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Convective heat transfer of viscous fluid over a stretching sheet embedded in a thermally stratified medium

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In this article we have investigated the heat transfer of an electrically conducting viscous fluid over a porous stretching sheet in a thermally stratified medium. The governing non-linear partial differential equations are reduced to ordinary differential equations using appropriate similarity transformations. The resulting ordinary differential equations are then solved in the form of a confluent hyper-geometric function for an exact solution. The developed exact solutions of the velocity and temperature fields are graphically sketched and examined for various values of pertinent parameters including the Prandtl number, stratification parameter, suction/injection parameter and the magnetic parameter. The skin friction coefficient and the local Nusselt number are tabulated and thoroughly discussed.

Keywords: Convective heat transfer; Suction/Injection; Exact solution

INTRODUCTION

The study of the boundary layer flow on a stretching sheet has been done by a large number of researchers during the last few decades with the aidof significant applications of industrial and technological processes. To name a few; these applications include manufacturing of glass fiber, drawing plastic films and wires, the condensation process, crystal growing polymer extrusion and others. These processes are highly dependent on the subject of heat transfer of stretching surfaces. In his ground breaking work, Sakiadis [1] presented the studies on the boundary flow layer over continuously moving surfaces and obtained the numerical solution. Natarjaet al. [2]obtained the closed form solution for the boundary layer flow of walters' B-type fluid, over a stretching sheet for the heat transfer and obtained the coefficients of skin friction. Meanwhile, Crane [3] provided the closed form solution for the boundary layer flow of a stretching sheet. The heat transfer in hydrodynamic flow of viscoelastic fluid over a stretching sheet was analyzed by Char [4]. Liao [5]studied the analytic solution of unsteady boundary layer flows caused by an impulsively stretching plate. Khan and Sanjayanand [6]presented the analytic solution for the heat transfer of visco-elastic boundary layer flow with viscous dissipation. Devi and Ganga [7]bring into account the non-linear MHD flow in a porous medium over a stretching porous surface including the effects of viscous dissipation. Abel et al. [8]have investigated the heat transfer over a stretching surface for second grade fluid through porous medium with viscous dissipation and a non-uniform heat source/sink. Cortell [9] investigated the flow and heat transfer through a porous medium over a stretching surface with heat generation/absorption and suction/blowing. Hayat et al. [10]provided the analytic solution for the axi-symmetric flow and heat transfer of second grade fluid past over a stretching sheet. Xu and Liao[11]considered the unsteady MHD flows of non-Newtonian fluids over impulsively a stretching plate. In another related article Cortell[12]studied MHD flow heat transfer of visco-elastic fluid by considering the effects of viscous dissipation.

Thermally stratified flows are of significant interest because of their importance in thermo-hydraulics, volcanic flows, geothermal systems and also in industrial thermal processes. Stratification of a medium arises due to temperature variation which resultsin density variationof the medium. Stratification may also arise due to the presence of different fluids so that a stable situation arises when the lighter fluid lies over the denser one. Keeping in view these applications of stratified mediums several studies have been carried out. Hayat et al. [13]studied the thermally stratified flow of third grade fluid over a stretching sheet including radiation. Kandasamy and Khamis [14]discussed the effect of thermal stratification on heat transfer across a porous vertical stretching sheet. Ishak et al.[15]studied the mixed convection flow to a vertical plate in a thermally stratified medium. Mukhopadhyay and Ishak [16]examined mixed convection flow along a stretching cylinder in a

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thermally stratified medium. MHD boundary layer flow and heat transfer over an exponentially stretching sheet in a thermally stratified medium have also been investigated by the same author Mukhopadhyay [17].

It is clear that the suction/injection of fluid can play a significant role in changing the flow field. Roughly speaking, suction tends to enhance the skin friction, whereas injection acts in the opposite manner. These processes have great importance in many engineering activities like the design of thrust bearing and radial diffusers, thermal oil recovery and many more.

In manufacturing processes the properties of the final product highly depend on the rate of cooling. In this scenario, an electrically conducting fluid proves to be beneficial for industrial application. The applied magnetic field may play a key role in heat transfer and momentum of the boundary layer flow. Keeping all these factsin view, recently Chen [18] examined the analytic solution of MHD flow and heat transfer for two types of visco-elastic fluid over a stretching sheet, also bring under consideration the energy dissipation, internal heat source and thermal radiation. Liu [19] presented an analytic solution for heat transfer of second grade MHD flow subject to the transverse magnetic field across a stretching sheet with power law surface heat flux. Shahzad and Ali [20, 21] contributed a couple of articles on an approximate solution for MHD flow of a non-Newtonian Power law fluid over a vertical stretching sheet with convective boundary conditions and radiation effects, respectively. Kar et al. [22]studied the heat and mass transfer effects on dissipative and radiative visco-elastic MHD flow over a stretching porous sheet.

The exact solution for the flow problem with heat transfer is highly demanding in many research areas. The exact solutions are handy to compare with the numerical counter parts in the study of several flow problems. The purpose of the present study is, to give the exact solution of fluid flow and heat transfer of an electrically conducting viscous fluid over a stretching sheet in a thermally stratified medium with suction/injection. We derived a closed form analytic solution in the form of a confluent hyper-geometric function for non-dimensional velocity and temperature profiles. The skin friction coefficient and heat flux at the wall with a constant wall temperature are brought into account. The influence of different non-dimensional parameters like the Prandtl number, magnetic number, stratification parameter and the surface suction/injection are graphically discussed with respect to velocity and temperature profiles.

MATHEMATICAL FORMULATION

We consider the steady two-dimensional boundary layer flow of an incompressible electrically conducting viscous fluid, which is passed over a stretching sheet in the presence of a magnetic field coinciding with the plane $y = 0.$ The flow is generated due to linearly stretching of sheet by applying two equal and opposite forces along the $x-$axis keeping the origin fixed as observed in figure (1). A variable magnetic field B_o is applied normal to the sheet. It is assumed that the surface temperature of the sheet is $T_w(x) = T_0 + a(\frac{x}{l})^2$ and is embedded in a thermally stratified medium of a variableambient fluid temperatures $T_\infty = T_0 + b(\frac{x}{l})^2$ where $T_w > T_\infty$, T_0 is the reference temperature, $a > 0,$ $b \geq 0$ are constants.

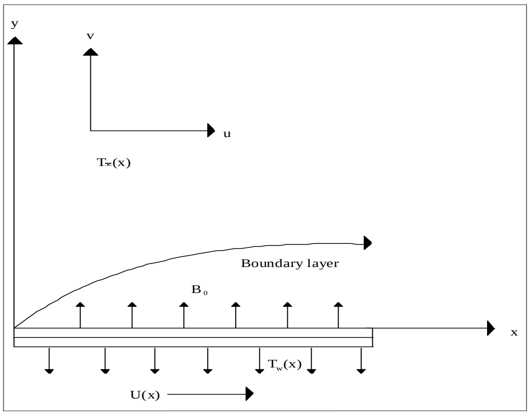


Fig. 1. Sketch of the physical problem.

Under these assumptions the steady state boundary layer equations governing the flow and heat transfer of viscous fluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_\circ^2 u}{\rho}, \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \quad (3)$$

where x and y are the directions along and perpendicular to the sheet respectively, with u and v the velocity components along the x and y directions, respectively, T the temperature of the fluid , υ the kinematic viscosity, ρ the density of the fluid, σ the electrical conductivity, $B_\circ$ the applied magnetic field and c_p is the specific heat at constant pressure. The corresponding boundary conditions for the momentum equation are

$$u = U = Ex \qquad v = -v_\circ \quad \text{at} \quad y = 0, \qquad u \to 0 \text{ as } y \to \infty, \tag{4}$$

where $E > 0$ is called stretching rate, $v_\circ > 0$ is the velocity of suction and $v_\circ < 0$ is the velocity of injection. To facilitate the analysis we introduce the following transformations:

$$u = Uf'(\eta), \quad v = -\sqrt{E\upsilon}\, f(\eta), \quad \eta = \sqrt{\frac{E}{\upsilon}}\, y, \tag{5}$$

clearly u and v satisfy Eq. (1), here $f(\eta)$ is the dimensionless stream function, η is the similarity variable and denote the differentiation with respect to η. Making use of Eq. (5) in Eq. (2) the following third order non-linear differential equation is obtained:

$$f''' - f'^2 + ff'' - M_n f' = 0, \tag{6}$$

where $M_n = \frac{\sigma B_\circ^2}{\rho E}$, is the Magnetic parameter.

Similarly the boundary conditions in Eq. (4) can be written as:

$$f'(\eta) = 1, \quad f(\eta) = S \quad \text{at } \eta = 0, \qquad f'(\eta) \to 0 \text{ as } \eta \to \infty, \tag{7}$$

where $S = \frac{v_\circ}{\sqrt{E\upsilon}} > 0$ (or < 0) is the suction (or injection) parameter.

Lawrence and Rao [23] presented a general method and obtained an all non-unique solution of the modified Eq. (6). Recently Taha et al. [24] have discussed a unified compatibility method for the exact solutions of non-linear flow models of Newtonian and non-Newtonian fluids, we consider the following solution:

$$f(\eta) = S + \frac{1}{\beta}(1 - \exp[-\beta\eta]), \tag{8}$$

where $\beta = \frac{S + \sqrt{S^2 + 4(1 + M_n)}}{2}$ with $S^2 + 4(1 + M_n) \geq 0.$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number N_u defined as:

$$C_f = \frac{\tau_w}{\frac{\rho}{2}U^2}, \quad N_u = \frac{xq_w}{k(T_w - T_\circ)}, \tag{9}$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)\Big|_{y=0}$ is the wall shear stress and $q_w = -k\left(\frac{\partial T}{\partial y}\right)\Big|_{y=0}$ the wall heat flux. In terms of dimensionless variables defined in Eq. (5), we can write:

$$\mathrm{Re}^{1/2} C_f = f''(0), \quad \mathrm{Re}^{-1/2} N_u = -\theta'(0), \tag{10}$$

where $\mathrm{Re} = \frac{xU}{v}$ is the local Reynolds number.

SOLUTION OF THE HEAT TRANSFER EQUATION

In order to solve the governing heat transport Eq. (3), we consider the boundary with a prescribed surface temperature (PST). In this case we employ the following surface boundary conditions on temperature:

$$T = T_w(x) = T_\circ + a\left(\frac{x}{l}\right)^2 \quad \text{at} \quad y = 0, \qquad T = T_\infty(x) = T_\circ + b\left(\frac{x}{l}\right)^2 \quad \text{as} \quad y \to \infty, \tag{11}$$

where T_w and T_∞ are the temperatures at the wall and far away from the wall, respectively and $T_\circ$ is the reference temperature. In order to obtain the similarity solution we define the non-dimensional temperature variables as:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\circ}. \tag{12}$$

Making use of the transformations (5) and (12), we obtain the following non-dimensional form of temperature Eq. (3) as:

$$\theta'' + \Pr(f\theta' - 2f'\theta) = 2St\Pr f', \tag{13}$$

where inthe non-dimensional parameters $\Pr = \frac{\rho C_p \upsilon}{k}$ the Prandtl number and $St = \frac{b}{a}$ is the stratification parameter. We note that $St > 0$ for a stably stratified environment and $St = 0$ corresponds to an unstratified environment.

Thenon-dimensional form of the boundary conditions in Eq. (11) is:

$$\theta = 1 - St \quad \text{at} \quad \eta = 0, \qquad \theta \to 0 \quad \text{as} \quad \eta \to \infty. \tag{14}$$

Following the introduction of a new variable $\xi = -\frac{\Pr}{\beta^2}\exp(-\beta\eta)$, Eq. (13) becomes:

$$\xi\frac{d^2\theta}{d\xi^2} + (1 - \Pr^* - \xi)\frac{d\theta}{d\xi} + 2\theta = -2St, \tag{15}$$

and the boundary conditions presented in Eq. (14) reduce to:

$$\theta(\xi) = 1 - St \quad \text{at} \quad \xi = -\Pr^*, \qquad \theta(\xi) = 0 \quad \text{at} \quad \xi = 0, \tag{16}$$

where $\Pr^* = \frac{\Pr}{\beta^2}$ is the modified Prandtl number.

Equations (15) and (16) constitute a non-homogenous boundary value problem. Let us decompose the temperature $\theta(\xi)$ into two parts:

$$\theta(\xi) = \theta_c(\xi) + \theta_p(\xi), \tag{17}$$

where $\theta_c(\xi)$ stands for the complemenrty solution and $\theta_p(\xi)$ is a particular solution. The closed form particular solution is given by:

$$\theta_p(\xi) = -St. \quad (18)$$

The complementary factor $\theta_c(\xi)$ can be written in the form of a confluent hyper-geometric function [25] as:

$$\theta_c(\xi) = AM[-2,\ 1-\mathrm{Pr}^*,\ \xi] + B\xi^{\mathrm{Pr}^*} M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ \xi] \quad (19)$$

where M is the Kummer's function defined as:

$$M(a_\circ, b_\circ, z) = 1 + \sum_{n=1}^{\infty} \frac{(a_\circ)_n z^n}{(b_\circ)_n n!},$$
$$(a_\circ)_n = a_\circ(a_\circ+1)(a_\circ+2)\cdots(a_\circ+n-1),$$
$$(b_\circ)_n = b_\circ(b_\circ+1)(b_\circ+2)\cdots(b_\circ+n-1). \quad (20)$$

The solution of Eq. (15), subject to the boundary conditions of Eq. (16) is determined to be:

$$\theta(\xi) = StM[-2,\ 1-\mathrm{Pr}^*,\ \xi] + \left\{\frac{1-StM[-2,\ 1-\mathrm{Pr}^*,\ -\mathrm{Pr}^*]}{(-\mathrm{Pr}^*)^{\mathrm{Pr}^*} M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ -\mathrm{Pr}^*]}\right\} \xi^{\mathrm{Pr}^*} M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ \xi] - St. \quad (21)$$

The temperature profile in term of η is given by

$$\theta(\eta) = StM[-2,\ 1-\mathrm{Pr}^*,\ -\mathrm{Pr}^*\exp(-\beta\eta)] + \left\{\frac{1-StM[-2,\ 1-\mathrm{Pr}^*,\ -\mathrm{Pr}^*]}{M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ -\mathrm{Pr}^*]}\right\} \times \exp(-\beta\,\mathrm{Pr}^*\eta)\, M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ -\mathrm{Pr}^*\exp(-\beta\eta)] - St. \quad (22)$$

The derivative of the Kummer's function $M(a_\circ, b_\circ, z)$ with respect to z is given by:

$$\frac{d^n}{dz^n} M[a_\circ, b_\circ, z] = \frac{(a_\circ)_n}{(b_\circ)_n} M[a_\circ+n, b_\circ+n,\ z].$$

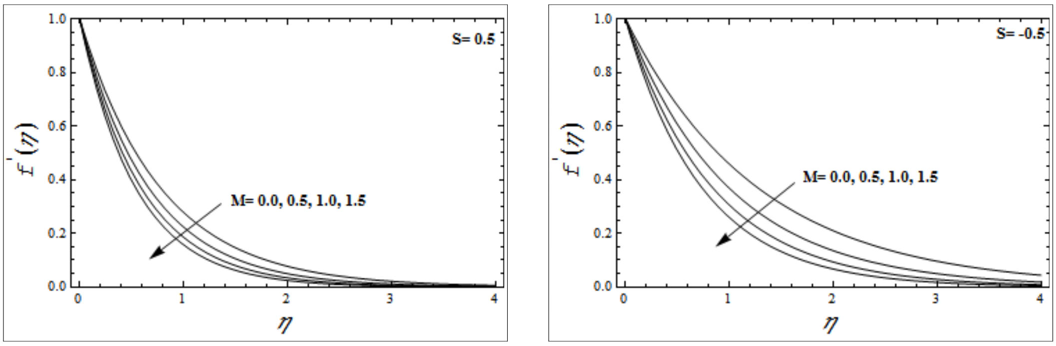
The dimensionless wall temperature gradient is obtained from Eq. (22)

$$\theta'(0) = \frac{-2St\beta\,\mathrm{Pr}^*}{1-\mathrm{Pr}^*} M[-1,\ 2-\mathrm{Pr}^*,\ -\mathrm{Pr}^*] - \left(\frac{1-StM[-2,\ 1-\mathrm{Pr}^*,\ -\mathrm{Pr}^*]}{M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ -\mathrm{Pr}^*]}\right) \times \left\{\begin{array}{l} -\frac{\beta\,\mathrm{Pr}^*(-2+\mathrm{Pr}^*)}{1+\mathrm{Pr}^*} M[-1+\mathrm{Pr}^*,\ 2+\mathrm{Pr}^*,\ -\mathrm{Pr}^*] \\ +\beta\,\mathrm{Pr}^* M[-2+\mathrm{Pr}^*,\ 1+\mathrm{Pr}^*,\ -\mathrm{Pr}^*] \end{array}\right\}. \quad (23)$$

GRAPHICAL RESULTS AND DISCUSSION

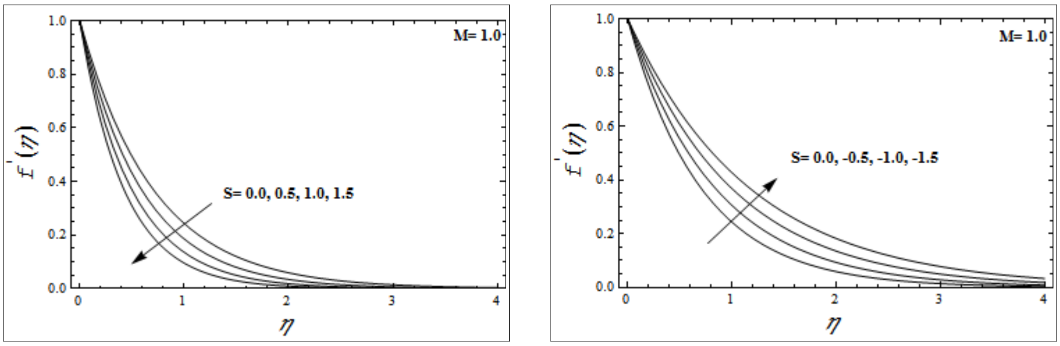
Momentum and heat transfer in a boundary layer flow of a viscous fluid over a stretching sheet in a thermally stratified medium have been discussed in this article. The governing non-linear partial differential equations have been reduced to a set of non-linear ordinary differential equations. The exact solutions are developed for the reduced problem in terms of the Kummer's function. In order to have a clear insight of a physical problem, the influence of various pertinent parameters on velocity and temperature profiles are shown graphically through figures $(2-9)$. Figures $(2a, 2b)$ depict the influence of the magnetic parameter M on the velocity profile $f'(\eta)$ for both suction/injection S cases. It is observed that the velocity profile $f'(\eta)$ decreases with an increase in the values of the magnetic parameter M for both suction/injection parameters S . The boundary layer thickness also decreases here. The variation of suction/injection parameter S on the velocity profile $f'(\eta)$ is shown in figures $(3a, 3b)$. The velocity profile $f'(\eta)$ decreases in the case of the suction parameter $(S>0)$, while the opposite behavior is noticed in case of injection $(S<0)$. Figures ($4a, 4b$) are plotted to see the effects of the Prandtl number Pr on the temperature profile $\theta(\eta)$ in the presence of suction/injection parameter S. It is obvious from these figures that an increase in the Prandtl number Pr results in a decrease in the temperature profile $\theta(\eta)$ for bothcases suction $(S>0)$ and injection $(S<0)$, however this decrease in the temperature profile $\theta(\eta)$ is more prominent in the case of suction $(S>0)$. The influence of the suction /injection parameter S on the temperature profile $\theta(\eta)$ is displayed in figures $(5a, 5b)$. These figures show that by increasing the suction parameter $(S>0)$ the temperature profile $\theta(\eta)$ increases while an adverse behavior is observed in the case of injection $(S<0)$.

In order to see the effects of the stratified medium St on the temperature profile $\theta(\eta)$ in the presence of the suctions/injection parameter S figures $(6a, 6b)$ are plotted. These figures illustrate that the temperature profile $\theta(\eta)$ decreases as the stratification parameter St increases for both suction/injection parameters S . As the increase in the stratification medium St implies an increase in ambient fluid temperature or a decrease in the surface temperature, which results in a decrease of the thermal boundary layer thickness.



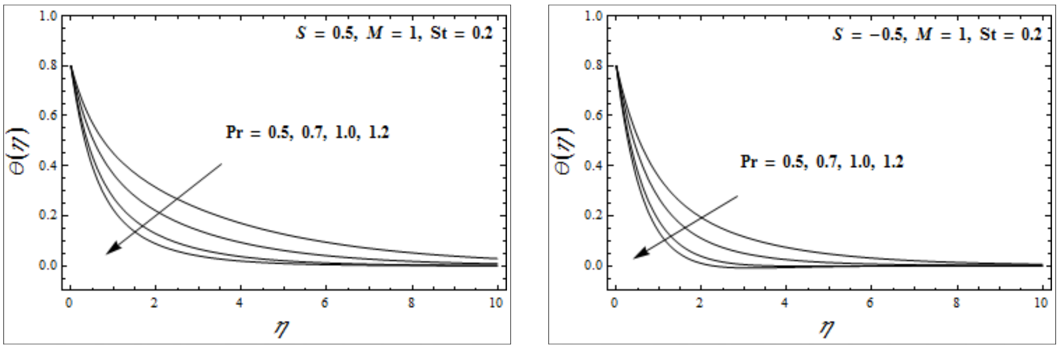
2a 2b

Fig.2. Variation of velocity profile $f'(\eta)$ with η for several values of magnetic parameter M in the presence of the suction/injection parameter S.



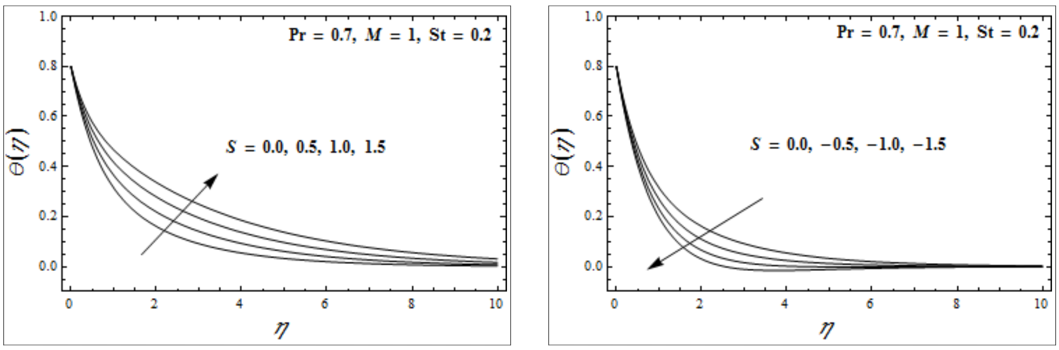
3a 3b

Fig. 3. Variation of the velocity profile $f'(\eta)$ with η for several values of the suction/injection parameter S.



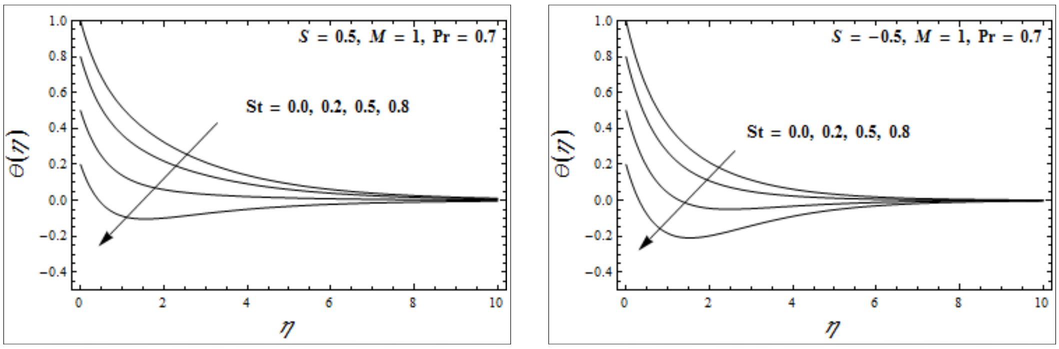
4a 4b

Fig. 4. Variation of the temperature profile $\theta(\eta)$ with η for several values of the Prandtl number $\Pr$.



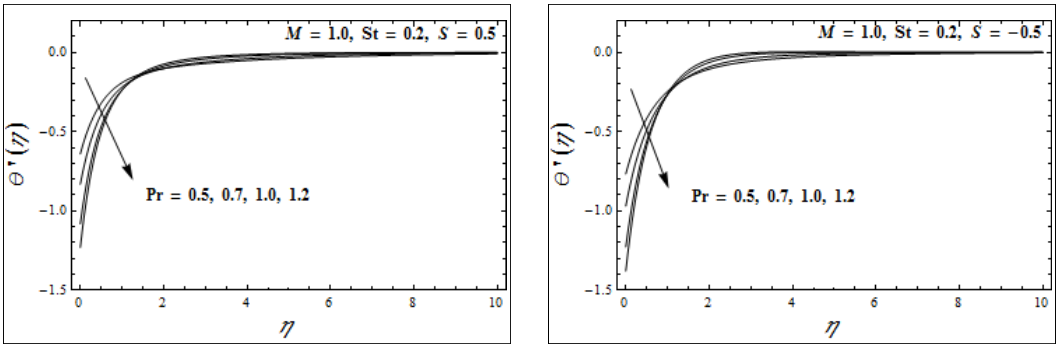
5a 5b

Fig. 5. Variation of the temperature profile $\theta(\eta)$ with η for several values of the suction parameter S.



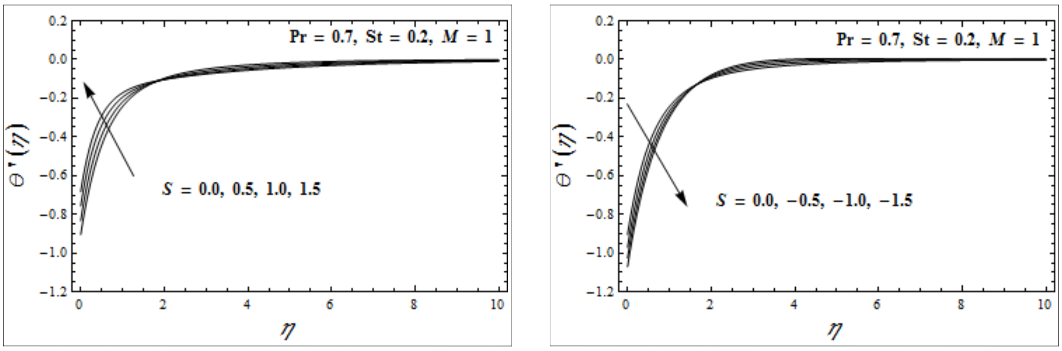
6a 6b

Fig. 6. Variation of the temperature profile $\theta(\eta)$ with η for several values of the thermal stratification parameter St inthe presence of suction/injection S.



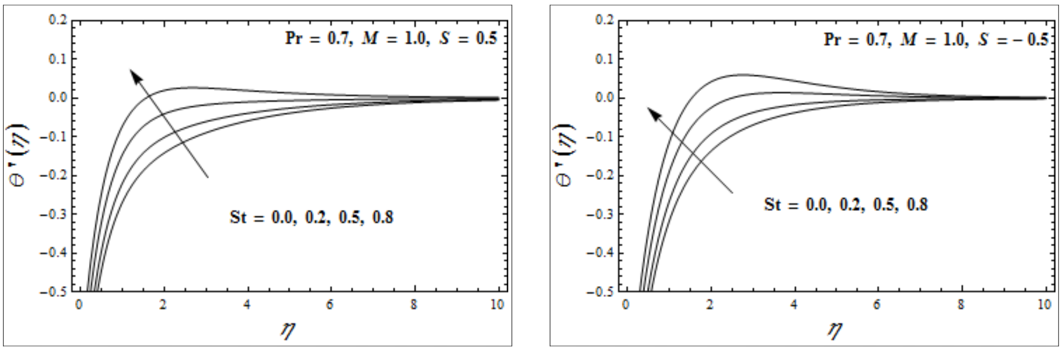
7a 7b

Fig. 7. Variation of the temperature gradient $\theta'(\eta)$ with η for several values of the Prandtlnumber $\Pr$.



8a 8b

Fig. 8. Variation of the temperature gradient $\theta'(\eta)$ with η for several values of the suction/injection parameter S.



9a 9b

Fig. 9. Variation of the temperature gradient $\theta'(\eta)$ with η for several values of the thermal stratification parameter St.

The influence of the Prandtl number Pr on the temperature gradient $\theta'(\eta)$ in the presence of suction/injection S is displayed in figures $(7a, 7b)$.

From these figures it is observed that initially an increase in the Prandtl number Pr results in a decrease in the temperature gradient $\theta'(\eta)$. It is also observed that the temperature gradient $\theta'(\eta)$ starts increasing after a certain distance η from the surface. Moreover this effect is more prominent in the case of injection $(S<0)$. From figures $(8a, 8b)$ it can be seen that by increasing the suction parameter $(S>0)$ the temperature gradient $\theta'(\eta)$ increases up to a certain distance from the surface and then decreases. Quite the opposite behavior is observed in the case of injection $(S<0)$. To analyze the effect of the stratification parameter St on the temperature gradient the figures $(9a, 9b)$ are plotted. It is noticed that the temperature gradient $\theta'(\eta)$ increases with an increase in the stratification parameter St for both suction andthe injection parameter S. Furthermore, it is also observed that this increase is more prominent in the case of injection $(S<0)$.

Table 1. Values of the skin friction coefficient C_f $\mathrm{Re}^{1/2} = f''(0)$ for several values of the material parameters.

M	S	$-f''(0)$
0.0	0.5	1.28078
0.5		1.50000
1.0		1.68614
0.5	−1.0	0.822876
	−0.5	1.00000
	0.0	1.22474
	0.5	1.50000
	1.0	1.82288

Table 1 is displayed in order to see the effect of the magnetic parameter M and the suction/injection parameter S on the skin friction coefficient. We can see that by increasing the values of the magnetic parameter M and the suction/injection parameter S, the values of the skin friction coefficient increase. Table 2 shows the effect of the Prandtlnumber Pr, suction/injection S and stratification parameter St on the dimensionless heat transfer rate at the wall. It has been observed that the increase in thePrandtl number Pr and stratification parameter, St results,an increase in the dimensionless heat transfer rate at the wall. While increasing values ofsuction/injection parameter S decreasesthe dimensionless heat transfer rate at the wall.

Table 2. Values of the Nusselt number N_u $\mathrm{Re}^{-1/2} = -\theta'(0)$ for several values of the material parameters for $M = 1$.

Pr	S	St	$-\theta'(0)$
0.5	0.5	0.2	0.640382
0.7			0.831958
1.0			1.08195
0.7	0.5	0.0	0.87966
		0.2	0.831958
		0.5	0.760405
0.7	−1.0	0.2	1.02462
	−0.5		0.969256
	0.0		0.904259
	0.5		0.831958
	1.0		0.756596

CONCLUDING REMARKS

In this contribution, we have articulated the exact solutions of a viscous fluid over a stretching sheet in a thermally stratified medium in the presence of a magnetic field. The modeled non-linear partial equations were transformed into a system of non-dimensional ordinary differential equations using appropriate transformations. The exact solutions were found in the form of confluent hyper-geometric functions (Kummer's function). The influence of pertinent parameters on the velocity and temperature profiles were shown graphically and discussed in details. Numerical values concerning the skin friction coefficient and Nusselt numbers with several respective parameters were provided in tabular form.

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КОНВЕКТИВНО ТОПЛОПРЕНАСЯНЕ ВЪВ ВИСКОЗЕН ФЛУИД НАД РАЗТЕГНАТ ЛИСТ, ПОСТАВЕН В ТЕРМИЧНО ЕДНОРОДНА СРЕДА

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(Резюме)

В тази статия ние изследвахме топлопренасянето в електропроводящ вискозен флуид над разтегнат порьозен лист в термично еднородна среда. Не-линейните частни диференциални уравнения на преноса са сведени до обикновени с помощта на автомоделни трансформации. Получените обикновени диференциални уравнения са решени точно във вид на изродени хипергеометрични функции.Получените точни решения за скоростното и температурното поле са представени графично и са изследвани за различни съществени параметри на течението, включвайки числото на Прандтл, параметъра на стратификация, параметъра за всмукване/впръскване и за магнитните свойства. Табулирани са и подробно са обсъдени коефициента на триене и числото на Нуселт.