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Heating of the solar corona by Alfvén waves – self-induced opacity

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Static distributions of temperature and wind velocity at the transition region are calculated within the framework of magnetohydrodynamics (MHD) of completely ionized hydrogen plasma. The numerical solution of the derived equations gives the width of the transition layer between the chromosphere and the corona as a self-induced opacity of high-frequency Alfvén waves (AW). The domain wall is direct consequence of the self-consistent MHD treatment of AW propagation. The low-frequency MHD waves coming from the Sun are strongly reflected by the narrow transition layer, while the high-frequency waves are absorbed – that is why we predict considerable spectral density of the AW in the photosphere. The numerical method allows consideration of incoming AW with arbitrary spectral density. The idea that Alfvén waves might heat the solar corona belongs to Alfvén, we simply solved the corresponding MHD equations. The comparison of the solution to the experiment is crucial for revealing the heating mechanism.

Key words: Alfvén waves, Magnetohydrodynamics, Plasma, Numerical Methods, Solar corona

ALFVÉN MODEL FOR CORONA HEATING

The discovery of the lines of the multiply ionized iron in the solar corona spectrum [1] posed an important problem for the fundamental physics - what is the mechanism of the heating of the solar corona and why the temperature of the corona is 100 times larger than the temperature of the photosphere.

The first idea by Alfvén [2] was that Alfvén waves (AW) [3] are the mechanism for heating the corona. AW are generated by the turbulence in the convection zone and propagate along the magnetic field lines. Absorption is proportional to ω^2 and the heating comes from high-frequency AW. Alfvén's idea for the viscous heating of plasma by absorption of AW was analyzed in the theoretical work by Heyvaerts [4]. In support of this idea is the work by Chitta [5] (Figures 8, 9 therein). The authors came to the conclusion that the spectral density satisfies a power law with an exponent of 1.59. This gives a strong hint that this scaling can be extrapolated in the nearest spectral range for times less than 1 s and frequencies in the 1 Hz range. We should also mention two recent papers [6, 7] investigating the possibility of heating the Solar corona by AW. Furthermore, Tomczyk [8] states that there exist very few direct measurements of the strength and orientation of coronal magnetic fields, meaning that the mechanisms responsible for heating the corona, driving the solar wind, and initiating coronal mass ejections remain poorly understood. However, recently discovered spatially and temporally ubiquitous waves in the solar corona [9] gave strong support for Alfvén's idea. A clear presence of outward and inward propagating waves in the corona was noted. $k-\omega$ diagnostics revealed coronal wave power spectrum with an exponent of $\approx -\frac{3}{2}$ (cf. Fig. 2 of [8]). The low frequency AW, on the other hand, reach the Earth orbit and thanks to the magnetometers on the various satellites we "hear" the bass of the great symphony of solar turbulence.

The purpose of the present work is to examine whether the initial Alfvén idea is correct and to solve the MHD equations which give the dependence of the temperature on the height $T(x)$ and the related velocity of the solar wind $U(x)$ supposing static density of the incoming AW. Due to the high density of the transition layer MHD is an adequate tool to analyze the beginning of the process. Without a doubt the kinetic approach is indispensable for the treatment of low density solar corona but this problem is beyond the purpose of the present work. When the plasma gets hotter and more dilute the MHD treatment is not sufficient and a detailed kinetic theory is required.

Our starting point are the MHD equations for the velocity field $\mathbf{v}$ and magnetic field $\mathbf{B}$

$$\partial_t \rho + \mathrm{div}\mathbf{j} = 0, \qquad \mathbf{j} = \rho\mathbf{v}, \tag{1}$$

$$\partial_t \left(\frac{1}{2}\rho\mathbf{v}^2 + \varepsilon + \frac{\mathbf{B}^2}{2\mu_0}\right) + \mathrm{div}\,\mathbf{q} = 0, \tag{2}$$

$$\partial_t(\rho\mathbf{v}) + \nabla\cdot\Pi = 0, \tag{3}$$

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where

$$\mathbf{q}=\rho\left(\frac{1}{2}\mathbf{v}^2+h\right)\mathbf{v}+\mathbf{v}\cdot\Pi^{(\mathrm{visc})}-\varkappa\nabla T+\mathbf{S} \quad (4)$$

is the energy density flux, ρ is the mass density, ε is the internal energy density, $\varkappa$ is the thermal conductivity, h is the enthalpy per unit mass;

$$\mathbf{S}=\frac{1}{\mu_0}\left[\mathbf{B}\times(\mathbf{v}\times\mathbf{B})-\nu_{\mathrm{m}}\mathbf{B}\times(\nabla\times\mathbf{B})\right], \quad (5)$$

is the Poynting vector and $\nu_{\mathrm{m}}\equiv c^2\varepsilon_0\rho_{\Omega}$ is the magnetic diffusion determined by Ohmic resistance ρ_{Ω} and vacuum susceptibility ε_0; vacuum permeability is μ_0. For hot enough plasma ρ_{Ω} is negligible and we ignore it hereafter. The total momentum flux

$$\Pi=\rho\mathbf{v}\mathbf{v}+P\mathbb{1}+\Pi^{(\mathrm{visc})}+\Pi^{(\mathrm{Maxw})} \quad (6)$$

is a sum of the inviscid part $\rho\mathbf{v}\mathbf{v}+P$ of the fluid, with pressure P,

$$\Pi_{ik}^{(\mathrm{visc})}=-\eta\left(\partial_i v_k+\partial_k v_i-\frac{2}{3}\delta_{ik}\nabla\cdot\mathbf{v}\right)-\zeta\,\delta_{ik}\nabla\cdot\mathbf{v}, \quad (7)$$

the viscous part of the stress tensor, with viscosity η and second viscosity ζ, and lastly, the Maxwell stress tensor

$$-\Pi_{ik}^{(\mathrm{Maxw})}=\frac{1}{\mu_0}\left(B_iB_k-\frac{1}{2}\mathbf{B}^2\delta_{ik}\right), \quad (8)$$

with δ_{ik} the Kronecker delta. We model coronal plasma with completely ionized hydrogen plasma

$$\varkappa=0.9\frac{T^{5/2}}{e^4m_e^{1/2}\Lambda},\quad \eta=0.4\frac{m_p^{1/2}T^{5/2}}{e^4\Lambda},\quad \zeta\approx 0, \quad (9)$$

$$\Lambda=\ln\left(\frac{r_{\mathrm{D}}T}{e^2}\right),\quad \frac{1}{r_{\mathrm{D}}^2}=\frac{4\pi e^2 n_{\mathrm{tot}}}{T},\quad e^2\equiv\frac{q_e^2}{4\pi\varepsilon_0},$$

where q_e is the electron charge, m_e is the mass of electron, m_p is the proton mass, T is the temperature and $n_{\mathrm{tot}}=n_e+n_p$ is the total density of electrons and protons; $\rho=m_pn_p$. We suppose that $\mu_0=4\pi$ and $\varepsilon_0=1/4\pi$, but in the practical system all formulae are the same; as well as in Heaviside-Lorentz units where $\mu_0=1$ and $\varepsilon_0=1$. As we mentioned above

$$\nu_{\mathrm{m}}=\frac{c^2}{4\pi}\frac{e^2m_e^{1/2}\Lambda}{0.6T^{3/2}}\ll\nu_{\mathrm{k}}\equiv\frac{\eta}{\rho}=\frac{0.4T^{5/2}}{e^4m_p^{1/2}n_p\Lambda}; \quad (10)$$

i.e. the hot hydrogen plasma is sticky, dilute, and "superconducting" $\nu_{\mathrm{m}}\approx 0$. Let us mention also the relations $\varkappa\rho_{\Omega}=1.5T/q_e^2$ and $\eta/\varkappa\approx\frac{4}{9}\sqrt{m_em_p}$,

$$\rho_{\Omega}=\frac{1}{4\pi\varepsilon_0}\frac{e^2m_e^{1/2}\Lambda}{0.6T^{3/2}}. \quad (11)$$

For weakly ionized plasma the magnetic viscosity definitely dominates $\nu_{\mathrm{m}}>\nu_{\mathrm{k}}$. As the plasma gets hotter absorbing AW the ohmic resistance becomes negligible. For illustrative purposes we shall completely ignore it $\nu_{\mathrm{m}}=0,\rho_{\Omega}=0$ in order to emphasize the importance of the viscosity at high final temperatures.

MHD EQUATIONS AND ENERGY FLUXES

The time derivative $\partial_t\mathbf{B}$ which implicitly participates in the energy conservation Eq. (2) at zero Ohmic resistivity obeys the equation

$$\mathrm{d}_t\mathbf{B}=\mathbf{B}\cdot\nabla\mathbf{v}-\mathbf{B}\,\mathrm{div}\,\mathbf{v},\quad \mathrm{d}_t\equiv\partial_t+\mathbf{v}\cdot\nabla. \quad (12)$$

Analogously the momentum equation Eq. (3) can be rewritten by the substantial derivative

$$\rho\,\mathrm{d}_tv_i=-\partial_iP+\partial_k\left\{\eta\left(\partial_kv_i+\partial_iv_k-\frac{2}{3}\delta_{ik}\partial_jv_j\right)\right\}+\partial_i(\zeta\partial_jv_j)-\frac{1}{\mu_0}(\mathbf{B}\times\mathrm{rot}\,\mathbf{B})_i\,. \quad (13)$$

In our model we consider AW propagating along magnetic field lines $\mathbf{B}_0$. We focus our attention on the narrow transition layer, where the static magnetic field is almost homogeneous and the waves are within acceptable accuracy one dimensional. For the velocity and magnetic fields we assume

$$\mathbf{v}(t,x)=U(x)\hat{\mathrm{x}}+u(t,x)\hat{\mathrm{z}},\quad \mathbf{B}(t,x)=B_0\hat{\mathrm{x}}+b(t,x)\hat{\mathrm{z}}, \quad (14)$$

with homogeneous magnetic field $B_0\hat{\mathrm{x}}$ perpendicular to the surface of the Sun. The transverse wave amplitudes of the velocity $u(t,x)$ and magnetic field $b(t,x)$ we represent with the Fourier integrals

$$u(t,x)=\int_{-\infty}^{\infty}\tilde{u}(\omega,x)\mathrm{e}^{-\mathrm{i}\omega t}\frac{\mathrm{d}\omega}{2\pi}, \quad (15)$$

$$b(t,x)=\int_{-\infty}^{\infty}\tilde{b}(\omega,x)\mathrm{e}^{-\mathrm{i}\omega t}\frac{\mathrm{d}\omega}{2\pi}. \quad (16)$$

For illustrative purposes it is convenient to consider monochromatic AW with $u(t,x)=\hat{u}(x)\mathrm{e}^{-\mathrm{i}\omega t}$ and $b(t,x)=\hat{b}(x)\mathrm{e}^{-\mathrm{i}\omega t}$.

Wave equations

For linearized waves the general MHD equations (13) and (12) give the following system for $\widehat{u}(x)$ and $\widehat{\overline{b}}(x) \equiv \widehat{b}(x)/B_0$

$$(-\mathrm{i}\omega + U\mathrm{d}_x)\,\widehat{u} = V_\mathrm{A}^2 \mathrm{d}_x \widehat{\overline{b}} + \frac{1}{\rho}\mathrm{d}_x\left(\eta\,\mathrm{d}_x\widehat{u}\right), \quad (17)$$

$$-\mathrm{i}\omega\widehat{\overline{b}} = \mathrm{d}_x\widehat{u} - \mathrm{d}_x(U\widehat{\overline{b}}), \quad (18)$$

where

$$V_\mathrm{A}(x) = B_0/\sqrt{\mu_0\rho(x)} \quad (19)$$

is the Alfvén velocity. In our numerical analysis we solve the first order linear system of equations

$$\begin{aligned} -\mathrm{i}\mathrm{d}_x\ \Psi\ &= \frac{\mathrm{i}}{\nu_\mathrm{k} U}\,\mathsf{M}\Psi = \mathsf{K}\Psi, \quad (20)\\ \Psi\ &\equiv \begin{pmatrix}\widehat{u}\\ \widehat{\overline{b}}\\ \widehat{w}\end{pmatrix}, \quad \mathsf{K} = \frac{\mathrm{i}}{\nu_\mathrm{k} U}\mathsf{M},\\ \Psi^\dagger\ &= \left(\widehat{u}^*, \widehat{\overline{b}}^*, \widehat{w}^*\right), \end{aligned}$$

where $\widehat{w} \equiv \mathrm{d}_x\widehat{u}$, and

$$\mathsf{M} \equiv \begin{pmatrix} 0 & 0 & -\nu_\mathrm{k} U \\ 0 & \nu_\mathrm{k}(-\mathrm{i}\omega + \mathrm{d}_x U) & -\nu_\mathrm{k} \\ \mathrm{i}\omega & -V_\mathrm{A}^2(-\mathrm{i}\omega + \mathrm{d}_x U) & \left(V_\mathrm{A}^2 - U^2\right) + \frac{U}{\rho}\mathrm{d}_x\eta \end{pmatrix}.$$

For homogeneous medium with constant η, ρ, V_A, and U, in short for constant wave-vector matrix K, the exponential substitution $\Psi \propto \exp(\mathrm{i}kx)$ in Eq. (20) or equivalently Eq. (17) and Eq. (18) gives the secular equation

$$\begin{aligned} &\mathrm{i}\,\nu_\mathrm{k} U \det\left(\mathsf{K} - k\mathbb{1}\right)\\ &\quad = \omega_\mathrm{D}\left(\omega_\mathrm{D} + \mathrm{i}\nu_\mathrm{k} k^2\right) - V_\mathrm{A}^2 k^2 = 0, \quad (21) \end{aligned}$$

where $\omega_\mathrm{D} \equiv \omega - kU$ is the Doppler shifted frequency. This secular equation gives the well-known dispersion

$$\omega_\mathrm{D}\left(\omega_\mathrm{D} + \mathrm{i}\nu_\mathrm{k} k^2\right) = V_\mathrm{A}^2 k^2$$

of the AW. This equation is quadratic with respect to ω and cubic with respect to k.

Wind variables

We solve the wave equation Eq. (20) from "Sun's surface" $x = 0$ to some distance large enough $x = l$, where the short wavelength AW are almost absorbed. This distance is much bigger than the *width of the transition layer* λ, but much smaller than solar radius. The considered one-dimensional $0 < x < l$ time-independent problem has three integrals corresponding to the three conservation laws related to mass, energy and momentum. The mass conservation law Eq. (1) gives the constant flow

$$j = \rho(x)U(x) = \rho_0 U_0 = \rho_l U_l = \mathrm{const}, \quad (22)$$

where $\rho_0 = \rho(0)$, $\rho_l = \rho(l)$, $U_0 = U(0)$, and $U_l = U(l)$. The energy conservation law reduces to a constant flux along the x-axis

$$\begin{aligned} q =& q_\mathrm{wind}^\mathrm{ideal}(x) + \widetilde{q}(x)\\ =& \rho U\left(\frac{1}{2}U^2 + h\right) + \widetilde{q} = \mathrm{const}. \quad (23) \end{aligned}$$

Here the first term describes the energy of the ideal wind, i.e. a wind from an ideal (inviscid) fluid. The second term $\widetilde{q}(x)$ includes all other energy fluxes; in our notations tilde will denote sum of the non-ideal (dissipative) terms of the wind and wave terms. In detail the non-ideal part of the energy flux $\widetilde{q}(x)$ consists of: the wave kinetic energy $\propto |\widehat{u}|^2$, viscosity (wind $\propto \frac{4}{3}\eta + \zeta$ and wave $\propto \eta$ components), heat conductivity $\propto \varkappa$, and Poynting vector $\propto \widehat{b}^*$,

$$\begin{aligned} \widetilde{q}(x)\ \equiv&\ \frac{j}{4}|\widehat{u}|^2 - \xi U \mathrm{d}_x U - \frac{1}{4}\eta\,\mathrm{d}_x|\widehat{u}|^2 - \varkappa\,\mathrm{d}_x T\\ &+ \frac{1}{2\mu_0}\left(U\left|\widehat{b}\right|^2 - B_0\,\mathrm{Re}(\widehat{b}^*\widehat{u})\right), \quad (24) \end{aligned}$$

where $\xi \equiv \frac{4}{3}\eta + \zeta$. Here time averaged energy flux is represented by the amplitudes of the monochromatic oscillations, this is a standard procedure for alternating current processes. In our case we have, for example, $\langle\widehat{u}^2\rangle_t = \left\langle(\mathrm{Re}\,\widehat{u})^2\right\rangle_t = \left\langle\frac{1}{4}(\widehat{u} + \widehat{u}^*)^2\right\rangle_t = \frac{1}{2}|\widehat{u}|^2$. The other terms from Eq. (4) are averaged in a similar way in the equation above.

The momentum conservation law Eq. (6) gives constant flux $\Pi = \Pi_{xx}$

$$\Pi = \Pi_\mathrm{wind}^\mathrm{ideal}(x) + \widetilde{\Pi}(x) = \rho UU + P + \widetilde{\Pi}, \quad (25)$$

the sum of the ideal wind fluid and the other terms

$$\widetilde{\Pi}(x) \equiv \frac{1}{4\mu_0}\left|\widehat{b}\right|^2 - \xi\,\mathrm{d}_x U, \quad (26)$$

which take into account the wave part of the Maxwell stress tensor $\propto b^2$ and viscosity of the wind $\propto \xi$.

We have to solve the hydrodynamic problem for calculation of wind velocity and temperature at known energy and momentum fluxes. The problem is formally reduced to analogous one for a jet engine, cf. Ref. [10]. We approximate the corona as completely ionized hydrogen plasma, i.e. electrically neutral mixture of electrons and protons. The experimental data tells us that proton temperature T_p is higher than electron one T_e. This is an important hint that heating goes through the viscosity determined mainly by protons. However for illustration purpose and simplicity we assume proton and electron temperatures to be equal $T_e = T_p = T$. For such an ideal (in thermodynamic sense) gas the local sound velocity is

$$
\begin{aligned}
&c_\mathrm{s}^2(x) = \frac{c_P}{c_V}\frac{P}{\rho} = \gamma\frac{T}{\langle m\rangle}, \quad \gamma = \frac{c_P}{c_V} = \frac{5}{3}, \qquad (27)\\
&\langle m\rangle = \frac{n_p m_p + n_e m_e}{n_p + n_e} \approx \frac{1}{2}m_p, \quad n_e = n_p = \frac{1}{2}n_\mathrm{tot},\\
&P = n_\mathrm{tot}T = \frac{\rho T}{\langle m\rangle} = \frac{j}{U}\frac{T}{\langle m\rangle}, \quad h = c_P\frac{T}{\langle m\rangle} = \frac{\varepsilon + P}{\rho},
\end{aligned}
$$

where, as we mentioned earlier, h is the enthalpy per unit mass and ε is the density of internal energy.

In order to alleviate the final formulae we introduce two dimensionless variables χ and τ which represent the non-ideal part of the energy and momentum flux respectively

$$
\chi(x) \equiv \left.\frac{\widetilde{q}(x)}{\rho_0 U_0^3}\right|_x^0, \quad \tau(x) \equiv \left.\frac{\widetilde{\Pi}(x)}{\rho_0 U_0^2}\right|_x^0, \qquad (28)
$$

and analogously for the wind velocity and temperature

$$
\begin{aligned}
\overline{U}(x) &\equiv \frac{U(x)}{U_0},\\
\Theta(x) &\equiv \frac{T(x)}{\langle m\rangle U_0^2}, \qquad (29)\\
\Theta_0 &= \Theta(0),
\end{aligned}
$$

where $U_0 = U(0)$. The energy and momentum constant fluxes Eq. (23) and Eq. (25) in the new notation take the form

$$
\frac{q - \widetilde{q}(0)}{\rho_0 U_0^3} = \frac{1}{2}\overline{U}^2 + c_p\Theta - \chi = \frac{1}{2} + c_p\Theta_0, \qquad (30)
$$

$$
\frac{\Pi - \widetilde{\Pi}(0)}{\rho_0 U_0^2} = \overline{U} + \Theta/\overline{U} - \tau = 1 + \Theta_0. \qquad (31)
$$

From the second equation we express the dimensionless temperature Θ and substitute in the first one. Solving the quadratic equation for the wind velocity U we derive

$$
\begin{aligned}
U &= U_0\overline{U},\\
\overline{U}(x) &= \frac{1}{\gamma+1}\left(\gamma + s^2 + \tau(x) - \sqrt{\mathscr{D}(x)}\right), \qquad (32)
\end{aligned}
$$

where for the discriminant we have

$$
\begin{aligned}
\mathscr{D} &= (s^2-1)^2 - 2(\gamma^2-1)(\chi+1)\\
&\qquad + \gamma\tau\left[\gamma\tau + 2\left(\gamma + s^2\right)\right],\\
s^2 &\equiv \frac{c_\mathrm{s}^2(0)}{U_0^2} = \gamma\Theta_0, \qquad (33)\\
c_\mathrm{s}^2(x) &= \left(\frac{\partial P}{\partial\rho}\right)_S = \frac{\gamma T(x)}{\langle m\rangle}.
\end{aligned}
$$

Here γ is the constant ratio of the heat capacities, and $s \equiv c_\mathrm{s}(0)/U_0$ is the ratio of the sound and wind velocity at $x = 0$. We suppose that initial wind velocity is very small $U(0) \ll c_\mathrm{s}(0)$. The velocity distribution Eq. (32) can be substituted in Eq. (31) and we derive the dimensionless equation for the temperature distribution

$$
\begin{aligned}
&T(x) = \langle m\rangle U_0^2\Theta(x), \qquad (34)\\
&\Theta(x) = \overline{U}(x)\left(1 + \Theta_0 + \gamma\tau(x) - \overline{U}(x)\right), \ \Theta_0 = \frac{s^2}{\gamma}.
\end{aligned}
$$

The solutions for velocity $\overline{U}(x)$ Eq. (32) and temperature $\overline{T}(x)$ Eq. (34) distributions are important ingredients in our analysis and derivation of the self-consistent picture of the solar wind. We use a one dimensional approximation and in addition the constant flux of mass, energy and momentum gives 3 integrals of motion. This enables us to solve the nonlinear part of the problem analytically. That is why we do not solve the differential equations for the density $\rho(x) = \rho(0)U(0)/U(x)$, temperature $T(x)$ and wind velocity $U(x)$, and use analytical expressions containing the energy and momentum fluxes. Thus the numerical problem is reduced to a system of three linear differential equations.

Boundary conditions for the waves

At known background wind variables $U(x)$ and $T(x)$ we can solve the wave equation Eq. (20) for run-away AW at $x = l$. As we will see later the run-away boundary condition Eq. (49) corresponds to right propagating AW at the right boundary of the interval. The wave equation Eq. (20) is extremely stiff at small viscosity, and numerical solution is possible to be obtained only downstream from $x=0$ to $x=l$. We have to find the linear combination of left and right propagating waves at $x=0$, which gives the run-away condition at $x=l$.

The solution of wave equation according to Eq. (23) determines the energy flux related to the propagation of AW

$$\widetilde{q}_{\mathrm{wave}}(\Psi(x)) \equiv \Psi^\dagger g \Psi = \frac{j}{4}|\widehat{u}|^2 - \frac{1}{2}\eta\,\mathrm{Re}\left(\widehat{u}^*\widehat{w}\right) + \frac{B_0^2}{2\mu_0}\left(U\left|\widehat{b}\right|^2 - \mathrm{Re}\left(\widehat{b}^*\,\widehat{u}\right)\right), \quad (35)$$

where

$$g(x) \equiv \begin{pmatrix} \frac{1}{4}j & -\frac{B_0^2}{4\mu_0} & -\frac{1}{4}\eta \\ -\frac{B_0^2}{4\mu_0} & \frac{UB_0^2}{2\mu_0} & 0 \\ -\frac{1}{4}\eta & 0 & 0 \end{pmatrix}. \quad (36)$$

Here j-term represents kinetic energy of the wave, η-term comes from the viscous part of the wave energy flux, and B_0-terms describe the Poynting vector of the wave.

In order to take into account the boundary condition at $x=l$ we calculate the eigenvectors of the matrix K, which according to Eq. (20) determine the wave propagation in a homogeneous fluid with amplitude $\propto \exp(\mathrm{i}kx)$. Then the eigenvalues of K give the complex wave-vectors

$$k = k' + \mathrm{i}k'' = \mathrm{eigenvalue}(\mathsf{K}), \quad (37)$$

i.e.

$$\det(\mathsf{K} - k\mathbb{1}) = 0. \quad (38)$$

The three eigenvectors L, D and R are ordered by spatial decrements of their eigenvalues

$$k''_{\mathrm{L}} < 0 < k''_{\mathrm{R}} < k''_{\mathrm{D}}, \quad (39)$$

and are normalized by the conditions

$$-\mathrm{L}^\dagger g \mathrm{L} = \mathrm{R}^\dagger g \mathrm{R} = \mathrm{D}^\dagger g \mathrm{D} = 1, \quad (40)$$

where the sign corresponds to the direction of wave propagation. Notation L corresponds to left propagating wave, R to right propagating wave, and D for an overdamped at small viscosity mode.

For technical purposes we introduce the matrix notations

$$\mathrm{L} = \begin{pmatrix} \mathrm{L}_u(x) \\ \mathrm{L}_b(x) \\ \mathrm{L}_w(x) \end{pmatrix}, \ \mathrm{R} = \begin{pmatrix} \mathrm{R}_u(x) \\ \mathrm{R}_b(x) \\ \mathrm{R}_w(x) \end{pmatrix}, \ \mathrm{D} = \begin{pmatrix} \mathrm{D}_u(x) \\ \mathrm{D}_b(x) \\ \mathrm{D}_w(x) \end{pmatrix}. \quad (41)$$

For low enough frequencies $\omega \to 0$ and wind velocities the modes describe: 1) right-propagating AW with $k'_{\mathrm{R}} \approx \omega/V_{\mathrm{A}}$ and small $k''_{\mathrm{R}} \approx \nu_{\mathrm{k}}\omega^2/2V_{\mathrm{A}}^3 \ll k'_{\mathrm{R}}$, 2) left propagating wave $k_{\mathrm{L}} = -k_{\mathrm{R}}$, and a diffusion overdamped mode $k''_{\mathrm{D}} \approx V_{\mathrm{A}}^2/\nu_{\mathrm{k}}U \gg k'_{\mathrm{D}}$ which describes the drag of a static perturbation by the slow wind $U \ll V_{\mathrm{A}}$ in a fluid with small viscosity. In this low frequency and long wavelength limit the stiffness ratio of the eigenvalues is very large

$$r_{\mathrm{DR}} = \frac{|k_{\mathrm{D}}|}{|k_{\mathrm{R}}|} \approx \frac{k''_{\mathrm{D}}}{k'_{\mathrm{R}}} \approx \frac{V_{\mathrm{A}}^3}{\nu_{\mathrm{k}}U\omega} \gg 1. \quad (42)$$

The strong inequality is applicable to the chromosphere where the viscosity of the cold plasma is very low. As we emphasized the wave equations Eq. (20) form a very stiff system and indispensably has to be solved downstream from the chromosphere $x=0$ to the corona $x=l$ using algorithms for stiff systems. Let

$$\psi_{\mathrm{L}}(x) = \begin{pmatrix} u_{\mathrm{L}}(x) \\ b_{\mathrm{L}}(x) \\ w_{\mathrm{L}}(x) \end{pmatrix}, \quad \psi_{\mathrm{R}}(x) = \begin{pmatrix} u_{\mathrm{R}}(x) \\ b_{\mathrm{R}}(x) \\ w_{\mathrm{R}}(x) \end{pmatrix} \quad (43)$$

are the solutions of the wave equation Eq. (20) with boundary conditions

$$\psi_{\mathrm{L}}(0) = \mathrm{L}(0), \quad \psi_{\mathrm{R}}(0) = \mathrm{R}(0). \quad (44)$$

We look for a solution as a linear combination

$$\psi(x) = \psi_{\mathrm{R}}(x) + r\,\psi_{\mathrm{L}}(x), \quad (45)$$

in other words we suppose that from the low viscosity chromosphere plasma do not come overdamped diffusion modes. The strong decay rate make them negligible at $x=0$. Physically this means that AW (R-modes) are coming from the Sun and some of them are reflected from the transition layer (L-modes)

$$\psi(0) = \mathrm{R}(0) + r\mathrm{L}(0). \quad (46)$$

Analogously for the configuration of open corona we have to take into account the run-away boundary condition for which we suppose zero amplitude for the wave coming from infinity

$$\psi(l) = \widetilde{t}\,\mathrm{R} + \widetilde{c}\,\mathrm{D}(l). \quad (47)$$

Written by components

$$\begin{pmatrix} u_{\rm R}(l) \\ b_{\rm R}(l) \\ w_{\rm R}(l) \end{pmatrix} + r \begin{pmatrix} u_{\rm L}(l) \\ b_{\rm L}(l) \\ w_{\rm L}(l) \end{pmatrix} = \widetilde{t} \begin{pmatrix} \mathrm{R}_u(l) \\ \mathrm{R}_b(l) \\ \mathrm{R}_w(l) \end{pmatrix} + \widetilde{c} \begin{pmatrix} \mathrm{D}_u(l) \\ \mathrm{D}_b(l) \\ \mathrm{D}_w(l) \end{pmatrix}. \quad (48)$$

This boundary condition gives a linear system of equation for the reflection coefficient r, transmission coefficient $\widetilde{t}$ and the mode-conversion coefficient $\widetilde{c}$.

For $l \to \infty$ when $\exp[-k''_{\rm D}(l)l] \ll 1 \exp[-k''_{\rm R}(l)l]$ the amplitude of D-mode is negligible and the run-away boundary condition reads

$$\psi(l) = \psi_{\rm R}(l) + r\psi_{\rm L}(l) \approx \widetilde{t}\mathrm{R}(l), \quad (49)$$

or by components

$$\begin{pmatrix} u_{\rm R}(l) \\ b_{\rm R}(l) \end{pmatrix} + r \begin{pmatrix} u_{\rm L}(l) \\ b_{\rm L}(l) \end{pmatrix} = \widetilde{t} \begin{pmatrix} \mathrm{R}_u(l) \\ \mathrm{R}_{\overline{b}}(l) \end{pmatrix}. \quad (50)$$

These systems give the amplitudes of the reflected wave r and transmitted wave $\widetilde{t}$ in the solution Eq. (45). For this solution we have the energy fluxes

$$\begin{aligned} \mathscr{T} &\equiv \psi^\dagger(l)\, g(l)\, \psi(l) \\ &= |\widetilde{t}|^2 + |\widetilde{c}|^2 + (\widetilde{t}\widetilde{c}^* \mathrm{D}^\dagger(l) g(l) \mathrm{R}(l) + \text{c.c.}), \end{aligned} \quad (51)$$

$$\begin{aligned} 1 - \mathscr{R} &\equiv \psi^\dagger(0)\, g(0)\, \psi(0) \\ &= 1 - |r|^2 + \left(r^* \mathrm{L}^\dagger(0)\, g(0)\, \mathrm{R}(0) + \text{c.c.}\right). \end{aligned} \quad (52)$$

Then we introduce the absorption coefficient

$$\mathscr{A} \equiv -\psi^\dagger(x)\, g(x)\, \psi(x)\Big|_0^l = 1 - \mathscr{R} - \mathscr{T}. \quad (53)$$

The described solution is normalized by unit energy flux of the R-wave. If we wish to fix energy flux of the right propagating wave to be $q_{\rm wave}(0)$ we have to make the renormalization

$$\Psi(x) = A_{\rm wave}\, \psi(x). \quad (54)$$

In this section we have described Absorbing Boundary Conditions (ABC) well known from radar calculations, but realization for AW is more complicated and require eigenvector analysis. Now using $\Psi(x)$ we can calculate the wave part of the energy flux Eq. (35) and the wave part of the momentum flux

$$\widetilde{\Pi}_{\rm wave}(x) \equiv \frac{1}{4\mu_0} \left|\widehat{b}(x)\right|^2. \quad (55)$$

This section is written in dimensional variables, but all equations can be easily converted in dimensionless variables as is done in the next sub-sub-section.

Dimensionless wave variables, convenient for numerical calculations

Using mechanical units for length l, velocity U_0 and density ρ_0 we can convert all equations in dimensionless form. The formulae remain almost the same and we wish to mention only the differences. Introducing dimensionless density

$$\overline{\rho}(x) = \rho(x)/\rho_0 = 1/\overline{U}(\overline{x}) \quad (56)$$

and wave energy flux

$$\mathscr{Q}_{\rm wave}(0) = \frac{q_{\rm wave}(0)}{\rho_0 U_0^3} = (1 - \mathscr{R})\, |A_{\rm wave}|^2, \quad (57)$$

we have dimensionless matrices

$$\overline{\mathsf{M}} = \begin{pmatrix} 0 & 0 & -\overline{v}\overline{U} \\ 0 & \overline{v}(-\mathrm{i}\overline{\omega} + \overline{W}) & -\overline{v} \\ \mathrm{i}\overline{\omega}\overline{U} & -\overline{V}_{\rm A}^2(-\mathrm{i}\overline{\omega} + \overline{W}) & \left(\overline{V}_{\rm A}^2 - \overline{U}^2\right) + \overline{U}^2 \mathrm{d}_{\overline{x}}\overline{\eta} \end{pmatrix},$$

and

$$\overline{g} \equiv \frac{1}{4} \begin{pmatrix} 1 & -a^2 & -\overline{\eta}(\overline{x}) \\ -a^2 & 2a^2\overline{U}(\overline{x}) & 0 \\ -\overline{\eta}(\overline{x}) & 0 & 0 \end{pmatrix}, \quad (58)$$

$$\overline{V}_{\rm A}^2(\overline{x}) = a^2 \overline{U}(\overline{x}), \quad \overline{V}_{\rm A}^2(0) = a^2 \overline{U}(0) = a^2. \quad (59)$$

For the dimensionless energy flux we have (Fig. 1b)

$$\begin{aligned} \mathscr{Q}_{\rm wave}(\overline{x}) = \frac{1}{4}\left|\widehat{\overline{u}}\right|^2 + \frac{a^2}{2}\left(\overline{U}\left|\widehat{\overline{b}}\right|^2 - \mathrm{Re}(\widehat{\overline{b}}^* \widehat{\overline{u}})\right) \\ - \frac{1}{2}\overline{\eta}\, \mathrm{Re}\left(\widehat{\overline{u}}^* \widehat{\overline{w}}\right) = \overline{\Psi}^\dagger \overline{g}\, \overline{\Psi}, \end{aligned} \quad (60)$$

where (Fig. 1a)

$$\widehat{\overline{u}} = \frac{\widehat{u}}{U_0}, \quad \widehat{\overline{w}} = \frac{l \mathrm{d}_x \widehat{u}}{U_0}, \quad \overline{\omega} = \frac{l\omega}{U_0}. \quad (61)$$

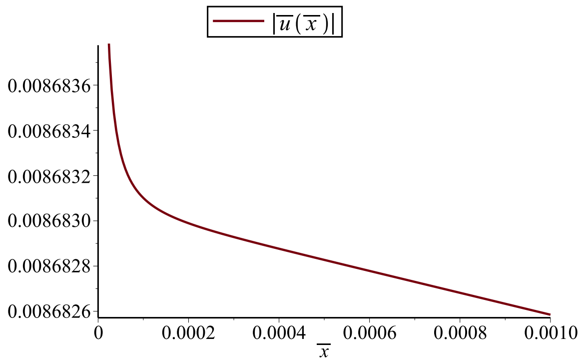
Then for the dimensional wave energy flux we have

$$q_{\rm wave}(x) = \rho_0 U_0^3\, \mathscr{Q}_{\rm wave}(\overline{x}), \quad (62)$$

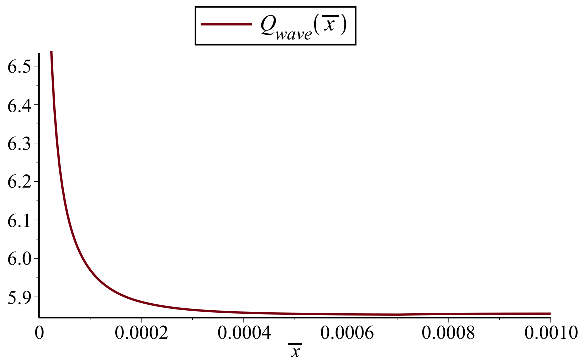
and analogously for the momentum flux of the wave (Fig. 1c)

$$\Pi_{\rm wave}(x) = \rho_0 U_0^2\, \mathscr{P}_{\rm wave}(\overline{x}), \quad \mathscr{P}_{\rm wave} = \frac{1}{4}\left|\widehat{\overline{b}}\right|^2. \quad (63)$$

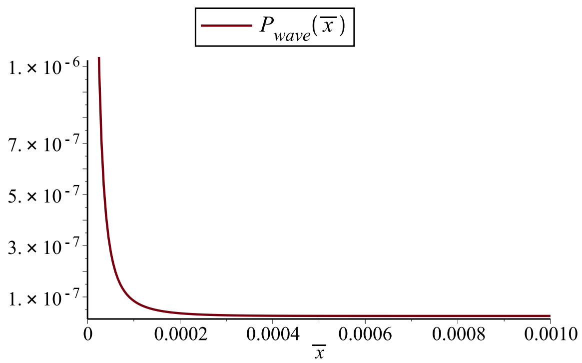
in the next section we will consider all parts of the energy and momentum fluxes.



(a)



(b)



(c)

Fig. 1. AW profiles of velocity (a), energy flux (b) and momentum flux (c).

Total wave fluxes

The total energy and momentum fluxes are integrals over all frequencies. From Eq. (60) we have

$$\mathscr{Q}_{\mathrm{wave},\overline{\omega}}(\overline{x}) = \Psi^{\dagger}_{\overline{\omega}} \overline{g} \Psi_{\overline{\omega}} = \mathscr{W}_{\overline{\omega}} \psi^{\dagger}_{\overline{\omega}}(\overline{x}) \overline{g}(\overline{x}) \psi_{\overline{\omega}}(\overline{x}), \quad (64)$$

where $\mathscr{W}_{\overline{\omega}}$ is the spectral density of the waves. We can construct the total energy flux of the waves as

$$\begin{aligned}\mathscr{Q}^{\mathrm{total}}_{\mathrm{wave}}(\overline{x}) &= \int_{\overline{\omega}=0}^{\infty} \mathscr{W}_{\overline{\omega}} \psi^{\dagger}_{\overline{\omega}}(\overline{x}) \overline{g}(\overline{x}) \psi_{\overline{\omega}}(\overline{x}) \frac{\mathrm{d}\overline{\omega}}{2\pi} \\ &= \sum_{\overline{\omega}>0} \mathscr{W}_{\overline{\omega}} \psi^{\dagger}_{\overline{\omega}}(\overline{x}) \overline{g}(\overline{x}) \psi_{\overline{\omega}}(\overline{x}), \quad (65)\end{aligned}$$

$$\mathscr{Q}^{\mathrm{total}}_{\mathrm{wave}}(0) \equiv \mathscr{Q}_0 = \sum_{\overline{\omega}>0} \mathscr{W}_{\overline{\omega}} \psi^{\dagger}_{\overline{\omega}}(0) \overline{g}(0) \psi_{\overline{\omega}}(0). \quad (66)$$

Observational data gives a power law dependence of the spectral density of AW in the solar corona. One can suppose that the spectral density of the waves coming from the chromosphere has the same power law dependence, *i.e.* $\mathscr{W}_{\overline{\omega}} = C/\overline{\omega}^{\alpha}$. α is between 1.5 and 2: $\frac{3}{2}$ [8], 1.59 [5], 2 [11]. C is the unknown parameter of the theory, which we vary for fixed α in order to reproduce the temperature increase in the transition layer. Note that here we have used the dimensionless frequency. If we want to use the dimensional one, then the parameter C also has to become dimensional. If we know the initial total energy flux of the waves, we can calculate the spectral density as

$$\mathscr{W}_{\overline{\omega}} = Q_0 / \sum_{\overline{\omega}>0} \psi^{\dagger}_{\overline{\omega}}(0) \, \overline{g}(0) \, \psi_{\overline{\omega}}(0). \quad (67)$$

Analogously to Eq. (65), the total momentum flux is calculated from Eq. (63) as

$$\begin{aligned}\mathscr{P}^{\mathrm{total}}_{\mathrm{wave}}(\overline{x}) &= \sum_{\overline{\omega}>0} \mathscr{W}_{\overline{\omega}} \mathscr{P}_{\mathrm{wave},\overline{\omega}}(\overline{x}) \\ &= \int_{\overline{\omega}=0}^{\infty} \mathscr{W}_{\overline{\omega}} \mathscr{P}_{\mathrm{wave},\overline{\omega}}(\overline{x}) \frac{\mathrm{d}\overline{\omega}}{2\pi}. \quad (68)\end{aligned}$$

In order to simulate plasma heating by AW with power law spectral density, in the work by Topchiyska [12] an illustration is given with 8 AW with different frequencies. Wave propagation can be easily seen for moderate of $T(l)/T_0 = 3$. In order to concentrate our attention on a realistic temperature increase $T(l)/T_0 = 100$ in the present work we take into account only one wave with frequency 300 Hz. No doubt waves in the Hz range do not exist in the solar corona because they are absorbed during the heating, but we wish to emphasize the importance of Hz range waves in the solar photosphere, which are not observable at the moment.

Mass, energy and momentum fluxes

In the one-dimensional model which we analyze the conservation laws Eq. (1), Eq. (2) and Eq. (3) are converted in three integrals of our dynamic system describing the mass $j = \rho_0 U_0 \overline{j}$, energy $q = \rho_0 U_0^3 \mathscr{Q}$, and momentum $\Pi = \rho_0 U_0^2 \mathscr{P}$ fluxes

$$\overline{j} = \overline{U}\overline{\rho} = 1, \tag{69}$$

$$\mathscr{Q} = \frac{1}{2}\overline{U}^2 + c_p\Theta_0\overline{T} - \overline{\varkappa}\Theta_0 d_{\bar{x}}\overline{T} - \left(\frac{4}{3}\overline{\eta} + \overline{\zeta}\right)\overline{U}\, d_{\bar{x}}\overline{U} + \sum_{\varpi>0}\mathscr{W}_{\varpi}\left(\frac{1}{4}\left|\widehat{\overline{u}}_{\varpi}\right|^2 + \frac{a^2}{2}\left(\overline{U}\left|\widehat{\overline{b}}_{\varpi}\right|^2 - \mathrm{Re}\left(\widehat{\overline{b}}^*_{\varpi}\widehat{\overline{u}}_{\varpi}\right)\right)\right) - \sum_{\varpi>0}\mathscr{W}_{\varpi}\frac{1}{2}\overline{\eta}\,\mathrm{Re}\left(\widehat{\overline{u}}^*_{\varpi}\widehat{\overline{w}}_{\varpi}\right) = \mathrm{const}, \tag{70}$$

$$\mathscr{P} = \overline{U} + \frac{\Theta_0\overline{T}}{\overline{U}} - \left(\frac{4}{3}\overline{\eta} + \overline{\zeta}\right) d_{\bar{x}}\overline{U} + \sum_{\varpi>0}\mathscr{W}_{\varpi}\frac{1}{4}\left|\widehat{\overline{b}}_{\varpi}\right|^2 = \mathrm{const}. \tag{71}$$

Here we can recognize the energy flux of an ideal inviscid gas

$$\mathscr{Q}^{\mathrm{ideal}}_{\mathrm{wind}} = \frac{1}{2}\overline{U}^2 + c_p\Theta_0\overline{T}, \qquad \Theta = \Theta_0\overline{T}, \tag{72}$$

dissipative energy flux of the wind related to heat conductivity and viscosity

$$\mathscr{Q}^{\mathrm{diss}}_{\mathrm{wind}} = -\overline{\varkappa}\Theta_0 d_{\bar{x}}\overline{T} - \left(\frac{4}{3}\overline{\eta} + \overline{\zeta}\right)\overline{U}\, d_{\bar{x}}\overline{U}, \tag{73}$$

the non-absorptive part of the wave energy flux

$$\mathscr{Q}^{\mathrm{ideal}}_{\mathrm{wave}} = \sum_{\varpi>0}\mathscr{W}_{\varpi}\left\{\frac{1}{4}\left|\widehat{\overline{u}}_{\varpi}\right|^2 + \frac{a^2}{2}\left[\overline{U}\left|\widehat{\overline{b}}_{\varpi}\right|^2 - \mathrm{Re}\left(\widehat{\overline{b}}^*_{\varpi}\widehat{\overline{u}}_{\varpi}\right)\right]\right\}, \tag{74}$$

and the absorptive part of the wave energy flux

$$\mathscr{Q}^{\mathrm{diss}}_{\mathrm{wave}} = -\sum_{\varpi>0}\mathscr{W}_{\varpi}\frac{1}{2}\overline{\eta}\,\mathrm{Re}\left(\widehat{\overline{u}}^*_{\varpi}\widehat{\overline{w}}_{\varpi}\right) \tag{75}$$

proportional to the viscosity. Analogously for the momentum flux we have:

$$\mathscr{P}^{\mathrm{ideal}}_{\mathrm{wind}} = \overline{U} + \frac{\Theta_0\overline{T}}{\overline{U}}, \tag{76}$$

$$\mathscr{P}^{\mathrm{diss}}_{\mathrm{wind}} = -\left(\frac{4}{3}\overline{\eta} + \overline{\zeta}\right) d_{\bar{x}}\overline{U}, \tag{77}$$

$$\mathscr{P}^{\mathrm{ideal}}_{\mathrm{wave}} = \sum_{\varpi>0}\mathscr{W}_{\varpi}\frac{1}{4}\left|\widehat{\overline{b}}_{\varpi}\right|^2, \tag{78}$$

$$\mathscr{P}^{\mathrm{diss}}_{\mathrm{wave}} = 0 \tag{79}$$

for the transversal AW. As a rule the dissipative fluxes are against the non-dissipative ones. One can introduce non-ideal wind energy flux

$$\widetilde{\mathscr{Q}} \equiv \mathscr{Q}^{\mathrm{nonideal}}_{\mathrm{wind}} = \mathscr{Q}^{\mathrm{diss}}_{\mathrm{wind}} + \mathscr{Q}^{\mathrm{total}}_{\mathrm{wave}} = \mathscr{Q} - \mathscr{Q}^{\mathrm{ideal}}_{\mathrm{wind}} = -\overline{\varkappa} d_{\bar{x}}\overline{T} - \left(\frac{4}{3}\overline{\eta} + \overline{\zeta}\right)\overline{U} d_{\bar{x}}\overline{U} - \sum_{\varpi>0}\mathscr{W}_{\varpi}\left(\frac{1}{2}\overline{\eta}\mathrm{Re}\left(\widehat{\overline{u}}^*_{\varpi}\widehat{\overline{w}}_{\varpi}\right)\right) + \sum_{\varpi>0}\mathscr{W}_{\varpi}\left[\frac{1}{4}\left|\widehat{\overline{u}}_{\varpi}\right|^2 + \frac{a^2}{2}\left(\overline{U}\left|\widehat{\overline{b}}_{\varpi}\right|^2 - \mathrm{Re}\left(\widehat{\overline{b}}^*_{\varpi}\widehat{\overline{u}}_{\varpi}\right)\right)\right], \tag{80}$$

$$\mathscr{Q}^{\mathrm{total}}_{\mathrm{wave}} = \mathscr{Q}^{\mathrm{ideal}}_{\mathrm{wave}} + \mathscr{Q}^{\mathrm{diss}}_{\mathrm{wave}}. \tag{81}$$

The non-ideal wind momentum flux is

$$\widetilde{\mathscr{P}} \equiv \mathscr{P}^{\mathrm{nonideal}}_{\mathrm{wind}} = \mathscr{P}^{\mathrm{diss}}_{\mathrm{wind}} + \mathscr{P}^{\mathrm{total}}_{\mathrm{wave}} = \mathscr{P} - \mathscr{P}^{\mathrm{ideal}}_{\mathrm{wind}} = -\left(\frac{4}{3}\overline{\eta} + \overline{\zeta}\right) d_{\bar{x}}\overline{U} + \sum_{\varpi>0}\mathscr{W}_{\varpi}\frac{1}{4}\left|\widehat{\overline{b}}_{\varpi}\right|^2, \tag{82}$$

$$\mathscr{P}^{\mathrm{total}}_{\mathrm{wave}} = \mathscr{P}^{\mathrm{ideal}}_{\mathrm{wave}} + \mathscr{P}^{\mathrm{diss}}_{\mathrm{wave}}. \tag{83}$$

Then according to Eq. (28) we have

$$\chi(\bar{x}) \equiv \widetilde{\mathscr{Q}}\Big|_{\bar{x}}^{0} = \mathscr{Q}_{\rm wind}^{\rm ideal}\Big|_{0}^{\bar{x}}, \qquad (84)$$

$$\tau(\bar{x}) \equiv \widetilde{\mathscr{P}}\Big|_{\bar{x}}^{0} = \mathscr{P}_{\rm wind}^{\rm ideal}\Big|_{0}^{\bar{x}}. \qquad (85)$$

This analysis finally reveals the usefulness of dimensionless variables. The dimensional momentum flux τ and energy flux χ participate in the important analytical expressions for the wind variables Eq. (32) and Eq. (34).

Self-consistent procedure and results

First we fix the boundary condition, the temperature T_0 and proton density $n_p(0)$ for $x = 0$. For these parameters we calculate density ρ_0, Debye radius length $r_{\rm D}(0)$, Coulomb logarithm Λ_0, viscosity η_0, heat conductivity $\varkappa_0$, Ohmic resistivity $\varrho_{\Omega 0}$, and sound speed $c_{\rm s}(0)$. Initial velocity of the wind is better to be parameterized by the dimensionless parameter $s \gg 1$, i.e. $U_0 = c_{\rm s}(0)/s$. Analogously plasma beta parameter β_0 determines the Alfvén speed at $V_{\rm A}(0) = \sqrt{\frac{\gamma}{2\beta}}c_{\rm s}(0)$. Let us also fix the maximal frequency for which we will consider plasma waves ω and calculate the absorption rate of the energy density of Alfvén waves $2k''(0)$. One can choose the interval of the solution of MHD equations to be much larger than the AW mean free path $1/2k''(0)$, for example

$$l = \frac{10}{2k''(0)} = \frac{10V_{\rm A}^3(0)}{\nu_{\rm k}(0)\omega^2}. \qquad (86)$$

Having units for length l, velocity U_0 and density ρ_0 we can calculate dimensionless variables at $\bar{x} = 0$: $\bar{\varkappa}_0$, $\bar{\eta}_0$, and Θ_0.

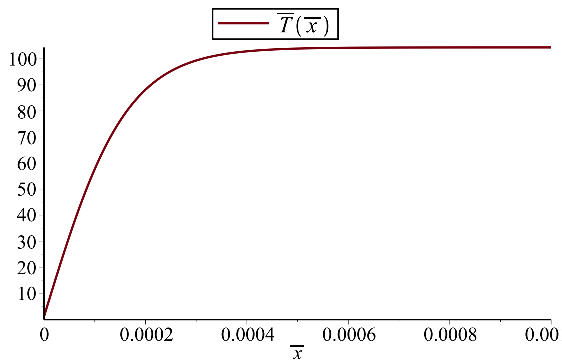


Fig. 2. Dimensionless temperature vs dimensionless distance. A hundred times increase of the plasma temperature by absorption of AW.

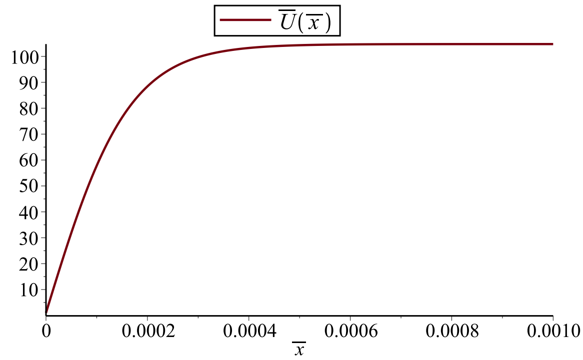


Fig. 3. Dimensionless wind velocity vs dimensionless distance. A hundred times increase of the plasma velocity by absorption of AW.

The input parameters of the program are T_0, $n_{\rm tot}(0)$, β_0, ω, and s which parameterizes $j = \langle m\rangle n_{\rm tot}(0)U_0$. We have obtained our results by setting $T_0 = 6000$ K, $n_{\rm tot}(0) = 5\times 10^{14}$ m^{-3}, $\beta_0 = 1$, $\omega/2\pi = 300\ {\rm s}^{-1}$, $s = 137$. We calculate l, a, $\overline{\eta}_0 = \overline{\nu}_0$, $\overline{\kappa}_0$ and choose some $A_{\rm wave}$ which finally determines increasing of the temperature $\overline{T}(1) = T(l)/T(0)$.

In our self-consistent calculation we use the nonlinear fit $\overline{T}(\bar{x}) = 1 + \left(\sum_{n=0}^{3} a_n\bar{x}^n\right)\tanh(b_1\bar{x})$ for the numerically calculated profile of the temperature $\overline{T}(\bar{x})$ (Fig. 2) as well as for the velocity $\overline{U}(\bar{x})$ (Fig. 3). In order to accelerate the convergence for the initial approximation we use $a_0 = 20$, $a_1 = a_2 = a_3 = 0$, $b_1 = 10$. Let us explain in detail the successive approximations.

1. At fixed wind profiles $\overline{T}(\bar{x})$ and $\overline{U}(\bar{x})$ we calculate $\Lambda = \Lambda_0 + \frac{3}{2}\ln\overline{T} + \frac{1}{2}\ln\overline{U}$, $\overline{\eta}$, $\overline{\varkappa}$, ${\rm d}_{\bar{x}}\overline{\eta}$, and dimensionless matrices $\overline{\mathsf{M}}$ and $\bar{g}$. Then we have to solve the wave equation and to renormalize the solution with some fixed dimensionless energy flux for the R-mode $\mathscr{Q}_{\rm wave}(0)$.

2. Using so obtained wave variables Ψ we have to solve equations (30) and (31) to find $\overline{U}(\bar{x})$ and $\overline{\Theta}(\bar{x})$, and respectively $\overline{T}(\bar{x})$ from Eq. (29). The variables $\overline{\eta}$, $\overline{\varkappa}$, ${\rm d}_{\bar{x}}\overline{\eta}$, and ${\rm d}_{\bar{x}}\overline{\varkappa}$, which participate in the coefficients have to be calculated simultaneously.

3. Having solved the equations for the wind variables and formerly the equations for the wave variables we can calculate the total energy and momentum flux. If the maximal relative difference between two successive temperature profiles is larger than some predetermined value we go to step 1 and repeat the procedure.

The width of the transition layer λ, and the increasing of the temperature $\overline{T}(1)$ are functions of the wave energy flux coming from the chromosphere $q_{\mathrm{wave}}(0)$.

$$\frac{l}{\lambda} = \max_x \left|\mathrm{d}_x \ln \overline{T}\right| \simeq \frac{1}{l\,b_1}. \tag{87}$$

The wave amplitude A_{wave} is determined in a way that ensures the desired temperature increase $\overline{T}(1;A_{\mathrm{wave}}) = \overline{T}(l)/\overline{T}(0)$ in the end (Fig. 2) of the space interval. In our numerical illustration we used one wave which corresponds to δ-like spectral density of AW. In general, at fixed temperature increase and chosen spectral density of the AW the MHD theory has no more free parameters. All that is left is to compare the calculations with other models and observational data for heating of the solar corona and launching of the solar wind.

CONCLUSION

Discussions

In spite that iron was suspicious from the very beginning the problem of Coronium was a 70 year standing mystery until unambiguous identification as Fe^{13+} by Grotrian and Edlen in 1939. The same 70 year time quantum was repeated. In 1947 Alfvén [2] advocated the idea that absorbtion of AW is the mechanism of heating of solar corona. Unfortunately the idea by Swedish iconoclast [13] was never realized in original form: what can be calculated, what is measured, what is explained and what is predicted. That is why there is a calamity of ideas still on the arena, for a contemporary review see the SOHO proceedings ([14]). The wide variety of effects in the physics of the corona is not directly related with hydrodynamic mechanism of its creation from the chromosphere as gerontology has few common points with tocology. From qualitative point of view the narrow width of the transition layer $\lambda = \min\,|\mathrm{d}x/\mathrm{d}\ln T(x)|$ is the main property which should be compared against the predictions of other scenarios. For example, in order for the nanoflare hypothesis to be vindicated [15] such reconnections are needed to explain the narrow width of the transition layer at the same boundary conditions of wind velocity and temperature. Moreover electric fields of the reconnections heats mainly the electron component of the plasma. How then proton temperature in the corona is higher? Launching of Hinode gave a lot of hints for the existence of AW in the corona [16], see also [17]. However most of the research was in the UV region where high frequency AW are already absorbed. All observations are for low frequency (mHz range) AW for which hot corona is transparent. The best that can be done is to extract the low frequency behavior of the spectral density of AW and to extrapolate to higher frequencies responsible for heating. The observed AW are irrelevant for the heating. In order to identify AW responsible for the heating it is necessary to investigate high frequency (1 Hz range) AW in the cold chromosphere using optical not UV spectral lines. We are unaware whether such type of experiments have been planned. One of the purposes of the present work is to focus the attention of experimentalists on the 1 Hz range AW in the chromosphere, which we predict on the basis of our MHD analysis. For such purposes we suggest Doppler tomography [18] of Hα; Ca lines are another possibility. Doppler tomography was successfully used for investigation of rotating objects, such as accretion disks [19] and solar tornados [20]. Here we wish to mention the Doppler tomography by Coronal Multi-channel Polarimeter build by Tomczyk [8]. For investigation of AW by Doppler tomography we suggest development of frequency dependent Doppler tomography operating as a lock-in voltmeter. The data from every space pixel should be multiplied by $\sin\omega t$ and integrated over many wave periods. Finally one can observe time averaged distribution of the AW amplitude. Systematic investigation of such frequency dependent Doppler tomograms will reveal that the Swedish iconoclast [13, 21] is again right that AW heat the solar corona, after another 70 years of dramatic launching of vast variety of ideas.

Plasma heating by AW – a historical perspective

What have we learned from the one-dimensional static MHD problem? We have demonstrated that qualitatively predicted self-induced opacity of plasma is an intrinsic property. Absorption of AW causes viscous heating of ions and that is why the proton temperature is higher than the electron one. In this way we have revealed an effective method for ion heating which can be applied to many plasma problems. Actually plasma heating by MHD waves is used in the MIT alcator [22]. We suggest however that the toroidal geometry can be replaced by Budker probkotron geometry, in which the energy of the AW will be focused in a narrow jet with a hundred times increased temperature. A de Laval nozzle will be realized by strong magnetic fields. We do believe that that this will be an effective method for navigation in the Solar System (cf. [23]). Electric power from a nuclear reactor will create a fast electron-proton jet and this will dramatically decrease the initial mass of the rocket. For large-scale Earth-based installations such a jet of high-temperature deuterium will inject a fresh idea in nuclear fusion physics.

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REFERENCES

[1] P. Swings, Astrophys. J. **98**, 116–128 (1943); B. Edlén, Arkiv f. Matem. Astr. och Fys. **28**, B No. 1 (1941); B. Edlén, Zs. f. Ap. **22**, 30 (1942)

[2] H. Alfvén, MNRAS **107**, 211–219 (1947)

[3] H. Alfvén, Nature **150**, 405–406 (1942)

[4] J. Heyvaerts and E.R. Priest, A&A, 117, 220 (1983)

[5] L. P. Chitta, A. A. Ballegooijen, L. Rouppe van der Voort, E.E. DeLuca, and R. Kariyappa, Asprophys. J; http://arxiv.org/abs/1204.4362

[6] P. Antolin and K. Shibata, ApJ **712**, 494–510 (2010)

[7] M. Hahn and D. W. Savin, ApJ **776**, 78 (2013)

[8] S. Tomczyk and S.W. McIntosh, Asprophys. J; http://arxiv.org/abs/0903.2002

[9] S. Tomczyk, S.W. McIntosh, S.L. Keil, P.G. Judge, T. Schad, D.H. Seeley, and J. Edmondson, Science, 317, 1192 (2007)

[10] R.P. Feynman, R.B. Leighton, and M. Sands, *The Feynman Lectures on Physics. Exercises* (Addison-Wesley, London, 1965), Problems 40.4-5

[11] L.F. Burlaga and W.H. Mish, J. Gephys. Res., 92, 1261 (1987); E. Marsch, in Physics and Chemistry in Space – Space and Solar Physics, Vol. 21, Series Editors: M.C.E. Huber et al., Physics of the Inner Heliosphere, Vol. 2, Editors: R. Schwenn and E. Marsch, Springer-Verlag Berlin (1991); Fig. 10.4

[12] R.V. Topchiyska, N.I. Zahariev, T.M. Mishonov, Bulg. J. Phys. 40 (2013) 56–77

[13] A. J. Dessler, Science **170**, 604 - 606 (1970); http://www.sciencemag.org/content/170/3958/604

[14] *Proceedings of the SOHO 15 Workshop – Coronal Heating* 6-9 September 2004, St. Andrews, Scotland (ESA SP-575, December 2004) Editors: R.W. Walsh, J. Ireland, D. Danesy, and B. Fleck

[15] Ch. Day, Phys. Today **62**, May 18–21 (2009)

[16] B. De Pontieu, *et al.,* Science **318**, 1574–1577 (2007); T.J. Okamoto *et al.,* Science **318**, 1577-1580 (2007); R. Erdélyi and V. Fedun, Science **318**, 1572-1574 (2007); Y. Katsukawa *et al.*, Science **318**, 1594-1597 (2007)

[17] D. Jess *et al.,* Science **323**, 1582–1585 (2009)

[18] T. Marsh, Europhysicsnews **36** No. 4, 133-138 (2005)

[19] T. R. Marsh and K. Horn, MNRAS **235**, 269–286 (1988)

[20] http://www.mps.mpg.de/en/forschung/sonne/

[21] *"Students using astrophysical textbooks remain essentially ignorant of even the existence of plasma concepts, despite the fact that some of them have been known for half a century. The conclusion is that astrophysics is too important to be left in the hands of astrophysicists who have gotten their main knowledge from these textbooks. Earthbound and space telescope data must be treated by scientists who are familiar with laboratory and magnetospheric physics and circuit theory, and of course with modern plasma theory."* Hannes Alfvén quoted by Anthony L. Peratt, 'Dean of the Plasma Dissidents', Washington Times, supplement: The World and I (May 1988), 197. http://todayinsci.com/A/Alfven_Hannes/AlfvenHannes-Quotations.htm

[22] J. A. Snipes *et al.,* Phys. Plasmas **12**, 056102 (2005); http://dx.doi.org/10.1063/1.1865012

[23] E. Y. Choueiri, Scientific American, **300**, 58-65 (2009); doi:10.1038/scientificamerican0209-58

[24] J. A. Araneda, Y. Maneva, and E. Marsch, Phys. Rev. Lett., **102**, 175001 (2009)

[25] Y. G. Maneva, J. A. Araneda, and E. Marsch, ed. by I. Zhelyazkov, AIP Conference Proceedings, vol. **1121**, pp. 122–126 (2009)

[26] Y. G. Maneva, J. A. Araneda, and E. Marsch, Twelfth International Solar Wind Conference, eds. M. Maksimovic, K. Issautier, N. Meyer-Vernet, M. Moncu-

quet and F. Pantellini, AIP Conference Proceedings, vol. **1216**, pp. 227–230, (2010)

[27] Y. G. Maneva, Ph.D. Thesis, (Göttingen, uni-edition, 2010), ISBN 978-3-942171-39-7

[28] T. M. Mishonov, M. V. Stoev, and Y. G. Maneva, Eur. Phys. J. D **44**, 533–536 (2007)

НАГРЯВАНЕ НА СЛЪНЧЕВАТА КОРОНА ЧРЕЗ АЛФВЕНОВИ ВЪЛНИ – САМОИНДУЦИРАНА НЕПРОЗРАЧНОСТ

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(Резюме)

За първи път Ханес Алфвен предлага идеята, че нагряването на слънчевата корона се осъществява благодарение на Алфвенови вълни (АВ) [1,2]. Тези вълни се генерират от турбуленцията в зоната на конвекция и се разпространяват по магнитните силови линии.

Цели. Пресметнато е статично разпределение на температурата и скоростта на вятъра в преходната зона, използвайки магнитохидродинамика (МХД) на напълно йонизирана водородна плазма.

Методи. Численото решение на изведените уравнения позволява да се определи дебелината на преходния слой между хромосферата и короната, в който се поражда явлението самоиндуцирана непрозрачност на високочестотни АВ. Обособяването на граница на зоната е директно следствие от самосъгласуваното МХД разглеждане на разпространението на АВ.

Резултати. Нискочестотните МХД вълни, идващи от Слънцето, се отразяват силно от тесния преходен слой, а високочестотните вълни се поглъщат. Това ни позволява да предскажем значителна спектрална плътност на АВ във фотосферата. Численото разглеждане позволява отчитането на падащи АВ с произволна спектрална плътност.

Заключение. Идеята, че АВ могат да нагреят слънчевата корона, принадлежи на Ханес Алфвен. Нашата работа се състои в решаването на съответните МХД уравнения. Сравняването на решението с експериментални данни е от съществено значение за разкриването на механизма на нагряване на короната.

1. H. Alfvén, *Nature* **150** (1942), 405-406.
2. H. Alfvén, *MNRAS* **107** (1947), 211-219.