$$\nabla \times \vec{F} = \frac{1}{2}\nabla \times \vec{E} + i\frac{1}{2}\nabla \times \vec{H} \tag{13}$$

$$\nabla \cdot \vec{F} = \frac{1}{2}\nabla \cdot \vec{E} + i\frac{1}{2}\nabla \cdot \vec{H} \tag{14}$$

Therefore, the equations (7) and (8) are reduced to an equation:

$$\left\{ \begin{array}{ccc} 0 & i\beta & \dfrac{\partial}{\partial y} \\ -i\beta & 0 & -\dfrac{\partial}{\partial x} \\ -\dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} & 0 \end{array} \right\} \left\{ \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \right\} = \frac{1}{2}[T_1] \left\{ \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \right\} + \frac{1}{2}[T_2] \left\{ \begin{array}{c} F_x^* \\ F_y^* \\ F_z^* \end{array} \right\}, \tag{15}$$

where $F_z = \frac{1}{2}E_z + i\frac{1}{2}H_z$ already known from expressions (9) and (10),

$$\begin{array}{l} [T_1] = \omega\mu_0[\mu] + \omega\varepsilon_0[\varepsilon] \\ [T_2] = -\omega\mu_0[\mu] + \omega\varepsilon_0[\varepsilon]. \end{array} \tag{16}$$

For the boundary conditions of considered anisotropic waveguide in the PML (perfected matched layer) regions, the permittivity and permeability tensors are taken to be

$$[\varepsilon_{PML}] = \mu_0 \begin{Bmatrix} \frac{s_y s_z}{s_x}\varepsilon_{xx} & s_z\varepsilon_{xy} & s_y\varepsilon_{xz} \\ s_z\varepsilon_{yx} & \frac{s_x s_z}{s_y}\varepsilon_{yy} & s_z\varepsilon_{yz} \\ s_y\varepsilon_{zx} & s_x\varepsilon_{zy} & \frac{s_x s_y}{s_z}\varepsilon_{zz} \end{Bmatrix} \qquad (17)$$

$$[\mu_{PML}] = \mu_0 \begin{Bmatrix} \frac{s_y s_z}{s_x}\mu_{xx} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y}\mu_{yy} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z}\mu_{zz} \end{Bmatrix}, \qquad (18)$$

where s_x, s_y and s_z are the complex PML parameters defined as

$$s_x = 1 - i\alpha_x\,, \quad s_y = 1 - i\alpha_y\,, \quad s_z = 1 - i\alpha_z \qquad (19)$$

witch appropriate values controlling the field attenuation in PML regions. As in [6] we may determine parameter s as follows

$$s = 1 - i\alpha = 1 - i\frac{\sigma_e}{\omega\varepsilon_0 n^2} = 1 - i\frac{\sigma_m}{\omega\mu_0}, \qquad (20)$$

where σ_e and σ_m are the electric and magnetic conductivities of the PML, respectively, and n is the refractive index of the adjacent computing domain. This relation means that the wave impedance of a PML medium exactly equals to that of the adjacent medium in the computing window regardless of the angle of propagation. Assume that the electric conductivity of the PML medium has an m-power profile as

$$\sigma_e(\rho) = \sigma_{\max}\left(\frac{\rho}{d}\right)^m, \qquad (21)$$

where ρ is the distance from the beginning of the PML and d is thickness. At the interface of the PML and the computing window, the theoretical reflection coefficient for the normal incident wave is

$$R = \exp\left[-2\frac{\sigma_{\max}}{\varepsilon_0 cn}\int_0^d (\rho/d)^m d\rho\right], \qquad (22)$$

and the maximum conductivity $\sigma_{\max}$ can then be determined as

$$\sigma_{\max} = \frac{m+1}{2}\frac{\varepsilon_0 cn}{d}\ln\left(\frac{1}{R}\right), \qquad (23)$$

where c is the speed of light in free space. For the case of $m = 2$

$$s = 1 - i\frac{3\lambda}{4\pi nd}\left(\frac{\rho}{d}\right)^2 \ln\left(\frac{1}{R}\right). \qquad (24)$$

In our considerations we choose

$$\alpha_j = \alpha_{j,\max}\left(\frac{\rho}{d}\right)^2 \qquad (25)$$

for $j = x$ and $j = y$, and $\alpha_z = 0$, where ρ represents the distance in the j-direction from the beginning of the PML region and $\alpha_{j,\max}$ is determined by the assumed reflectivity value from the PML layer.

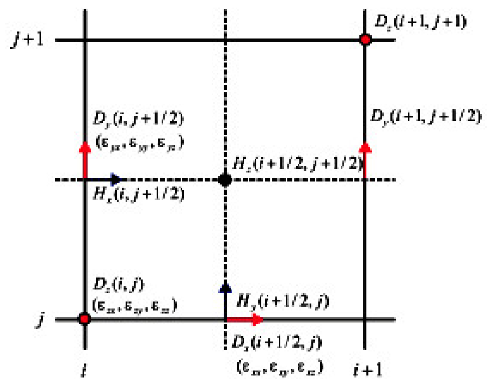
When the waveguide media can be represented with diagonal permittivity tensor it is more convenient to use modified differential operator in PML regions:

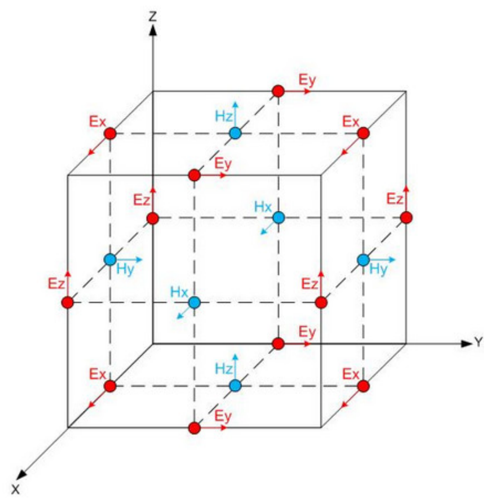
$$\nabla = \left(\frac{1}{s_x}\frac{\partial}{\partial x}, \frac{1}{s_y}\frac{\partial}{\partial y}, \frac{1}{s_z}\frac{\partial}{\partial z}\right) \qquad (26)$$

without changes of permittivity and permeability tensors.

NUMERICAL SCHEME REALISATION

For numerical discretisation of equations (15) is used (FD) finite difference method based Yee's mesh algorithm by applying the central difference scheme for the differential operators. In brackets i and j is not indexes of arrays, they presented Yee's cells coordinates.





Applying the Yee's mesh and the central difference scheme expressions for curl operators

$$\nabla \times (\vec{F}+\vec{F}^*) = \nabla \times \vec{E} = -i\omega\mu_0[\mu]\vec{H} = \omega\mu_0[\mu](\vec{F}-\vec{F}^*) \tag{27}$$

$$\nabla \times (\vec{F}-\vec{F}^*) = \frac{1}{i}\nabla \times \vec{H} = \omega\varepsilon_0[\varepsilon]\vec{E} = \omega\varepsilon_0[\varepsilon](\vec{F}+\vec{F}^*) \tag{28}$$

become

$$0\cdot E_{x,(i,j+1/2)} + i\beta E_{y,(i,j+1/2)} + \frac{E_{z,(i,j+1)} - E_{z,(i,j)}}{y_{(j+1)} - y_{(j)}} = -i\omega\mu_0\mu_{xx}H_{x,(i,j+1/2)} \tag{29}$$

$$-i\beta E_{x,(i+1/2,j)} + 0\cdot E_{y,(i+1/2,j)} + \frac{E_{z,(i+1,j)} - E_{z,(i,j)}}{x_{(i+1)} - x_{(i)}} = -i\omega\mu_0\mu_{yy}H_{y,(i+1/2,j)} \tag{30}$$

$$\frac{E_{y,(i+1,j+1/2)} - E_{y,(i,j+1/2)}}{x_{(i+1)} - x_{(i)}} - \frac{E_{x,(i+1/2,j+1)} - E_{x,(i+1/2,j)}}{y_{(j+1)} - y_{(j)}} + 0\cdot E_{z,(i+1/2,j+1/2)} = -i\omega\mu_0\mu_{zz}H_{z,(i+1/2,j+1/2)} \tag{31}$$

$$\begin{aligned} 0\cdot H_{x,(i+1/2,j)} + i\beta H_{y,(i+1/2,j)} + \frac{H_{z,(i+1/2,j+1/2)} - H_{z,(i+1/2,j-1/2)}}{y_{(j+1/2)} - y_{(j-1/2)}} &= i\omega\varepsilon_0\varepsilon_{xx}E_{x,(i+1/2,j)} \\ &+ i\omega\varepsilon_0\varepsilon_{xy}E_{y,(i+1/2,j)} + i\omega\varepsilon_0\varepsilon_{xz}E_{z,(i+1/2,j)} \end{aligned} \tag{32}$$

$$\begin{aligned} -i\beta H_{x,(i,j+1/2)} + 0\cdot H_{y,(i,j+1/2)} + \frac{H_{z,(i+1/2,j+1/2)} - H_{z,(i-1/2,j+1/2)}}{x_{(i+1/2)} - x_{(i-1/2)}} &= i\omega\varepsilon_0\varepsilon_{yx}E_{x,(i,j+1/2)} \\ &+ i\omega\varepsilon_0\varepsilon_{yy}E_{y,(i,j+1/2)} + i\omega\varepsilon_0\varepsilon_{yz}E_{z,(i,j+1/2)} \end{aligned} \tag{33}$$

$$\begin{aligned} -\frac{H_{x,(i,j+1/2)} + H_{x,(i,j-1/2)}}{y_{(j+1/2)} - y_{(j-1/2)}} + \frac{H_{y,(i+1/2,j)} - H_{y,(i-1/2,j)}}{x_{(i+1/2)} - x_{(i-1/2)}} + 0\cdot H_{z,(i,j)} &= i\omega\varepsilon_0\varepsilon_{zx}E_{x,(i,j)} \\ &+ i\omega\varepsilon_0\varepsilon_{zy}E_{y,(i,j)} + i\omega\varepsilon_0\varepsilon_{zz}E_{z,(i,j)} \end{aligned} \tag{34}$$

In our case $F_x = E_x + F_x^*$, $F_y = E_y + F_y^*$, $F_z = E_z + F_z^*$, $F_x^* = F_x - iH_x$, $\ldots$etc.
Assuming field vectors functions represents one column array according Yee's cells.
For example if $\Delta x = x_{i+1} - x_i$ where $i = 0,1,2,\ldots,n$ and $\Delta y = y_{j+1} - y_j$ where $j = 0,1,2,\ldots,m$

$$E = (E_{x_0,y_0}, E_{x_0+\Delta x,y_0}, \ldots, E_{x_0+n\Delta x,y_0}, E_{x_0,y_0+\Delta y}, E_{x_0+\Delta x,y_0+\Delta y}, \ldots, E_{x_0+n\Delta x,y_0+\Delta y}, \ldots, E_{x_0+n\Delta x,y_0+m\Delta y})^{tr} \tag{35}$$

$$H = (H_{x_0,y_0}, H_{x_0,y_0+\Delta y}, \ldots, H_{x_0,y_0+m\Delta y}, H_{x_0+\Delta x,y_0}, H_{x_0+\Delta x,y_0+\Delta y}, \ldots, H_{x_0+n\Delta x,y_0+\Delta y}, \ldots, H_{x_0+n\Delta x,y_0+m\Delta y})^{tr}, \tag{36}$$

where E can be any of E_x, E_y or E_z and H can be H_x, H_y or H_z.
Then we can define derivates of functions using matrices according [8]

$$\frac{\partial}{\partial x}E = \mathbf{U_x E} = \frac{1}{\Delta x}\begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & & \\ & & & -1 & 1 \\ & & & & -1 \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{n\times(m-1)} \\ E_{n\times m} \end{bmatrix}, \tag{37}$$

$$\frac{\partial}{\partial y}E = \mathbf{U_y E} = \frac{1}{\Delta y}\begin{bmatrix} -1 & \text{“n”}\ldots & 1 & & \\ & -1 & & \ddots & \\ & & \ddots & & 1 \\ & & & -1 & \\ & & & & -1 \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{(n-1)\times m} \\ E_{n\times m} \end{bmatrix}, \tag{38}$$

$$\frac{\partial}{\partial x}H = \mathbf{V_x}\mathbf{H} = \frac{1}{\Delta x}\begin{bmatrix} 1 & & & & \\ -1 & 1 & & & \\ & & \ddots & & \\ & & -1 & 1 & \\ & & & -1 & 1 \end{bmatrix}\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{(n-1)\times m} \\ H_{n\times m} \end{bmatrix}, \tag{39}$$

$$\frac{\partial}{\partial y}H = \mathbf{V_y}\mathbf{H} = \frac{1}{\Delta y}\begin{bmatrix} 1 & & & \\ & 1 & & \\ -1 & & \ddots & \\ & & & 1 \\ & -1 & & 1 \end{bmatrix}\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{n\times(m-1)} \\ H_{n\times m} \end{bmatrix}. \tag{40}$$

Final eigenvalue matrix equation for the four transverse field components can be derived as in the fooling form:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}\begin{bmatrix} E_x \\ H_x \\ E_y \\ H_y \end{bmatrix} = \beta \begin{bmatrix} E_x \\ H_x \\ E_y \\ H_y \end{bmatrix} \tag{41}$$

After some obvious algebraic simplification (41) can be written as:

$$\begin{bmatrix} B_{33} & B_{34} \\ B_{43} & B_{44} \end{bmatrix}\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \beta \begin{bmatrix} F_x \\ F_y \end{bmatrix}, \tag{42}$$
$$2F_x = E_x + iH_x, \qquad 2F_y = E_y + iH_y$$

and once solved this matrix equation, then can find four transverse field components by formulas (12). Or we can solve equation (41) and directly find field components. Note, if choose (42) matrix equations are twice less, which reduces memory and optimize iterative work of eigen solver.

Below are listed coefficients of matrix A. They are similar to [8], distinguished by their ranking

$$\begin{cases} A_{11} = -i\dfrac{\varepsilon_{zx}}{\varepsilon_{zz}}\mathbf{U_x}, \\ A_{12} = -\dfrac{1}{\omega\varepsilon_0\varepsilon_{zz}}\mathbf{U_x}\mathbf{V_y}, \\ A_{13} = -i\dfrac{\varepsilon_{zy}}{\varepsilon_{zz}}\mathbf{U_x}, \\ A_{14} = \dfrac{1}{\omega\varepsilon_0\varepsilon_{zz}}\mathbf{U_x}\mathbf{V_x} + \omega\mu_0\mu_{yy}\mathbf{I}, \end{cases} \tag{43}$$

$$\begin{cases} A_{21} = -\omega\varepsilon_0\varepsilon_{yx}\mathbf{I} + \dfrac{1}{\omega\mu_0\mu_{zz}}\mathbf{V_x}\mathbf{U_y} + \frac{\omega\varepsilon_{yz}\varepsilon_{zx}}{\varepsilon_{zz}}\mathbf{I}, \\ A_{22} = -i\dfrac{\varepsilon_{yz}}{\varepsilon_{zz}}\mathbf{V_y}, \end{cases} \tag{44}$$

$$\begin{cases} A_{23} = -\omega\varepsilon_0\varepsilon_{yy}\mathbf{I} - \dfrac{1}{\omega\mu_0\mu_{zz}}\mathbf{V_x}\mathbf{U_x} + \frac{\omega\varepsilon_{yz}\varepsilon_{zy}}{\varepsilon_{zz}}\mathbf{I}, \\ A_{24} = i\frac{\varepsilon_{yz}}{\varepsilon_{zz}}\mathbf{V_x}, \end{cases} \tag{45}$$

$$\begin{cases} A_{31} = -i\dfrac{\varepsilon_{zx}}{\varepsilon_{zz}}\mathbf{U_y}, \\ A_{32} = -\dfrac{1}{\omega\varepsilon_0\varepsilon_{zz}}\mathbf{U_y}\mathbf{V_y} - \omega\mu_0\mu_{xx}\mathbf{I}, \\ A_{33} = -i\dfrac{\varepsilon_{zy}}{\varepsilon_{zz}}\mathbf{U_y}, \\ A_{34} = \dfrac{1}{\omega\varepsilon_0\varepsilon_{zz}}\mathbf{U_y}\mathbf{V_x}, \end{cases} \tag{46}$$

$$\begin{cases} A_{41} = \omega\varepsilon_0\varepsilon_{xx}\mathbf{I} + \dfrac{1}{\omega\mu_0\mu_{zz}}\mathbf{V_y}\mathbf{U_y} - \dfrac{\omega\varepsilon_{xz}\varepsilon_{zx}}{\varepsilon_{zz}}\mathbf{I}, \\ A_{42} = i\dfrac{\varepsilon_{xz}}{\varepsilon_{zz}}\mathbf{V_y}, \end{cases} \tag{47}$$

$$\begin{cases} A_{43} = \omega\varepsilon_0\varepsilon_{xy}\mathbf{I} - \dfrac{1}{\omega\mu_0\mu_{zz}}\mathbf{V_y}\mathbf{U_x} - \dfrac{\omega\varepsilon_{xz}\varepsilon_{zy}}{\varepsilon_{zz}}\mathbf{I}, \\ A_{44} = -i\frac{\varepsilon_{xz}}{\varepsilon_{zz}}\mathbf{V_x}. \end{cases} \tag{48}$$

NUMERICAL EXAMPES AND RESULTS

To demonstrate the accuracy and applications of the proposed 3D method, we present two numerical examples, one with electrooptic polymer waveguide that is poled at an oblique angle relative to the direction [9 - 10], and the other involving LCs with arbitrary director orientation [11].

In first example the cladding layers are assumed to be nonpolar (and hence immune to the poling field) with an isotropic refractive index of $n_1 = n_3 = 1.60$. The middle layer is modeled as an electrooptic polymer with a refractive index of $n_2 = 1.65$ prior to poling. This waveguide structure provides an interesting challenge for mode calculations because both the orientation and strength of the birefringence are nonuniform throughout the waveguide core. Moreover,

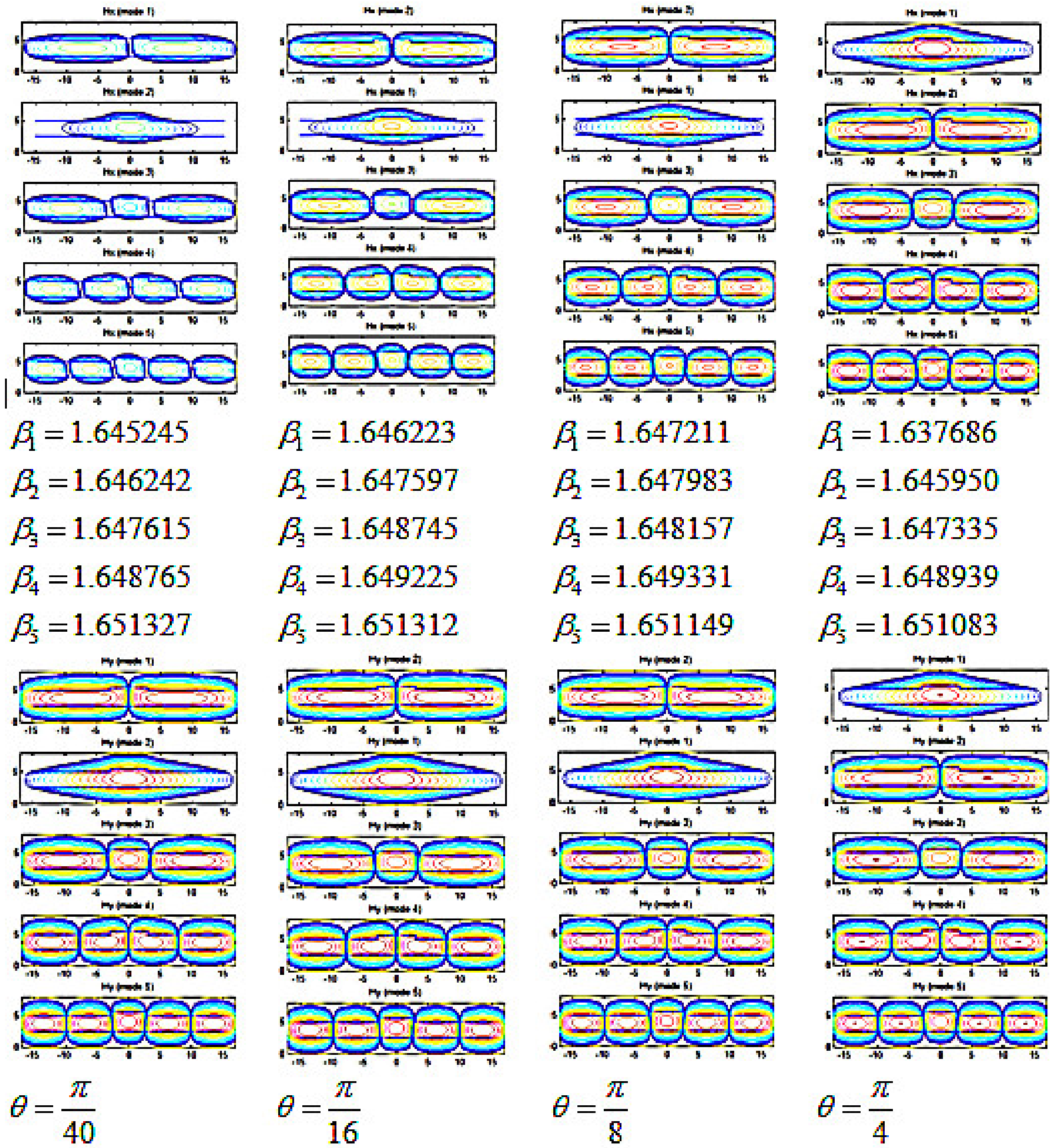


Fig. 2. H_x and H_y modes of electrooptic polymer waveguide.

the induced anisotropy, although small, plays an essential role in guaranteeing transverse mode confinement. The net birefringence is assumed to be split between the ordinary and extraordinary indices in the following way:

$$n_{2e} = n_2 + (2/3)\Delta n, \quad n_{2o} = n_2 - (1/3)\Delta n \tag{49}$$

with Δn given by eq (18) in [5]. The local permittivity tensor in the middle layer is then described by

$$\begin{bmatrix} n_{2e}^2\cos^2\theta + n_{2o}^2\sin^2\theta & (n_{2e}^2 - n_{2o}^2)\sin\theta\cos\theta & 0 \\ (n_{2e}^2 - n_{2o}^2)\sin\theta\cos\theta & n_{2e}^2\cos^2\theta + n_{2o}^2\sin^2\theta & 0 \\ 0 & 0 & n_{2o}^2 \end{bmatrix}, \tag{50}$$

where θ is an angle of axis of anisotropy.

Our results for first 5 modes (H_x, H_y) and for 4 different values of θ are shown in Fig. 2.

In [5] is shown only fundamental mode.

In second example we study an LC (liquid crystal) optical waveguide. The substrate being glass with the refractive index $n_1 = 1.45$ and the core region being filled with nematic LCs (5CB). The elements of the relative permittivity tensor of the nematic LCs are given as:

$$\varepsilon_{xx}/\varepsilon_0 = n_o^2 + (n_e^2 - n_o^2)\sin^2\theta\cos^2\phi \tag{51}$$
$$\varepsilon_{xy}/\varepsilon_0 = \varepsilon_{yx} = (n_e^2 - n_o^2)\sin^2\theta\sin\phi\cos\phi \tag{52}$$
$$\varepsilon_{xz}/\varepsilon_0 = \varepsilon_{zx} = (n_e^2 - n_o^2)\sin\theta\cos\theta\cos\phi \tag{53}$$
$$\varepsilon_{yy}/\varepsilon_0 = n_o^2 + (n_e^2 - n_o^2)\sin^2\theta\cos^2\phi \tag{54}$$
$$\varepsilon_{yz}/\varepsilon_0 = \varepsilon_{zy} = (n_e^2 - n_o^2)\sin\theta\cos\theta\sin\phi \tag{55}$$
$$\varepsilon_{zz}/\varepsilon_0 = n_o^2 + (n_e^2 - n_o^2)\cos^2\theta. \tag{56}$$

We have made many calculations at different rotation angles (θ, ϕ) defining the director of LC, where θ is the angle between the crystal c-axis and the z-axis, and ϕ is the angle between the projection of the crystal c-axis on the $x-y$ plane and the x-axis, as shown at Fig. 3.

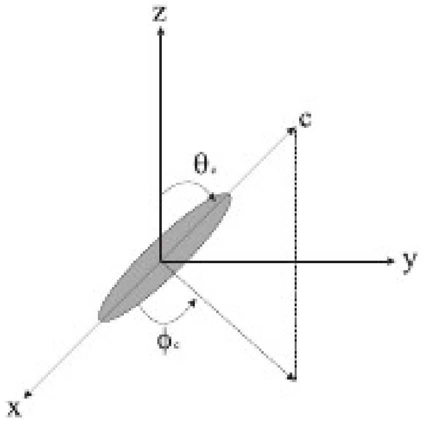


Fig. 3. Schematic definition of rotation angles for the LC molecular or director.

We use that where $n_o = 1.5292$ and $n_e = 1.7072$ are, respectively, the ordinary and extraordinary refractive indices of the nematic LCs.

To illustrate 3D features of our method we present all four field modes for selected values $\theta = 60°$, $\phi = 0$ and $\theta = 60°$, $\phi = 60°$. Results are shown at Fig. 4 and Fig. 5.

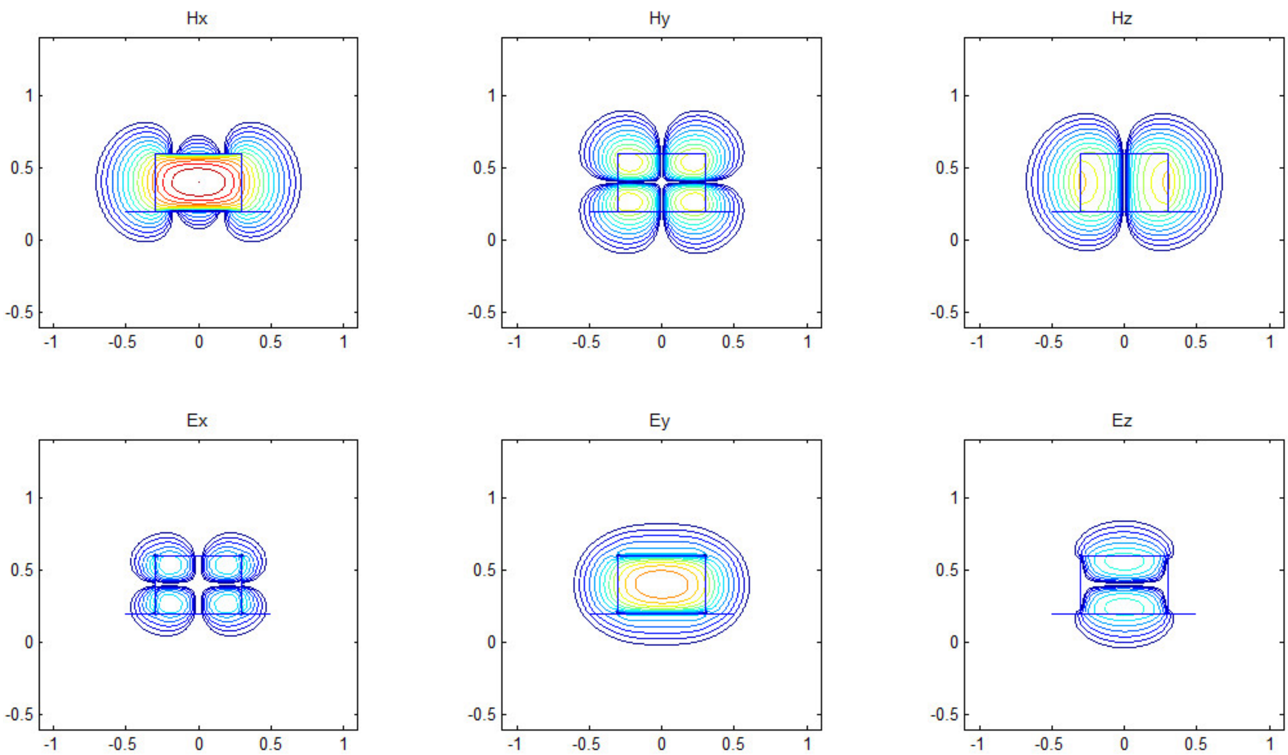


Fig. 4. All mode–field profiles for the first mode of the LC waveguide with $\theta = 60°$, $\phi = 0$.

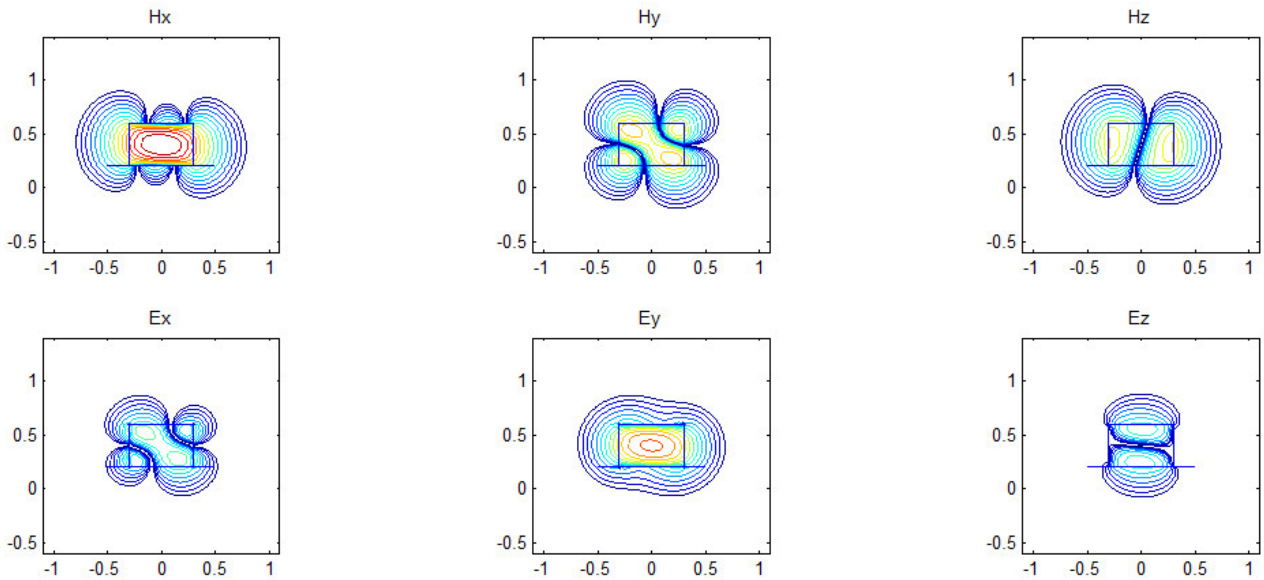


Fig. 5. All mode–field profiles for the first mode of the LC waveguide with $\theta = 60°$, $\phi = 60°$.

CONCLUSIONS

We have presented a new FD method based eigenvalue algorithm for computing guided modes of anisotropic optical waveguides with arbitrary permittivity tensor. Instead of using the standard eigenvalue matrix equation involving four transverse field components, we involve only two (complex vector of Riemann- Silberstein and its conjugated) witch halves the required computer memory. Algorithm has been used to solve guided modes on a liquid-crystal optical waveguide with arbitrary molecular director orientation. This established mode solver provides an efficient tool for studying and designing as ordinary waveguides and waveguides with complicated materials such as liquid crystals.

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ПРИЛОЖЕНИЕ НА МЕТОДА НА КРАЙНИТЕ РАЗЛИКИ ЗА НАМИРАНЕ НА СОБСТВЕНИТЕ СТОЙНОСТИ И ВЕКТОРИ НА МНОГОСЛОЙНИ АНИЗОТРОПНИ ОПТИЧНИ ВЪЛНОВОДИ

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(Резюме)

В работата се предлага числов модел на основата на 3D метода на крайните разлики за изследване на анизотропни оптични вълноводи с произволна конфигурация на тензора на диелектрична проницаемост. Прилагането му води до получаване на удобно матрично уравнение, от което се определят спектъра и собствените му функции, които представят четирите напречни полеви компоненти, подредени плъно едно под друго в едноразмерен масив. Намирането на възможните комплексни собствени стойности и вектори се извършва посредством shifted inverse power method, чието отместване се определя динамично посредством коефициента на Релей. Той е особено ефективен при разредени лентъчни матрици и притежава почти кубична сходимост. Алгоритмът е приложен за изчисление на модите на: 1) вълновод с електрооптичен полимерен слой; и 2) вълновод с течнокристален слой с произволна ориентация на направляващия вектор. Числовите резултати се съгласуват много добре с получени чрез други числови алгоритми.