

## Magnetohydrodynamic stability of self-gravitating compressible resistive rotating streaming fluid medium

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Magnetohydrodynamic stability of a gravitational medium with streams of variable velocity distribution for a general wave propagation in the presence of the rotation forces has been studied. The magnetic field has strong stabilizing influence but the streaming is a destabilizing. The rotating forces have a stabilizing influence under certain restrictions. It is proved that the gravitational Jean's instability criterion is not influenced by the electromagnetic force or the rotation force or even by the combined effect of them whether the fluid medium is streaming or not and whether the rotation in one or more dimension.

**Keywords:** Magnetogravitational, Resistive, Rotating, Streaming, Compressible

### INTRODUCTION

The self-gravitational instability of a homogeneous fluid medium at rest has been investigated since long time ago, for its practical application in astrophysics see Jeans [1]. It is founded that the model is unstable under the restriction

$$k^2 C_s^2 - 4\pi G \rho_o < 0$$

called after Jeans by Jeans' criterion, where  $k$  is the net wave number of the propagated wave,  $C_s^2$  is a sound speed in the fluid, of density  $\rho_o$ , and  $G$  is the self-gravitational constant. Chandrasekhar and Fermi [2], and later on Chandrasekhar [3] made several extensions. The Jeans' model of self-gravitational medium has been elaborated with streams of variable velocity distribution by Sengar [4]. Recently Radwan *et. al.* [5], developed the magnetogravitational stability of variable streams pervaded by the constant magnetic field  $(H_o, 0, 0)$ . The stability of different cylindrical models under the action of self-gravitating force in addition to other forces has been elaborated by Radwan and Hasan [6], [7]. Hasan [8] has investigated the stability of an oscillating streaming fluid cylinder subject to the combined effect of the capillary, self-gravitating and electrodynamic forces in all axisymmetric and non-axisymmetric perturbation modes. He [9] has investigated the stability of oscillating streaming self-gravitating dielectric incompressible

fluid cylinder surrounded by tenuous medium of negligible motion pervaded by transverse varying electric field for all the axisymmetric and non-axisymmetric perturbation modes. He [10] has studied the instability of a full fluid cylinder surrounded by self-gravitating tenuous medium pervaded by transverse varying electric field under the combined effect of the capillary, self-gravitating, and electric forces for all the modes of perturbations. He [11] the magnetodynamic stability of a fluid jet pervaded by transverse varying magnetic field while its surrounding tenuous medium is penetrated by uniform magnetic field.

Here in the present work we study the magnetodynamic stability of a self-gravitating rotating streaming viscous fluid medium pervaded by general magnetic field. Such studies have a correlation with the formation of sunspots. Also they have relevance in describing the condensation within astronomical bodies cf. Chandrasekhar and Fermi [2], and also Chandrasekhar [3].

### BASIC STATE

We consider an infinite self-gravitating fluid medium. The fluid is assumed to be homogeneous and viscous. The model is acting upon the following forces

(i) the pressure gradient force, (ii) electromagnetic force, (iii) self-gravitating force, (iv) the forces due to rotating factors and (v) the forces due to resistivity.

We shall utilize the cartesian coordinates  $(x, y, z)$  for investigating such problem. The required equations for the present problem

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$$\rho \left( \frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} \right) = -\nabla P + \mu (\nabla \wedge \underline{H}) \wedge \underline{H} + \rho \nabla V - 2\rho (\underline{u} \wedge \underline{\Omega}) + \frac{1}{2} \rho (\underline{\Omega} \wedge \underline{r})^2 \quad (1)$$

$$\frac{\partial \rho}{\partial t} + (\underline{u} \cdot \nabla) \rho = -\rho (\nabla \cdot \underline{u}) \quad (2)$$

$$\frac{\partial \underline{H}}{\partial t} = \nabla \wedge (\underline{u} \wedge \underline{H}) - \nabla (\eta \nabla \wedge \underline{H}) \quad (3)$$

$$\nabla \cdot \underline{H} = 0 \quad (4)$$

$$\nabla^2 V = -4\pi G \rho \quad (5)$$

$$P = K \rho^\Gamma \quad (6)$$

Here  $\rho$ ,  $\underline{u}$ , and  $P$  are the fluid density, velocity vector and kinetic pressure,  $\mu$  and  $\underline{H}$  are the magnetic field permeability and intensity,  $V$  and  $G$  are the self-gravitating potential and constant,  $\eta$  is the coefficient of resistivity,  $\underline{\Omega}$  is the angular velocity of rotation,  $K$  and  $\Gamma$  are constants where  $\Gamma$  is the polytropic exponent.

We assume that the medium: (i) rotates with the general uniform angular velocity

$$\underline{\Omega} = (\Omega_x, \Omega_y, \Omega_z) \quad (7)$$

(ii) be pervaded by the two dimensions homogeneous magnetic field

$$\underline{H}_0 = (0, H_{oy}, H_{oz}) \quad (8)$$

and (iii) posses streams moving in the  $x$ -direction with velocity

$$\underline{u}_0 = (U(z), 0, 0) \quad (9)$$

varying along the  $z$ -direction of the Cartesian coordinates  $(x, y, z)$ .

## PERTURBATION ANALYSIS

For small departures from the initial state, every variable quantity  $Q$  may be expressed as

$$Q = Q_0 + Q_1, \quad |Q_1| \ll Q_0 \quad (10)$$

where  $Q$  stands for each  $\rho, \underline{H}, P, \underline{u}$  and  $V$ . Based on the expansion (10), the perturbation equations could be obtained from (1)--(6) in the form:

$$\rho \left( \frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_0 \cdot \nabla) \underline{u}_1 + (\underline{u}_1 \cdot \nabla) \underline{u}_0 \right) = -\nabla P_1 + \mu (\nabla \wedge \underline{H}_1) \wedge \underline{H}_0 + \mu (\nabla \wedge \underline{H}_0) \wedge \underline{H}_1 + \rho \nabla V_1 - 2\rho_0 (\underline{u}_1 \wedge \underline{\Omega}) \quad (11)$$

$$\frac{\partial \underline{H}_1}{\partial t} = \nabla \wedge (\underline{u}_1 \wedge \underline{H}_0) + \nabla \wedge (\underline{u}_0 \wedge \underline{H}_1) - \nabla \wedge (\eta \nabla \wedge \underline{H}_1) \quad (12)$$

$$\frac{\partial \rho_1}{\partial t} + (\underline{u}_0 \cdot \nabla) \rho_1 + (\underline{u}_1 \cdot \nabla) \rho_0 + \rho_0 (\nabla \cdot \underline{u}_1) + \rho_1 (\nabla \cdot \underline{u}_0) = 0 \quad (13)$$

$$\nabla \cdot \underline{H}_1 = 0 \quad (14)$$

$$\nabla^2 V_1 = -4\pi G \rho_1 \quad (15)$$

$$\frac{dP_1}{dt} = C_s^2 \frac{d\rho_1}{dt} \quad (16)$$

where  $C_s = \left( \frac{\Gamma P_0}{\rho_0} \right)^{1/2}$  is the sound speed in the fluid.

By the use of the components of  $\underline{u}_1$  and  $\underline{H}_1$ , viz.

$$\underline{u}_1 = (u, v, w) \quad (17)$$

$$\underline{H}_1 = (h_x, h_y, h_z) \quad (18)$$

together with the assumptions (7)--(9), the system of equations (11)--(16) may be rewritten as

$$\rho_0 \left[ \frac{\partial u}{\partial t} + U_0 \frac{\partial u}{\partial x} + w \frac{dU_0}{dz} \right] = -\frac{\partial P_1}{\partial x} - \mu \frac{\partial}{\partial x} (H_{oy} h_y + H_{oz} h_z) - \mu \left( H_{oy} \frac{\partial}{\partial y} + H_{oz} \frac{\partial}{\partial z} \right) h_x + \rho_0 \frac{\partial V_1}{\partial x} + 2\rho_0 \Omega_y w - 2\rho_0 \Omega_z v \quad (19)$$

$$\rho_0 \left[ \frac{\partial v}{\partial t} + U_0 \frac{\partial v}{\partial x} \right] = -\frac{\partial P_1}{\partial y} - \mu \frac{\partial}{\partial y} (H_{oy} h_y + H_{oz} h_z) - \mu \left( H_{oy} \frac{\partial}{\partial y} + H_{oz} \frac{\partial}{\partial z} \right) h_y + \rho_0 \frac{\partial V_1}{\partial y} + 2\rho_0 \Omega_x u - 2\rho_0 \Omega_z w \quad (20)$$

$$\rho_0 \left[ \frac{\partial w}{\partial t} + U_0 \frac{\partial w}{\partial x} \right] = -\frac{\partial P_1}{\partial z} - \mu \frac{\partial}{\partial z} (H_{oy} h_y + H_{oz} h_z) - \mu \left( H_{oy} \frac{\partial}{\partial y} + H_{oz} \frac{\partial}{\partial z} \right) h_z + \rho_0 \frac{\partial V_1}{\partial z} + 2\rho_0 \Omega_y u - 2\rho_0 \Omega_x v \quad (21)$$

$$\frac{\partial h_x}{\partial t} + U_0 \frac{\partial h_x}{\partial x} = \left( H_{oy} \frac{\partial}{\partial y} + H_{oz} \frac{\partial}{\partial z} \right) u + h_z \frac{dU_0}{dz} - \left( \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} h_y - \frac{\partial}{\partial y} h_x \right) - \frac{\partial}{\partial z} \left( \frac{\partial}{\partial x} h_z - \frac{\partial}{\partial z} h_x \right) \right) \quad (22)$$

$$\frac{\partial h_y}{\partial t} + U_0 \frac{\partial h_y}{\partial x} = \left( H_{oy} \frac{\partial}{\partial y} + H_{oz} \frac{\partial}{\partial z} \right) v + H_{oy} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \left( \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} h_x - \frac{\partial}{\partial y} h_y \right) - \frac{\partial}{\partial z} \left( \frac{\partial}{\partial y} h_z - \frac{\partial}{\partial z} h_y \right) \right) \quad (23)$$

$$\frac{\partial h_z}{\partial t} + U_0 \frac{\partial h_z}{\partial x} = \left( H_{oy} \frac{\partial}{\partial y} + H_{oz} \frac{\partial}{\partial z} \right) w + H_{oz} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) - \left( \frac{\partial}{\partial x} \left( \frac{\partial}{\partial z} h_x - \frac{\partial}{\partial x} h_z \right) - \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} h_z - \frac{\partial}{\partial z} h_y \right) \right) \quad (24)$$

$$\frac{\partial \rho_1}{\partial t} + U_0 \frac{\partial \rho_1}{\partial x} = -\rho_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \quad (25)$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0 \quad (26)$$

$$\frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial y^2} + \frac{\partial^2 V_1}{\partial z^2} = -4\pi G \rho_1 \quad (27)$$

$$\frac{\partial P_1}{\partial t} + U_0 \frac{\partial P_1}{\partial x} = C_s^2 \left( \frac{\partial \rho_1}{\partial t} + U_0 \frac{\partial \rho_1}{\partial x} \right) \quad (28)$$

## EIGENVALUE RELATION

Apply sinusoidal wave along the fluid interface. Consequently, from the viewpoint of the stability approaches given by Chandrasekhar [3], we assume that the space-time dependence of the wave propagation of the form

$$Q_1 \sim \exp[i(k_x x + k_y y + k_z z + \sigma t)] \quad (29)$$

Here  $\sigma$  is gyration frequency of the assuming wave.

$k_x, k_y$  and  $k_z$  are (any real values) the wave numbers in the  $(x, y, z)$  directions. By an appeal to the time-space dependence (29), the relevant perturbation equations (19)-(28) are given by

$$n\rho_0 u + \rho_0 w D U_0 = -ik_x C_s^2 \rho_1 - i\mu(k_y H_{oy} + k_z H_{oz}) h_x + ik_x \mu(H_{oy} h_y + H_{oz} h_z) + ik_x \rho_0 V_1 + 2\rho_0 \Omega_y w - 2\rho_0 \Omega_z v \quad (30)$$

$$n\rho_0 v = -ik_y C_s^2 \rho_1 - i\mu(k_y H_{oy} + k_z H_{oz}) h_y + ik_y \mu(H_{oy} h_y + H_{oz} h_z) + ik_y \rho_0 V_1 - 2\rho_0 \Omega_x w + 2\rho_0 \Omega_z u \quad (31)$$

$$n\rho_0 w = -ik_z C_s^2 \rho_1 - i\mu(k_y H_{oy} + k_z H_{oz}) h_z + ik_z \mu(H_{oy} h_y + H_{oz} h_z) + ik_z \rho_0 V_1 + 2\rho_0 \Omega_x u - 2\rho_0 \Omega_y v \quad (32)$$

$$nh_x = i(H_{oy} k_y + H_{oz} k_z) u + h_z D U_0 + i\eta \left( -(k_x^2 + k_z^2) h_x + k_x k_y h_y + k_y k_z h_z \right) \quad (33)$$

$$nh_y = i(H_{oy} k_y + H_{oz} k_z) v - iH_{oy} (k_x u + k_y v + k_z w) + i\eta (k_x k_y h_x - (k_x^2 + k_z^2) h_y + k_y k_z h_z) \quad (34)$$

$$nh_z = i(H_{oy} k_y + H_{oz} k_z) w - iH_{oz} (k_x u + k_y v + k_z w) + i\eta (k_x k_z h_x + k_y k_z h_y - (k_x^2 + k_z^2) h_z) \quad (35)$$

$$n\rho_1 = -i\rho_0 (k_x u + k_y v + k_z w) \quad (36)$$

$$ik_x h_x + ik_y h_y + ik_z h_z = 0 \quad (37)$$

$$k^2 V_1 = -4\pi G \rho_1 \quad (38)$$

where

$$k^2 = k_x^2 + k_y^2 + k_z^2, \quad D = \frac{d}{dz} \quad (39)$$

$$n = i(\sigma + k U_o) \quad (40)$$

The foregoing system equations (30)-(38) could be rewritten in the matrix form

$$[a_{ij}][b_j] = 0 \quad (41)$$

where the elements  $[a_{ij}]$  of the matrix are given in the appendix I while the elements of the column matrix  $[b_j]$  are being  $u, v, w, h_x, h_y, h_z, \rho_1$  and  $V_1$ . One has to mention here that equation (37) is identically satisfied.

For non-trivial solution of the equations (41), setting the determinant of the matrix  $[a_{ij}]$  equal to zero (see Appendix I), we get the general eigenvalue relation of seven order in  $n$  in the form

$$A_7 n^7 + A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0 \quad (42)$$

where the compound coefficients  $A_i$  ( $i = 0, 1, 2, 3, 4, 5, 6, 7$ ) are calculated.

## DISCUSSIONS AND RESULTS

Equation (42) is a general MHD eigenvalue relation of a rotating self-gravitating fluid medium pervaded by magnetic field of two dimensions. Some previously publishing results may be obtained as limiting cases here. That confirms the present analysis.

In absence of the rotating, electromagnetic forces and for inviscid fluid i.e.  $\underline{\Omega} = 0$ ,  $\underline{H}_o = 0$  and  $\eta = 0$ , equation (42) yields

$$k^2 n^3 + k^2 (k^2 C_s^2 - 4\pi G \rho_o) n - k_x k_z (k^2 C_s^2 - 4\pi G \rho_o) D U_o = 0 \quad (43)$$

This relation coincides with the dispersion relation, of a pure self-gravitating fluid medium streams with variable streams  $(U_o(z), 0, 0)$  derived by Sengar [1]. For more details concerning stability of this case we may refer to Sengar (1981).

If

$$\underline{\Omega} = 0, \quad \underline{H}_o = 0, \quad \eta = 0 \quad \text{and} \quad U_o = 0,$$

equation (42) reduces to

$$n^2 = k^2 C_s^2 - 4\pi G \rho_o \quad (44)$$

This gives the same results given by Jean's (1902). For more details concerning the instability of this case we may refer to the discussions of Jean's (1902).

In absence of the magnetic field and we assume that the fluid medium is stationary i.e.  $\underline{H}_o = 0$ ,  $\eta = 0$  and  $U_o = 0$ , equation (42) gives another relation. The purpose of the present part is to determine the influence of rotation on the Jean's

criterion (44) of a uniform streaming fluid. So in order to carry out and to facilitate the present situation we may choose  $\underline{\Omega}_x = 0$ ,  $\underline{H}_o = 0$ ,  $\eta = 0$ ,  $k_x = 0$  and  $k_y = 0$ , equation (42), gives

$$n^4 + (4\pi G \rho_o - C_s^2 k_z^2 - 4\Omega^2) n^2 + 4\Omega_z^2 (C_s^2 k_z^2 - 4\pi G \rho_o) = 0 \quad (45)$$

with

$$\Omega^2 = \Omega_y^2 + \Omega_z^2 \quad (46)$$

Equation (45) indicates that there must be two modes in which a wave can be propagated in the medium. If the roots of (45) are being  $n_1^2$  and  $n_2^2$ , then we have

$$n_1^2 + n_2^2 = C_s^2 k_z^2 + 4\Omega^2 - 4\pi G \rho_o \quad (47)$$

$$n_1^2 n_2^2 = 4\Omega_z^2 (C_s^2 k_z^2 - 4\pi G \rho_o) \quad (48)$$

and so we see that both the roots  $n_1^2$  and  $n_2^2$  are real. The discussions of (45) indicate that if the Jean's restriction

$$C_s^2 k_z^2 - 4\pi G \rho_o < 0 \quad (49)$$

is valid, then one of the two roots  $n_1^2$  or  $n_2^2$  must be negative and consequently the model will be unstable. This means that under the Jean's restriction (49), the self-gravitating rotating fluid medium is unstable. This shows that the Jean's criterion for a self-gravitating medium is unaffected by the influence of the uniform rotation.

In order to examine the influence of the electromagnetic force on the instability of the present model we use the relation (42) with the assumptions  $\underline{\Omega} = 0$ ,  $\eta = 0$ ,  $k_x = 0$  and  $k_y = 0$ .

Then, equation (42)

$$n^4 + A n^2 + B = 0 \quad (50)$$

with

$$A = 4\pi G \rho_o - C_s^2 k_z^2 - \frac{\mu H_o^2 k_z^2}{\rho_o} \quad (51)$$

$$B = \frac{\mu H_o^2 k_z^2}{\rho_o} (C_s^2 k_z^2 - 4\pi G \rho_o) \quad (52)$$

where

$$H_o^2 = H_{\omega}^2 + H_{\eta}^2 \quad (53)$$

Again as in the previous case of rotation, we have here also two modes of wave propagation, and therefore, we obtain

$$n_1^2 + n_2^2 = C_s^2 k_z^2 - 4\pi G \rho_o + \frac{\mu H_o^2 k_z^2}{\rho_o} \quad (54)$$

$$n_1^2 n_2^2 = \frac{\mu H_o^2 k_z^2}{\rho_o} (C_s^2 k_z^2 - 4\pi G \rho_o) \quad (55)$$

By comparing (47) & (48) with (54) & (55) we may say that  $4\Omega^2$  is replaced by  $\frac{\mu H_o^2 k_z^2}{\rho_o}$ . Following

the same analysis of the rotating case we finally find out that Jean's self-gravitating instability restriction of a streaming fluid medium is not influenced by the effect of the electromagnetic force.

In order to determined the combined effect of the rotation and the electromagnetic force we may put  $k_x = 0$  and  $k_y = 0$  for simplicity but  $\underline{\Omega}$  and  $\underline{H}_o$  are still as they are given in their general forms given by equations (7) and (8). In such a case the relation (42) reduces to

$$n^6 - E_1 n^4 + E_2 n^2 - E_3 = 0 \tag{56}$$

with

$$E_1 = 4\Omega^2 + \frac{2\mu H_{\omega z}^2 k_z^2}{\rho_o} + \frac{2\mu H_{\omega y}^2 k_z^2}{\rho_o} + C_3^2 k^2 - 4\pi G \rho_o \tag{57}$$

$$E_2 = 4 \left[ \Omega_y \left( \frac{\mu H_{\omega z}^2 k_z^2}{\rho_o} \right)^{\frac{1}{2}} - \Omega_z \left( \frac{\mu H_{\omega y}^2 k_z^2}{\rho_o} \right)^{\frac{1}{2}} \right] + \frac{\mu H_{\omega z}^2 k_z^2}{\rho_o} \left[ 4\Omega_x^2 + \frac{\mu H_{\omega z}^2 k_z^2}{\rho_o} + \frac{\mu H_{\omega y}^2 k_z^2}{\rho_o} + C_3^2 k^2 - 4\pi G \rho_o \right] + (C_3^2 k^2 - 4\pi G \rho_o) \left[ 4\Omega_x^2 + \frac{\mu H_{\omega z}^2 k_z^2}{\rho_o} \right] \tag{58}$$

$$E_3 = \left( \frac{\mu H_{\omega z}^2 k_z^2}{\rho_o} \right)^2 (C_3^2 k^2 - 4\pi G \rho_o) \tag{59}$$

where

$$\Omega^2 = \Omega_x^2 + \Omega_y^2 + \Omega_z^2 \tag{60}$$

The relation (56) is of sixth order algebraic equation in  $n$ , so there will be three modes for which the proposing sinusoidal wave may be propagated in the medium. If we assume that the oscillation frequencies i.e. the roots of (56) are  $n_1$ ,  $n_2$  and  $n_3$ , then using the theory of equations we get

$$n_1^2 + n_2^2 + n_3^2 = E_1 \tag{61}$$

$$n_1^2 n_2^2 n_3^2 = E_3 = \left( \frac{\mu H_{\omega z}^2 k_z^2}{\rho_o} \right)^2 (C_3^2 k^2 - 4\pi G \rho_o) \tag{62}$$

By the use of equation of multiple roots  $n_1$ ,  $n_2$  and  $n_3$  we see that if Jeans's criterion (cf. equation 49) is satisfied then one of the three roots is negative and consequently the model will be unstable with respect to one of the three modes.

We conclude that the Jeans's self-gravitating restriction of a streaming medium is not affected by the combined influence of the electromagnetic and rotational forces.

If  $G = 0$ ,  $\underline{\Omega} = 0$ ,  $\underline{H}_o = 0$  and  $\eta \neq 0$ , equation (42) degenerates to a somewhat complicated relation. The purpose of the present part is to determine the effect of the viscosity of fluid. So in order to carry out and to facilitate the present situation we may choose  $k_x = 0$  and  $k_y = 0$  so equation (42), at once, yields

$$(\sigma + k^2 \eta)^2 (\sigma^5 k^2 + k^4 \sigma C_3^2) = 0 \tag{63}$$

Equation (63) indicates that resistivity has a destabilizing influence under certain restrictions.

### CONCLUSION

The gravitational Jeans instability criterion is not influenced by the electromagnetic force or the rotation forces or even by the combined effect of them whether the fluid medium is streaming or not and whether the rotation in one dimension or more. The resistivity has destabilizing influence under certain restrictions.

### REFERENCES

1. J. Jeans, H. Philos, *Trans. R. Soc. London*, **199**, 1 (1902).
2. S. Chandrasekhar, E. Fermi, *Astrophys J.*, **118**, 116 (1953).
3. S. Chandrasekhar, Hydrodynamic and hydromagnetic stability, Dover, New York, 1981.
4. R. S. Sengar, Gravitational instability of streams, *Proc. Acad. Sci., India*, **51A**, 39 (1981).
5. A.E. Radwan, H.A. Radwan, M. Hendi, *Chaos, Solitons and Fractals*, **12**, 1729 (2001).
6. A.E. Radwan, A.A. Hasan, *Int J Appl Math.* **38**(3), 113 (2008).
7. A.E. Radwan, A.A. Hasan, *Appl Math Model.*, **33**(4), 2121 (2009).
8. A.A. Hasan, *Physica B.*, **406**(2), 234 (2011).
9. A.A. Hasan, *Boundary Value Problems*, **31**, 1 (2011).
10. A.A. Hasan, *J Appl Mech ASME*, **79**(2), 1 (2012).
11. A.A. Hasan, *Mathematical Problems in Engineering*, 1 (2012).

### Appendix I

The elements  $a_{ij}$  ( $i = 1, 2, \dots, 8$  and  $j = 1, 2, \dots, 8$ ) of the matrix  $[a_{ij}]$  in equation (41) of the linear algebraic equations (30)-(38) are being

$$a_{11} = (n \rho_o),$$

$$a_{12} = (2 \rho_o \Omega_z),$$

$$a_{13} = (\rho_o D U_o - 2 \rho_o \Omega_y),$$

$$a_{14} = i \mu (k_y H_{oy} + k_z H_{oz}),$$

$$a_{15} = i k_x \mu H_{oy},$$

$$a_{16} = i k_x \mu H_{oz},$$

$$a_{17} = i k_x C^2,$$

$$a_{18} = -i \rho_o k_x,$$

$$a_{21} = (-2 \rho_o \Omega_z),$$

$$a_{22} = (n \rho_o),$$

$$\begin{aligned}
 a_{23} &= (2\rho_o \Omega_x), & a_{56} &= ik_y k_z, \\
 a_{24} &= 0, & a_{57} &= 0, a_{58} = 0 \\
 a_{25} &= i\mu(2k_y H_{oy} + k_z H_{oz}), & a_{61} &= -ik_x H_{oz}, \\
 a_{26} &= ik_y \mu H_{oz}, & a_{62} &= -ik_y H_{oz}, \\
 a_{27} &= ik_y C^2, & a_{63} &= -ik_y H_{oy}, \\
 a_{28} &= -i\rho_o k_y, & a_{64} &= ik_x k_z, \\
 a_{31} &= (-2\rho_o \Omega_y), & a_{65} &= ik_y k_z, \\
 a_{32} &= (2\rho_o \Omega_x), & a_{66} &= -(n + i(k_x^2 + k_y^2)), \\
 a_{33} &= (n\rho_o), & a_{67} &= 0, \\
 a_{34} &= 0, & a_{68} &= 0 \\
 a_{35} &= ik_z \mu H_{oy}, & a_{71} &= i\rho_o k_x, \\
 a_{36} &= i\mu(k_y H_{oy} + 2k_z H_{oz}), & a_{72} &= i\rho_o k_y, \\
 a_{37} &= ik_z C^2, \quad a_{38} = -i\rho_o k_z, & a_{73} &= i\rho_o k_z, \\
 a_{41} &= i(k_y H_{oy} + k_z H_{oz}), & a_{74} &= 0, \\
 a_{42} &= 0, & a_{75} &= 0, \\
 a_{43} &= 0, & a_{76} &= 0, \\
 a_{44} &= -(n + i(k_x^2 + k_y^2)), & a_{77} &= n, \\
 a_{45} &= ik_x k_y, & a_{78} &= 0 \\
 a_{46} &= DU_o + ik_x k_z, & a_{81} &= 0, \\
 a_{47} &= 0, & a_{82} &= 0, \\
 a_{48} &= 0, & a_{83} &= 0, \\
 a_{51} &= -ik_x H_{oy}, & a_{84} &= 0, \\
 a_{52} &= ik_z H_{oz}, & a_{85} &= 0, \\
 a_{53} &= -ik_z H_{oy}, & a_{86} &= 0, \\
 a_{54} &= ik_x k_y, & a_{87} &= -4\pi G, \\
 a_{55} &= -(n + i(k_x^2 + k_z^2)), & a_{88} &= k^2
 \end{aligned}$$

**МАГНИТОХИДРОДИНАМИЧНА СТАБИЛНОСТ НА САМОГРАВИТИРАЩА,  
СВИВАЕМА, РЕЗИСТИВНА, ВЪРТЯЩА СЕ ПОТОЧНА СРЕДА**

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(Резюме)

Изследвана е магнитохидродинамичната стабилност на гравитационна среда с потоци с променливо скоростно разпределение за общо разпространение на вълните в присъствие на ротационни сили. Магнитното поле има силно стабилизиращо влияние, но стриймингът е дестабилизиращ. Ротационните сили имат стабилизиращо влияние при определени ограничения. Доказва се, че гравитационният критерий за нестабилност на Jean не е повлиян от електромагнитната сила или от въртящата сила или дори от комбинираното въздействие на това дали течната среда тече или не и дали въртенето е в едно или повече измерения.