## Hall effects on MHD natural convection flow with heat and mass transfer of heat absorbing and chemically reacting fluid past a vertical plate with ramped temperature and ramped surface concentration

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This paper is concerned with studying the influence of Hall current on the unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, temperature dependent heat absorbing and chemically reacting fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium. The governing non-dimensional equations are solved analytically in closed form by Laplace Transform Technique. The expressions for skin friction, Nusselt number and Sherwood number are also derived. The variations in fluid velocity, fluid temperature and species concentration are displayed graphically, whereas numerical values of skin friction, Nusselt number and Sherwood number are presented in tabular form for various values of pertinent flow parameters. Numerical results of natural convection flow near a ramped temperature plate with ramped surface concentration are also compared with the corresponding flow near an isothermal plate with uniform surface concentration.

Keywords: Hall Current, Natural Convection, Heat Absorption, Chemical Reaction.

#### INTRODUCTION

Hydromagnetic natural convection flows in porous and non-porous media have been studied extensively due to the widespread applications abound in many areas of science and technology including fusion processes in electrical furnaces, problems of boundary layer control in the field of aerodynamics, geothermal energy extraction, metallurgy, plasma studies, mineral and petroleum engineering, etc. In addition to it. the Magnetohydrodynamic (MHD) natural convection flow of an electrically conducting fluid in a fluid saturated porous medium has also been successfully exploited in crystal formation. Oreper and Szekely [1] observed that the presence of magnetic field can suppress natural convection currents and the strength of magnetic field is one of the important factors in reducing non-uniform composition thereby enhancing quality of crystal. Following this, several researchers investigated the unsteady hydromagnetic natural convection flow of an electrically conducting fluid past bodies with different geometries under various initial and boundary conditions. Mention may be made of the research studies of Raptis and Kafousias [2], Raptis [3], Chamkha [4], Aldoss et al. [5], Helmy [6], Kim [7], Chaudhary and Jain [8], Rashidi et al. [9-10],

The study of hydromagnetic natural convection flow with heat and mass transfer, induced due to a moving surface with a uniform or non-uniform velocity, has drawn considerable attention of several researchers owing to its vast applications in numerous manufacturing processes in industry, which include boundary layer flow along with material handling conveyers, cooling of an infinite metallic plate in a cooling bath, extrusion of plastic sheets, distribution of temperature and moisture over agricultural fields and groves of trees, continuous casting and levitation, glass blowing, design of chemical processing equipments, formation and dispersion of fog, damage of crops due to freezing, common industrial sight, especially in power plants, etc. Keeping in view the significance of such study, Jha [14] investigated hydromagnetic natural convection and mass transfer flow past a uniformly accelerated moving vertical plate through a porous medium. Ibrahim et al. [15] examined the unsteady hydromagnetic natural convection flow of a micro-polar fluid and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Makinde and Sibanda [16] analyzed the hydromagnetic mixed convective flow with heat and mass transfer past a vertical plate embedded in a porous medium with constant wall suction. Makinde [17] studied the

Seth et al. [11-12] and Makinde and Tshehla [13].

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hydromagnetic mixed convection flow and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium. Makinde [18] also studied the hydromagnetic boundary layer flow with heat and mass transfer over a moving vertical plate under convective surface boundary conditions. Poonia and Chaudhary [19] investigated a unsteady two-dimensional hydromagnetic laminar mixed convective boundary layer flow with heat and mass transfer of a viscous, incompressible and electrically conducting fluid along an infinite vertical plate embedded in a porous medium. Eldabe et al. [20] analyzed the unsteady hydromagnetic convection flow of a viscous and incompressible fluid with heat and mass transfer in a porous medium near a moving vertical plate with time-dependent velocity. Prakash et al. [21] studied the diffusion-thermo and radiation effects on a unsteady hydromagnetic natural convection flow in a porous medium past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion.

The effect of heat absorption/generation on the hydromagnetic natural convection flow of a viscous, incompressible and electrically conducting fluid has a significant effect on heat transfer characteristics in numerous physical problems of practical interest such as fluids undergoing exothermic and/or endothermic chemical reaction [22], convection in Earth's mantle [23], post accident heat removal [24], fire and combustion modeling [25], development of metal waste from spent nuclear fuel [26], etc. This encouraged several researchers to undertake the study of a hydromagnetic natural convection flow in the presence of heat absorption/generation of a viscous, incompressible and electrically conducting fluid past bodies with various geometries. Kamel [27] investigated unsteady hydromagnetic convection flow due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate with temperature dependent heat sources and sinks. Chamkha [28] examined the unsteady hydromagnetic two-dimensional convective laminar boundary layer flow with heat and mass transfer of a viscous, incompressible, electrically conducting and temperature dependent heat absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field. Makinde [29] analyzed heat and mass transfer by a hydromagnetic mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation. In all these investigations, numerical/analytical solutions were

obtained by assuming simplified conditions that the velocity and temperature at the plate are uniform, continuous and well defined. However, the majority of problems of practical interest require the velocity and temperature to satisfy non-uniform, discontinuous or arbitrary conditions at the plate. Due to this fact, several researchers investigated the natural convection flow from a vertical plate with discontinuities in the surface temperature considering different aspects of the problem. Mention may be made of the research studies by Hayday et al. [30], Kelleher [31], Kao [32], Lee and Yovanovich [33] and Chandran et al. [34]. The effect of radiation on the natural convection flow of a viscous and incompressible fluid near a vertical flat plate with ramped temperature was investigated by Patra et al. [35]. They compared the effects of radiative heat transfer on the natural convection flow near a ramped temperature plate with the flow near an isothermal plate. Seth and Ansari [36] studied the unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and temperature dependent heat absorbing fluid past an impulsively moving vertical plate with ramped temperature in a porous medium taking into account the effects of thermal diffusion. Subsequently, Seth et al. [37] extended the problem studied by Seth and Ansari [36] to consider the effects of rotation on flow-field. Seth et al. [38] analyzed the hydromagnetic natural convection flow past an accelerated moving vertical plate with ramped temperature in a porous medium with thermal diffusion and heat absorption.

The practical applications of a hydromagnetic convection flow with heat and mass transfer in the presence of chemically reactive species showed its importance in different areas of science and engineering. In many chemical engineering processes, there occurs a chemical reaction between a foreign mass and fluid. Classically, chemical reactions encompass changes that strictly involve the motion of electrons in forming and breaking of chemical bonds, although the general concept of a chemical reaction is applicable to transformations of elementary particles, as well as to nuclear reactions. Chemical reactions can be classified as either heterogeneous or homogeneous processes depending on, whether they occur at an interface or as a single-phase volume reaction. These processes take place in numerous industrial applications, viz. curing of plastics, cleaning and chemical processing of materials, manufacturing of pulp and insulated cables, polymer production, manufacturing of ceramics or glassware, food processing, etc. Keeping into consideration such study, Afify [39] investigated the effect of radiation on the natural convection flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Muthucumaraswamy and Chandrakala [40] studied radiative heat and mass transfer effects on a moving isothermal vertical plate in the presence of chemical reaction. Zueco and Ahmed [41] analyzed the combined heat and mass transfer by a mixed convection hydromagnetic flow along a porous plate with chemical reaction in the presence of heat source. Suneetha and Reddy [42] investigated radiation and Darcy effects on the unsteady hydromagnetic heat and mass transfer flow of a chemically reacting fluid past an impulsively started vertical plate with heat generation. Chamkha et al. [43] discussed the effects of Joule heating, chemical reaction and thermal radiation on the hydromagnetic natural unsteady convection boundary layer flow with heat and mass transfer of a micro polar fluid from a semi-infinite heated vertical porous plate in the presence of a uniform transverse magnetic field. Bhattacharya and Layek [44] obtained a similarity solution of MHD boundary layer flow with mass diffusion and chemical reaction over a porous flat plate with suction/blowing. Mohamed et al. [45] investigated the unsteady MHD natural convection heat and mass transfer boundary layer flow of a viscous, incompressible, optically thick and electrically conducting fluid through a porous medium along an impulsively moving hot vertical plate in the presence of homogeneous chemical reaction of first order and temperature-dependent heat sink. They obtained analytical solution of the governing equations in closed form by the Laplace transform technique. Hernandez and Zueco [46] studied the unsteady hydromagnetic free convection by a laminar, viscous, electrically conducting and heat generating/absorbing fluid on a continuously moving vertical permeable surface in the presence of radiation effect, chemical reaction and mass flux. Nandkyeolyar et al. [47] analyzed the unsteady hydromagnetic heat and mass transfer flow of a radiating and chemically reactive fluid past a flat porous plate with ramped wall temperature.

It is found that, when the density of an electrically conducting fluid is low and/or the applied magnetic field is strong, a current is induced in a direction which is normal to both the electric and magnetic fields. Thus, if an electric field be applied at the right angle to the magnetic field, the total current will not flow along the electric field. This tendency of the electric current to flow across an electric field in the presence of a magnetic field is called Hall effect and the resulting current is known as Hall current which reduces the electrical conductivity normal to the lines of force so that the electrical conductivity becomes anisotropic [48]. One of the important characteristics of Hall current is to induce a secondary flow in the flow-field. Hall effects are significant in science and engineering, namely, MHD power generation, nuclear power reactors, accelerator, Hall current magnetometers, underground energy storage system, Hall effect sensors and spacecraft population and in several areas of astrophysics and geophysics. Taking into consideration this fact, Takhar and Ram [49] studied the effects of Hall current on the hydromagnetic free convection boundary layer flow of a heat generating fluid past an infinite plate in a porous medium using harmonic analysis. Aboeldahab and Elbarbary [50] analyzed the influence of Hall current on a MHD natural convection flow with heat and mass transfer over a vertical plate in the presence of a strong external magnetic field. Seth et al. [51] considered the Hall effects on a hydromagnetic natural convection flow past an impulsively moving vertical plate with ramped temperature taking into account thermal diffusion and heat absorption.

Keeping in view the above literature survey, our objective is to study the effects of Hall current on a unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, heat absorbing and chemically reacting fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium. To the best of our knowledge, this problem has not yet received any attention from the researchers despite having its applications in the recovery of petroleum products and gases (e.g. CBM: Coal Bed Methane and UCG: Underground Coal Gasification), fire dynamics in insulations, solar collection systems, nuclear waste repositories, geothermal energy systems, catalytic reactors, etc.

# PROBLEM FORMULATION AND ITS SOLUTION

Consider the unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, temperature dependent heat absorbing and chemically reacting fluid past an accelerated moving vertical plate through a porous medium taking into account Hall effects. The flow is assumed to be in the x' direction, which is taken

along the length of the plate in upward direction, y'-axis is normal to it and z'-axis is perpendicular to the x'y'-plane. A uniform transverse magnetic field  $B_0$  is taken to be acting along the y'-axis. Firstly, i.e. at time  $t' \leq 0$ , both the fluid and plate are at rest and maintain uniform temperature  $T'_{\infty}$  and uniform surface concentration  $C'_{\infty}$ . At time t' > 0, the plate begins to move in the x' direction against the gravitational field with time dependent velocity U(t'). Instantaneously, the temperature of the plate is raised or lowered to  $T'_{\infty} + (T'_{w} - T'_{\infty})t' / t_{0}$ and the level of concentration at the surface of the plate is raised or lowered to  $C'_{\infty} + (C'_{w} - C'_{\infty})t'/t_{0}$ when  $0 < t' \le t_0$ . Thereafter, i.e. at time  $t' > t_0$ , the plate is maintained at uniform temperature  $T'_w$  and the level of concentration at surface of the plate is uniform concentration  $C'_w$ . preserved at A homogeneous chemical reaction of first order with constant rate  $K'_2$  is supposed to exist between the diffusing species and the fluid. Physical model of the problem is shown in Figure 1. It is assumed that the plate is of infinite extent in x' and z' directions and is electrically non-conducting, all quantities except pressure are functions of y' and t' only. For liquid metals and partially ionized fluid, the magnetic Reynolds number is very small and hence the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one [52] so that the magnetic field  $B = (0, B_0, 0)$ . Also the effect of polarization of fluid is negligible when no external electric field is applied [52]; that is E = (0,0,0). This corresponds to the case where no energy is added or extracted from the fluid by electrical means.

Owing to the above mentioned assumptions, the governing equations for the unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible and temperature dependent heat absorbing and chemically reacting fluid through a porous medium taking Hall effects into account, under Boussinesq approximation, are reduce to

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial {y'}^2} - \frac{\sigma B_0^2}{\rho} \left( \frac{u' + mw'}{1 + m^2} \right) - \frac{\upsilon}{K_1'} u' + g \beta' (T' - T_{\infty}') + g \beta^* (C' - C_{\infty}'), \dots \quad (1)$$

$$\frac{\partial w'}{\partial t'} = \upsilon \frac{\partial^2 w'}{\partial {y'}^2} + \frac{\sigma B_0^2}{\rho} \left( \frac{mu' - w'}{1 + m^2} \right) - \frac{\upsilon}{K_1'} w', \qquad (2)$$

$$\frac{\partial T'}{\partial t'} = \alpha' \frac{\partial^2 T'}{\partial {y'}^2} - \frac{Q_0}{\rho C_p} (T' - T'_{\infty}), \qquad \dots \dots \dots (3)$$

where

 $u', w', T', C', v, \sigma, \rho, K'_1, g, \beta', \beta^*, \alpha', Q_0, D, \omega_e, \tau_e$  and  $m = \omega_e \tau_e$  are, respectively, fluid velocity in x' direction, fluid velocity along z' direction, fluid temperature, species concentration, kinematic coefficient of viscosity, electrical conductivity, fluid density, permeability of porous medium, acceleration due to gravity, coefficient of thermal expansion, volumetric coefficient of thermal expansion, thermal diffusivity, heat absorption coefficient, chemical molecular diffusivity, cyclotron frequency, electron collision time and Hall current parameter.



**Fig.1.** Physical model of the problem.

Initial and boundary conditions for fluid velocity, fluid temperature and species concentration for the model are

$$u' = 0, w' = 0, T' = T'_{\infty}, C' = C'_{\infty} \text{ for } y' \ge 0 \text{ and } t' \le 0,$$

$$u' = U(t'), w' = 0 \text{ at } y' = 0 \text{ for } t' > 0,$$

$$T' = T'_{\infty} + (T'_{w} - T'_{\infty})t'/t_{0}, C' = C'_{\infty} + (C'_{w} - C'_{\infty})t'/t_{0}$$

$$at y' = 0 \text{ for } 0 < t' \le t_{0},$$

$$T' = T'_{w}, C' = C'_{w} \text{ at } y' = 0 \text{ for } t' > t_{0},$$

$$u' \to 0, w' \to 0, T' \to T'_{\infty}, C' \to C'_{\infty} \text{ as } y' \to \infty \text{ for } t' > 0.$$
(5)

In order to convert Eqn. (1) to (4) along with the initial and boundary conditions in non-dimensional form, the following non-dimensional quantities and parameters are introduced

$$y = y'/U_{0}t_{0}, u = u'/U_{0}, w = w'/U_{0}t = t'/t_{0},$$

$$T = (T' - T'_{\infty})/(T'_{w} - T'_{\infty}), C = (C' - C'_{\infty})/(C'_{w} - C'_{\infty}),$$

$$G_{r} = \upsilon g \beta'(T'_{w} - T'_{\infty})/U_{0}^{3}, G_{c} = \upsilon g \beta^{*}(C'_{w} - C'_{\infty})/U_{0}^{3},$$

$$M = \sigma B_{0}^{2}\upsilon / \rho U_{0}^{2}, K_{1} = K'_{1}U_{0}^{2} / \upsilon^{2}, P_{r} = \upsilon / \alpha',$$

$$S_{c} = \upsilon / D, K_{2} = \upsilon K'_{2} / U_{0}^{2} \text{ and } \phi = \upsilon Q_{0} / \rho C_{p} U_{0}^{2},$$
(6)

where  $G_r, G_c, M, K_1, P_r, S_c, K_2$  and  $\phi$  are respectively, thermal Grashof number, solutal Grashof number, magnetic parameter, permeability parameter, Prandtl number, Schmidt number, chemical reaction parameter and heat absorption parameter.

The governing equations (1) to (4), in nondimensional form, become

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M\left(\frac{u+mw}{1+m^2}\right) - \frac{u}{K_1} + G_r T + G_c C, \quad (7)$$
$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + M\left(\frac{mu-w}{1+m^2}\right) - \frac{w}{K_1}, \quad \dots \dots \dots (8)$$

$$\frac{\partial T}{\partial t} = \frac{1}{P} \frac{\partial^2 T}{\partial v^2} - \phi T, \qquad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - K_2 C .$$
(10)

It may be noted that the characteristic time  $t_0$  may be defined according to the non-dimensional process mentioned above, i.e.

$$t_0 = \upsilon / U_0^2$$
,

where  $U_0$  is the characteristic velocity.

The initial and boundary conditions in nondimensional form are

$$u = 0, w = 0, T = 0, C = 0 \text{ for } y \ge 0 \text{ and } t \le 0,$$
  

$$u = G(t), w = 0 \text{ at } y = 0 \text{ for } t > 0,$$
  

$$T = t, C = t \text{ at } y = 0 \text{ for } 0 < t \le 1,$$
  

$$T = 1, C = 1 \text{ at } y = 0 \text{ for } t > 1,$$
  

$$u \to 0, w \to 0, T \to 0, C \to 0 \text{ as } y \to \infty \text{ for } t > 0,$$
  
(11)

where  $G(t) = U(t')/U_0$ .

The equations (7) to (10) subject to initial and boundary conditions (11) showing fluid flow, are quite general. We now take a particular case of interest, namely, uniformly accelerated movement of the plate, i.e. G(t) = Rt where R is a nondimensional constant, to explore the flow features of the fluid flow.

Eqns. (7) and (8) are presented in compact form as

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - \lambda F + G_r T + G_c C, \qquad (12)$$

where

F = u + iw and  $\lambda = M(1 - im)/(1 + m^2) + 1/K_1$ .

Initial and boundary conditions (11) in compact form become

$$F = 0, T = 0, C = 0 \text{ for } y \ge 0 \text{ and } t \le 0,$$
  

$$F = Rt \text{ at } y = 0 \text{ for } t > 0,$$
  

$$T = t, C = t \text{ at } y = 0 \text{ for } 0 < t \le 1,$$
  

$$T = 1, C = 1 \text{ at } y = 0 \text{ for } t > 1,$$
  

$$F \to 0, T \to 0, C \to 0 \text{ as } y \to \infty \text{ for } t > 0.$$
(13)

The exact solutions for fluid velocity F(y,t), fluid temperature T(y,t) and species concentration C(y,t) are obtained by using Laplace transform technique, which are expressed in the following form after simplification

$$F(y,t) = Rf_{1}(y,1,\lambda,t) + \frac{a_{1}}{b_{1}^{2}} \{F_{1}(y,t) - H(t-1)F_{1}(y,t-1)\}$$
$$-\frac{a_{2}}{b_{2}^{2}} \{G_{1}(y,t) - H(t-1)G_{2}(y,t-1)\},$$
(14)

$$T(y,t) = f_1(y,P_r,\phi,t) - H(t-1)f_1(y,P_r,\phi,t-1),$$
(15)

 $C(y,t) = f_1(y,S_c, K_2, t) - H(t-1)f_1(y,S_c, K_2, t-1), (16)$ where

$$\begin{split} F_1(y,t) &= e^{b_t} \left\{ f_2(y,P_r,\phi,b_1,t) - f_2(y,1,\lambda,b_1,t) \right\} \\ &+ b_1 \left\{ f_3(y,P_r,\phi,b_1,t) - f_3(y,1,\lambda,b_1,t) \right\}, \\ G_1(y,t) &= e^{b_2 t} \left\{ f_2(y,S_c,K_2,b_2,t) - f_2(y,1,\lambda,b_2,t) \right\} \\ &+ b_2 \left\{ f_3(y,S_c,K_2,b_2,t) - f_3(y,1,\lambda,b_2,t) \right\}, \\ a_1 &= G_r / (1-P_r), a_2 &= G_c / (1-S_c), b_1 = (P_r \phi - \lambda) / (1-P_r) \\ \text{and } b_2 &= (S_c K_2 - \lambda) / (1-S_c). \end{split}$$

The expressions for  $f_i$  (i = 1, 2, 3) are given in the Appendix A.

S. M. Hussain et al.: Hall effects on MHD natural convection flow with heat and mass transfer of heat absorbing and ...

#### SOLUTION WHEN THE FLUID IS IN CONTACT WITH AN ISOTHERMAL PLATE WITH UNIFORM SURFACE CONCENTRATION

In order to put emphasis on the influence of ramped temperature and ramped surface concentration on the flow-field, it may be justified to compare such a flow with the one near an accelerated moving vertical isothermal plate with uniform surface concentration. Keeping in view the assumptions made in this paper, the solutions for fluid velocity, fluid temperature and species concentration for a natural convection flow past an accelerated moving vertical isothermal plate with uniform surface concentration are obtained and are presented in the following form

#### SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The expressions for primary skin friction  $\tau_x$ , secondary skin friction  $\tau_z$  which are measures of the shear stress at the plate due to primary flow, shear stress at the plate due to secondary flow, the Nusselt number  $N_u$ , which measures the rate of heat transfer at the plate and the Sherwood number  $S_h$ , which measures the rate of mass transfer at the plate, are presented in the following form for a ramped temperature plate with ramped surface concentration and an isothermal plate with uniform surface concentration:

For the plate with ramped temperature and ramped surface concentration

$$\tau = \tau_{x} + i\tau_{z} = R \left[ \left( t\sqrt{\lambda} + \frac{1}{2\sqrt{\lambda}} \right) \left\{ erfc\left(\sqrt{\lambda}t\right) - 1 \right\} - \sqrt{\frac{t}{\pi}} e^{-\lambda t} \right] + \frac{a_{1}}{b_{1}^{2}} \left[ F_{2}(0,t) - H(t-1)F_{2}(0,t-1) \right] + \frac{a_{2}}{b_{2}^{2}} \left[ G_{2}(0,t) - H(t-1)G_{2}(0,t-1) \right],$$
(20)

$$N_{u} = f_{6}(P_{r}, \phi, t) - H(t-1)f_{6}(P_{r}, \phi, t-1), \qquad (21)$$

$$S_h = f_6(S_c, K_2, t) - H(t-1)f_6(S_c, K_2, t-1), \quad (22)$$

where 664

$$\begin{split} F_{2}(0,t) &= e^{b_{1}t} \left\{ f_{4}(P_{r},\phi,b_{1},t) - f_{4}(1,\lambda,b_{1},t) \right\} \\ &- b_{1} \left\{ f_{5}(P_{r},\phi,b_{1},t) - f_{5}(1,\lambda,b_{1},t) \right\}, \\ G_{2}(0,t) &= e^{b_{2}t} \left\{ f_{4}(S_{c},K_{2},b_{2},t) - f_{4}(1,\lambda,b_{2},t) \right\} \\ &- b_{1} \left\{ f_{5}(S_{c},K_{2},b_{2},t) - f_{5}(1,\lambda,b_{2},t) \right\}, \end{split}$$

and for the isothermal plate with uniform surface concentration

$$\begin{split} \tau &= \tau_x + i\tau_z = \left\{ \left( \frac{a_1}{b_1} + \frac{a_2}{b_2} + Rt \right) \sqrt{\lambda} + \frac{R}{2\sqrt{\lambda}} \right\} \left\{ erfc\left(\sqrt{\lambda t}\right) - 1 \right\} \\ &- \left( \frac{a_1}{b_1} + \frac{a_2}{b_2} + Rt \right) \frac{e^{-\lambda t}}{\sqrt{t\pi}} + \frac{a_1}{b_1} \left[ e^{b_1 t} \left\{ f_4\left(P_r, \phi, b_1, t\right) - f_4\left(1, \lambda, b_1, t\right) \right\} - f_4\left(P_r, \phi, 0, t\right) \right] + \frac{a_2}{b_2} \left[ e^{b_2 t} \left\{ f_4\left(S_c, K_2, b_2, t\right) \right\} \\ &- f_4\left(1, \lambda, b_2, t\right) \right\} - f_4\left(S_c, K_2, 0, t\right) \right\} \right], \end{split}$$

$$N_{u} = f_{4}(P_{r}, \phi, 0, t), \qquad (24)$$

$$S_h = f_4(S_c, K_2, 0, t).$$
 (25)

Expressions for  $f_i(i=4,5,6)$  are provided in the Appendix A.

#### **RESULTS AND DISCUSSION**

In order to analyze the effects of Hall current, thermal buoyancy force, solutal buoyancy force, field, heat absorption, chemical reaction and time on the flow-field, the numerical values of primary and secondary fluid velocities in the boundary layer region, computed from the analytical solutions reported in the previous sections, are displayed graphically versus boundary layer coordinate y in Figures 2-13 for various values of Hall current parameter *m*, thermal Grashof number  $G_r$ , solutal Grashof number  $G_c$ , heat absorption parameter  $\phi$ , chemical reaction parameter  $K_2$  and time t taking magnetic parameter M = 10, permeability parameter  $K_1 = 0.5$ , Prandtl number  $P_r = 0.71$ (ionized air), Schmidt number  $S_c = 0.22$  and R = 1. It is noticed from Figures 2-13 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, the primary fluid velocity u and the secondary fluid velocity w attain a maximum distinctive value near the surface of the plate and then decrease properly on increasing the boundary layer coordinate y to approach the free stream value. Also the primary and secondary fluid velocities are faster in case of isothermal plate with uniform surface concentration than those of ramped ramped temperature plate with surface concentration. Figures 2-7 reveal the influence of Hall current, thermal and solutal buoyancy forces on the primary and secondary fluid velocities *u* and *w*, respectively.



Fig. 2. Primary velocity for varying *m*.



Fig.3. Secondary velocity profiles for varying m.



**Fig. 5.** Secondary velocity profiles for varying  $G_r$ .

0.00







Fig. 7. Secondary velocity profiles for varying  $G_c$ .

The thermal Grashof number  $G_r$  signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. The solutal Grashof number  $G_c$  characterizes the ratio of the solutal buoyancy force and viscous hydrodynamic force. As expected, it is noticed from Figures 2-7 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, u and w increase with the increase in either Hall current parameter m, Grashof number  $G_r$  or solutal Grashof number  $G_c$ . This implies that Hall current, thermal and solutal buoyancy forces tend to accelerate primary and fluid velocities secondary for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration. Figures 8-11 depict the effects of heat absorption and chemical reaction on the primary and secondary fluid velocities. It is observed from Figures 8-11 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, u and w decrease with the increase in either heat absorption parameter  $\phi$  or chemical reaction parameter  $K_2$ . This implies that heat absorption and chemical reaction have the retarding influence on the primary and secondary fluid velocities for both ramped temperature plate



**Fig. 8.** Primary velocity profiles for varying  $\phi$ .



**Fig. 9.** Secondary velocity profiles for varying  $\phi$ .



**Fig. 10.** Primary velocity profiles for varying  $K_2$ .



Fig. 11. Secondary velocity profiles for varying K<sub>2</sub>.

with ramped surface concentration and isothermal plate with uniform surface concentration. This may be attributed to the fact that the tendency of heat absorption (thermal sink) is to reduce the fluid temperature which causes the strength of thermal buoyancy force to decrease resulting in a net reduction in the fluid velocity. Figures 12 and 13 demonstrate the effects of time on the primary and secondary fluid velocities. It is evident from Figures 12 and 13 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, u and w increase on increasing time t. This implies that there is enrichment in the primary and secondary fluid velocities for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration with the progress of time.







Fig. 13. Secondary velocity profiles for varying t.

The numerical values of species concentration C, computed from the analytical solution mentioned in the previous sections, are depicted graphically versus the boundary layer coordinate y in Figures 14-16 for various values of the chemical reaction parameter  $K_2$ , Schmidt number  $S_c$  and time t. It is observed in Figures 14-16 that, for both ramped plate temperature with ramped surface concentration and isothermal plate with uniform surface concentration, species concentration

decreases on increasing either  $K_2 \text{ or } S_c$  whereas it increases on increasing time t. Since the Schmidt number  $S_c$  is the ratio of momentum diffusivity to mass diffusivity, an increase in  $S_c$  implies a decrease in the mass diffusion rate. This implies that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, chemical reaction tends to reduce species concentration, whereas mass diffusion has a reverse effect on it and there is an enhancement in the species concentration with the progress of time throughout the boundary layer region. It is also noticed from Figures 14-16 that species concentration is maximal at the surface of the plate and decreases properly with the increase of boundary layer coordinate y to approach free stream value.



Fig. 16. Concentration profiles for varying *t*.

The numerical values of non-dimensional skin frictions  $\tau_x$  and  $\tau_z$  due to primary and secondary flows, respectively, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, computed from the analytical expressions reported in the paper, are presented in tabular form in Tables 1 to 6 for various values of  $m, \phi, G_r, G_c, K_2$  and t taking R = 1, M = 10, $K_1 = 0.5, P_r = 0.71$  and  $S_c = 0.22$  whereas those of Sherwood number  $S_h$ , calculated from the analytical expressions presented in the paper, are exhibited in tabular form in Table 7 for different values of  $K_2$  and t.

It is observed from Tables 1 to 6 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, primary skin friction  $-\tau_{x}$  increases on increasing either  $\phi$  or  $K_{2}$  or t and it decreases on increasing either m or  $G_r$  or  $G_c$ whereas secondary skin friction  $\tau_z$  increases on increasing either *m* or  $G_r$  or  $G_c$  or *t* and it decreases on increasing either  $\phi$  or  $K_2$ . This implies that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, heat absorption and chemical reaction have a tendency to enhance primary skin friction whereas Hall current, thermal and solutal buoyancy forces have the reverse effect on it. Hall current, thermal and solutal buoyancy forces have a tendency to enhance secondary skin friction, whereas heat absorption and chemical reaction have the reverse effect on it, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration. Both the primary and secondary skin frictions are getting enhanced with the progress of time, for both ramped temperature plate with ramped surface concentration and with uniform isothermal plate surface concentration.

It is found from Table 7 that, for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, Sherwood number  $S_h$  increases on increasing  $K_2$ . On increasing time t,  $S_h$  increases for the ramped temperature plate with ramped surface concentration, whereas it decreases for the isothermal plate with uniform surface concentration. This implies that, for both ramped

S. M. Hussain et al.: Hall effects on MHD natural convection flow with heat and mass transfer of heat absorbing and...

······································		<b>D</b> 1	, - r · · , - c		T (1 1	
$\phi \downarrow m \rightarrow$	0.5	Ramped	15	0.5	Isothermal	15
φ τ <i>in</i> γ	0.5	1.0	1.5	0.5	1.0	1.5
1	1.01741	0.622583	0.278910	0.154959	0.144327	0.100579
3	1.05740	0.667453	0.329253	0.249978	0.236641	0.176908
5	1.08762	0.701253	0.367049	0.293849	0.264610	0.194591
able 2. Seconda	ary skin friction	$\tau_z$ when $K_2 = 0.5$	$2, G_r = 4, G_c$	= 5  and  t = 0.	.7	
		Ramped			Isothermal	
$\phi \star m \rightarrow$	0.5	1.0	1.5	0.5	1.0	1.5
1	0.579536	0.862241	0.931667	0.760465	1.19187	1.3681
3	0.571497	0.848668	0.915488	0.736848	1.14988	1.3150
5	0.565602	0.838733	0.903647	0.721472	1.12255	1.2803
<b>Fable 3.</b> Primary	skin friction –	$\tau_x$ when $m = 0.5$ , $R$	$K_2 = 0.2, \ \phi = 1$	and $t = 0.7$		
		Ramped			Isothermal	
$G_r \downarrow G_c \rightarrow$	3	5	7	3	5	7
2	1.62883	1.29846	0.968091	1.13613	0.599886	0.463638
4	1.34778	1.01741	0.687043	0.67120	0.434959	0.401288
6	1.06673	0.73636	0.405995	0.40628	0.329967	0.266214
Fable 4. Seconda	ary skin friction	$\tau_z$ when $m = 0.5$	$K_2 = 0.2, \phi$	=1 and $t = 0$ .	7	
		Ramped			Isothermal	
$G_r \downarrow G_c \rightarrow$	3	5	7	3	5	7
2	0.503644	0.547908	0.592173	0.60533	0.69448	0.78363
4	0.535271	0.579536	0.623800	0.67131	0.76046	0.84961
6	0.566899	0.611163	0.655428	0.73730	0.82645	0.91560
Fable 5. Primary	skin friction –	$\tau_x$ when $m = 0.5$ ,	$G_r = 4, G_c =$	5 and $\phi = 1$		
		Ramped	, c		Isothermal	
$t \downarrow K_2 \rightarrow$	0.2	2	5	0.2	2	5
0.5	0.82261	0 84418	0.87252	0 129931	0 181550	0.289501
0.2	1 01741	1.05636	1 10406	0 134959	0.229325	0.329780
0.9	1.20486	1.26388	1.33210	0.752093	0.857801	0.962095
Table 6. Seconda	ary skin friction	$\tau$ when $m = 0.5$	G = 4.G =	= 5 and $\phi = 1$		
		Pampad	$, o_r  ., o_c$		acthormal	
t	$\downarrow K_2 \rightarrow 0$	12 $2$	5	0.2	2 5	
		<u>, 2 2</u> 6746 0.382200	0.376203	0.610284	$\frac{2}{0.580456}$ $\frac{5}{0.564850}$	)
	0.5 0.50	0.50/40 0.502209	0.570295	0.010264	0.309430  0.304033	7
	0.7 0.57	$(9530 \ 0.570095)$	0.558750	0.700405	0.732020  0.703327	/
	0.9 0.77	/0110 0./00500	0.743094	0.898134	0.805251 0.855098	<u>s</u>
Table 7. Sherwood	od number $-S_h$	when $S_c = 0.22$				
$\boldsymbol{V}$	- <b>-</b>	Ramped		a <b>F</b>	Isothermal	
$\frac{\kappa_2 \star l \rightarrow}{2}$	0.5	0.7	0.9	0.5	0.7	0.9
0.2	0.386593	0.463189	0.531694	0.428415	0.379505	0.349641
2	0.488076	0.625355	0.760246	0.785973	0.757863	0.738408
5	0.628694	0.838894	1.04877	1.12945	1.09522	1.07538
perature pla	te with	ramped surf	ace		CONCLUSIONS	

**Table 1.** Primary skin friction  $-\tau_x$  when  $K_2 = 0.2, G_r = 4, G_c = 5$  and t = 0.7

temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration, chemical reaction tends to enhance the rate of mass transfer at the plate. For the ramped temperature plate with ramped surface concentration, the rate of mass transfer at the plate is getting enhanced, whereas for the isothermal plate with uniform surface concentration it is getting reduced with the progress of time.

The present study brings out the following significant findings of the effects of Hall current on the unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, temperature dependent heat absorbing and chemically reacting fluid past an accelerated moving vertical plate with ramped temperature and ramped surface concentration through a porous medium: 1. For both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration.

Hall current, thermal and solutal buoyancy forces tend to accelerate primary and secondary fluid velocities whereas heat absorption and chemical reaction have a reverse effect on it. There is enrichment in the primary and secondary fluid velocities with the progress of time. Chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it and there is an enhancement in the species concentration with the progress of time throughout the boundary layer region. Heat absorption and chemical reaction have a tendency to enhance primary skin friction, whereas Hall current, thermal and solutal buoyancy forces have a reverse effect on it. Hall current, thermal and solutal buoyancy forces have a tendency to enhance secondary skin friction, whereas chemical reaction has a reverse effect on it. Both the primary and secondary skin frictions are getting enhanced with the progress of time. Chemical reaction tends to enhance the rate of mass transfer at the plate.

2. For a ramped temperature plate with ramped surface concentration, the rate of mass transfer at the plate is getting enhanced, whereas for an isothermal plate with uniform surface concentration, it is getting reduced with the progress of time.

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$$\begin{split} \textbf{APPENDIX A} \\ f_1(x_1, x_2, x_3, x_4) &= \frac{1}{2} \Biggl[ \Biggl( x_4 + \frac{x_1}{2} \sqrt{\frac{x_2}{x_3}} \Biggr) e^{x_1 \sqrt{x_2 x_3}} erfc \Biggl( \sqrt{x_3 x_4} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_4}} \Biggr) + \Biggl( x_4 - \frac{x_1}{2} \sqrt{\frac{x_2}{x_3}} \Biggr) e^{-x_1 \sqrt{x_2 x_3}} \times \\ erfc \Biggl( -\sqrt{x_3 x_4} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_4}} \Biggr) \Biggr], \\ f_2(x_1, x_2, x_3, x_4, x_5) &= \frac{1}{2} \Biggl[ e^{x_1 \sqrt{x_2 (x_3 + x_4)}} erfc \Biggl( \sqrt{(x_3 + x_4) x_5} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_5}} \Biggr) + e^{-x_1 \sqrt{x_2 (x_3 + x_4)}} \times \\ erfc \Biggl( -\sqrt{(x_3 + x_4) x_5} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_5}} \Biggr) \Biggr], \\ f_3(x_1, x_2, x_3, x_4, x_5) &= \frac{1}{2} \Biggl[ \Biggl( x_5 + \frac{1}{x_4} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_3}} \Biggr) e^{x_1 \sqrt{x_2 x_3}} erfc \Biggl( \sqrt{x_3 x_5} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_5}} \Biggr) \Biggr], \\ f_3(x_1, x_2, x_3, x_4, x_5) &= \frac{1}{2} \Biggl[ \Biggl( x_5 + \frac{1}{x_4} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_3}} \Biggr) e^{x_1 \sqrt{x_2 x_3}} erfc \Biggl( \sqrt{x_3 x_5} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_5}} \Biggr) + \Biggl( x_5 + \frac{1}{x_4} - \frac{x_1}{2} \sqrt{\frac{x_2}{x_3}} \Biggr) \times \\ e^{-x_1 \sqrt{x_2 x_3}} erfc \Biggl( -\sqrt{x_3 x_5} + \frac{x_1}{2} \sqrt{\frac{x_2}{x_5}} \Biggr) \Biggr], \\ f_4(x_1, x_2, x_3, x_4) &= \Biggl[ \sqrt{x_1 (x_2 + x_3)} \Biggl\{ erfc \Biggl( \sqrt{(x_2 + x_3) x_4} \Biggr) - 1 \Biggr\} - \sqrt{\frac{x_1}{\pi x_4}} e^{-(x_2 + x_3) x_4} \Biggr], \\ f_5(x_1, x_2, x_3, x_4) &= \Biggl[ \Biggl\{ \frac{1}{2} \sqrt{\frac{x_1}{x_2}} + \Bigl( x_4 + \frac{1}{x_3} \Biggr) \sqrt{x_1 x_2} \Biggr\} \Biggl\{ erfc \Biggl( \sqrt{x_2 x_4} \Biggr) - 1 \Biggr\} - \Biggl( x_4 + \frac{1}{x_3} \Biggr) \sqrt{\frac{x_1}{\pi x_4}} e^{-x_2 x_4} \Biggr], \\ f_6(x_1, x_2, x_3) &= \frac{1}{2} \Biggl[ \Biggl[ \Biggl( \sqrt{\frac{x_1}{x_2}} + 2x_3 \sqrt{x_1 x_2} \Biggr) \Biggl\{ erfc \Biggl( \sqrt{x_2 x_3} \Biggr) - 1 \Biggr\} - 2x_3 \sqrt{\frac{x_1}{\pi x_3}} e^{-x_2 x_3} \Biggr].$$

### ЕФЕКТ НА ХОЛ ПРИ ЕСТЕСТВЕНА КОНВЕКЦИЯ ПРИ МАГНИТО-ХИДРОДИНАМИЧЕН ПОТОК С ТОПЛО–МАСОПРЕНАСЯНЕ ЗАД ВЕРТИКАЛНА ПЛОСКОСТ ПРИ ПРОМЕНЛИВИ ПОВЪРХНОСТНИ ТЕМПЕРАТУРА И КОНЦЕНТРАЦИЯ С ОТНЕМАНЕ НА ТОПЛИНА И ХИМИЧЕСКИ РЕАГИРАЩ ФЛУИД

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#### (Резюме)

В тази работа се изследва влиянието на ефекта на Хол върху нестационарна естествена конвекция при магнитохидродинамично течение с топло и масопренасяне в електропроводяща, вискозна и несвиваема течност. Разглежда се температурно зависимо поглъщане на топлина и химична реакция във флуида зад вертикална подвижна плоскост с променливи температура и концентрации на повърхността й. Водещите безизмерни уравнения са решавани аналитично с решения в затворена форма чрез Лапласова трансформация. Получени са зависимости за числата на Нуселт и Шервуд. Измененията на скоростта на флуида, температурата му и концентрацията на пренасяното вещество са представени графично, докато коефициентът на триене и числата на Нуселт и Шервуд са представени таблично за различни стойности на параметрите на течението. Числените резултати са сравнени с тези за подобно течение в блзиост до изотермична плоскост с постоянна повърхностна концентрация.