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Stagnation point nanofluid flow along a stretching sheet with non-uniform heat generation/absorption and Newtonian heating

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The non-uniform heat generation/absorption and Newtonian heating effects on the steady two-dimensional laminar stagnation point boundary layer nanofluid flow past a stretching sheet in the presence of an external uniform magnetic field is investigated. The nanofluid is assumed to be viscous, incompressible and electrically conducting. The effects of Brownian motion and thermophoretic diffusion are taken into account. The governing non-linear partial differential equations are transformed to a set of ordinary differential equations in similarity form which are then solved using Spectral Relaxation Method (SRM). The effects of pertinent flow parameters on the flow, heat and nanoparticle concentration are studied with the help of graphs and tables. The nanofluid model presented in the paper has significant applications in the fluid engineering process where simultaneous effects of heat generation and convecting heating of the bounding surface take place such as in heat exchangers and nuclear reactor cooling.

Keywords: Brownian motion, Magnetic field, Nanofluid, Newtonian heating, Non-uniform heat generation/absorption, Stagnation point, Thermophoresis.

INTRODUCTION

Nanofluids are fluids with suspended nanosized (typically 1-100nm in size) particles of metals such as copper and gold, oxides such as alumina, silica, copper oxide, carbides and carbon nanotubes etc. The base fluids are water, oils, ethylene glycol, bio-fluids, polymer solutions and some lubricants. Nanofluids are potentially useful in heat transfer devices such as in microelectronics, fuel cells, automobiles, pharmaceutical processes, and hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, nuclear reactor coolant, in grinding, in space technology, and in defense and ships [1]. Nanofluids are reported to exhibit enhanced thermal conductivity and convective heat transfer coefficient compared to the base fluid [2]. Choi et al. [3] concluded that addition of small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid upto approximately two times. Choi [4] was the first who used the term nanofluids to refer the fluids with suspended nano-sized particles. Later, it was revealed by Buongiorno [5] that the enhancement in the thermal conductivity occurs due to the presence of Brownian and thermophoretic diffusions in the flow field.

Flow of a viscous, incompressible, and electrically conducting fluid over a continuously stretching surface under the influence of a transverse magnetic field finds applications in a variety of engineering processes such as in polymer extrusion process which involves cooling of a molten liquid being stretched into a cooling systems. The cooling liquid used and the rate of stretching are the main factors for the fluid mechanical properties of the penultimate product in these processes. Flow of the fluids, having better electromagnetic properties, such as polyethylene oxide, polyisobutylene solution in cetane can be controlled by applying external magnetic fields. A considerable attention must be given to control the rate of stretching of the extrudate rather than cooling liquids to accomplish the properties expected for outcome. Crane [6] was the first to analyze boundary layer flow of the Newtonian fluid caused solely by the linear stretching of an elastic sheet moving in its own plane with velocity proportional to the distance from the fixed point. Nadeem et al [7] presented a numerical investigation of three-dimensional flow of a water-based nanofluid over an exponentially stretching sheet. Ibrahim and Shankar [8] studied the boundary layer flow and heat transfer of a nanofluid over a permeable stretching sheet in the presence of an external magnetic field, slip boundary condition and thermal radiation. The effect of magnetic field on stagnation point nanofluid flow and heat transfer over a stretching sheet was discussed by Ibrahim et al. [9]. They showed that the heat transfer rate at the surface increases with the magnetic parameter when

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the free stream velocity exceeds the stretching sheet velocity. Qasim et al. [10] investigated magnetohydrodynamic flow of a ferrofluid along a stretching cylinder with velocity slip and prescribed surface heat flux. The boundary layer flow of a nanofluid due to an exponentially permeable stretching sheet with external magnetic field was studied by Bhattacharyya and Layek [11].

The temperature dependent heat source/sink has significant contributions on the heat transfer characteristics. Temperature differences between the surface and ambient-fluid are encountered in several engineering applications. Nandepannavar et al. [12] studied the flow and heat transfer of a viscous and incompressible fluid over a non-linearly stretching sheet in the presence of non-uniform heat source and variable wall temperature. They numerically solved the problem for the case of non-linearly stretching sheet by shooting technique with a fourth-order Runge-Kutta method while an analytical solution was presented for the case of linearly stretching sheet. It was concluded that the cumulative effect of the temperature dependent and space dependent heat generation/absorption is significant on heat transfer characteristics. Pal [13] investigated the influence of Hall current and thermal radiation on the flow and heat transfer characteristics of a viscous, incompressible, and electrically conducting fluid over an unsteady stretching permeable surface in the presence of an externally applied magnetic field. The two-dimensional stagnation point flow of nanofluid toward an exponentially stretching sheet with non-uniform heat generation/absorption was investigated by Malvandi et al. [14]. They observed that the increase in the heat generation causes the heat transfer rate to decrease. The problem of heat and mass transfer in unsteady MHD boundary layer flow of a nanofluid over a stretching sheet with a non-uniform heat source/sink was considered by Shankar and Yirga [15]. There are several other studies [16, 17] which discussed the effects of non-uniform heat generation/absorption under different conditions. In these studies the fluid was considered to be visco-elastic. Goyal and Bhar- gava [18] presented numerical simulation of the MHD boundary layer flow of a viscoelastic nanofluid past a stretching sheet in the presence of partial slip and temperature dependent heat source/sink. In a comment, Mastroberardino [19] demonstrated that the analytical results reported by Nandeppanavar et al. [20] were incorrect. He then presented the valid solutions of the governing ordinary differential equations for the fluid flow and temperature field using the homotopy analysis method.

There are several practical situations where the bottom wall is subjected to convective heating using a hot fluid situated on the other side of the wall. The heat transfer taking place due to such arrangement is referred as Newtonian heating. Newtonian heating process from the bottom wall has applications in many engineering devices such as, in heat exchanger where the conduction in the solid tube wall is influenced by the convection in the fluid past it, conjugate heat transfer around fins where the conduction within the fin and the convection surrounding the fluid must be analyzed simultaneously to obtain important design information, and convection flows setup when the bounding surfaces absorbs heat by solar radiation. Uddin et al. [21] investigated the steady two dimensional MHD laminar free convective boundary layer flow of an electrically conducting Newtonian nanofluid over a vertical plate in a quiescent fluid taking into account the Newtonian heating boundary condition. Makinde et al. [22] presented the combined effects of buoyancy force, convecting heating, Brownian motion and thermophoresis on the stagnation point flow and heat transfer of an electrically conducting nanofluid towards a stretching sheet under the influence of transverse magnetic field. The complex interaction between the electrical conductivity and that of nanoparticles in the presence of a magnetic field in a boundary layer flow past a convectively heated flat surface was discussed by Makinde and Mutuku [23]. They observed that the presence of nanoparticles greatly enhance the magnetic susceptibility of nanofluids as compared to the conventional base fluid.

Motivated from the above studies, the authors intend to investigate the effects of non- uniform heat generation and Newtonian heating effects on the steady two dimensional laminar stagnation point boundary layer nanofluid flow past a stretching sheet in the presence of an external uniform magnetic field. The nanofluid model includes the effect of Brownian diffusion and thermophoresis forces. The governing nondimensionalized equations in similarity form are solved using spectral relaxation method (SRM).

FORMULATION OF THE PROBLEM

We consider the steady two dimensional stagnation point laminar boundary layer flow of a viscous, incompressible, and electrically conducting nanofluid past a stretching sheet. The x axis is taken along the stretching sheet while the y axis is normal to the sheet. The surface of the sheet is stretched with a velocity proportional to the distance along x axis keeping the origin as fixed. Thus, the fluid flow is induced due to the stretching velocity, say $Uw = ax$, of the sheet. The fluid flow is permitted by an external uniform magnetic field $B0$ which acts in $y-$ direction. The fluid out side the boundary layer is also assumed to have a velocity $U\infty$. The surface of the sheet is convectively heated by a hot fluid of temperature Tf and the concentration of the nanoparticle at the surface is Cw while the values at free stream temperature and nanoparticle concentration are $T\infty$ and $C\infty$, respectively.

Assuming that the induced magnetic field effects are negligible and there is no external electric field applied to the system, so that the effect of polarization of electric field is neglected, the equations governing the nanofluid velocity, nanofluid temperature and nonopar- ticle volume friction, are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}=U_\infty\frac{dU_\infty}{dx}+v\frac{\partial^2 x}{\partial y^2}-\frac{\sigma B_0^2}{\rho_0}(u-U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial X}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{q'''}{(\rho c)_f} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively, C and T are nanoparticle concentration and nanofluid temperature, respectively, v is the kinematic viscosity, σ is electrical conductivity, α is the thermal diffusivity of the fluid, DB is the Brownian diffusion coefficient, DT is the thermophoretic diffusion coefficient and $\tau=\frac{(pc)_p}{(pc)_f}$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, and c being the specific heat at constant pressure. The subscripts p and f are used to denote the physical properties of nanoparticles and base fluid, respectively.

In Eq. (3), $q///$ is the space and temperature dependent internal heat generation/absorption (non-uniform heat source/sink) which can be expressed as

$$q'''=\left(\frac{kU_w(x)}{xv}\right)\left[A^*\frac{u(T_f-T_\infty)}{U_\infty(x)}+B^*(T-T_\infty)\right] \tag{5}$$

where $A*$ and $B*$ are the parameters of the space and temperature dependent internal heat generation/absorption. It is to be noted that $A*$ and $B*$ are positive to internal heat source and negative to internal heat sink, v is the kinematic viscosity.

The boundary conditions for the problem are

$$u=U_x(x)=ax,\ v=0,\ -k\frac{\partial T}{\partial y}=h\big(T_f-T\big),C=C_w\ at\ y=0;u\to U_\infty(x)=bx,v\to 0,\ T\to T_\infty,\ C\to C_\infty,\ as\ y\to\infty. \tag{6}$$

We introduce the following dimensionless quantities

$$\psi(x,y)=\sqrt{bv}xf(\eta),\ \theta=\frac{(T-T_\infty)}{(T_f-T_\infty)},\ \phi=\frac{(C-C_\infty)}{(C_w-T_\infty)}$$

$$\text{where } \eta=\sqrt{\frac{b}{v}}y, \tag{7}$$

where η is the dimensionless stream function, and the above transformation is chosen in such a way that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

Using the above transformation, the equation of continuity (1) is automatically satisfied and we obtain from Eqs. (2)-(5), as

$$f=0,\quad f'=\epsilon,\quad \theta'=-Bi(1-\theta),\quad \phi=1\ at\ \eta=0,\quad f'\to 1,\quad \theta\to 0,\quad \phi\to 0\ as\ \eta\to\infty. \tag{11}$$

In the above equations, primes denote differentiation with respect to η and the parameters are defined as

$$Pr=\frac{v}{\alpha},\ Nb=\frac{(pc)_p D_B(c_w-c_\infty)}{(pc)_f v},$$

$$\epsilon=\frac{a}{b},\ Le=\frac{v}{D_B},\ Nt=\frac{(pc)_p D_T\big(T_f-T_\infty\big)}{(pc)_f T_\infty v},$$

$$Bi=\frac{h}{k}\sqrt{\frac{v}{b}},$$

where $E > 0$ is for a stretching sheet and $E < 0$ is for a shrinking sheet. Further, Pr is the Prandtl number, Le is the Lewis number, Nb is the Brownian motion parameter, and Nt is the thermophoresis parameter.

The physical quantities of interest are the local skin friction coefficient Cfx , the local Nusselt number Nux and the local Sherwood number Shx which are defined as

$$C_{f_x}=\frac{\tau_w}{\rho U_\infty^2},\ Nu_x=\frac{xq_w}{k(T_f-T_\infty)},\ Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)},$$

where the surface shear stress τw, the local heat flux qw, and the local mass flux qm are given by

$$\tau_w=\mu\left(\frac{\partial u}{dy}\right)_{y=0},\ q_w=-k\left(\frac{\partial T}{dy}\right)_{y=0},$$

$$q_m=-D_B\left(\frac{\partial C}{dy}\right)_{y=0},$$

with μ and k being the dynamic viscosity and thermal conductivity of the nanofluids, re- spectively. Using the similarity variables (7), we obtain

$$(Re_x)^{1/2}C_{f_x}=f''(0), \tag{12}$$

$$(Re_x)^{-1/2}Nu_x = -\theta'(0), \quad (13)$$
$$(Re_x)^{-1/2}Sh_x = -\phi'(0), \quad (14)$$

where $Re_x = \frac{U_\infty x}{v}$ is the local Reynolds number.

SOLUTION TECHNIQUE

The spectral relaxation method (SRM) (see Motsa and Makukula [24], Kameswaran et al. [25]) is employed to solve the Eqs. (8)-(10) subject to the boundary conditions (11). The method uses the Gauss-Seidel approach to decouple the system of equations. In the framework of SRM method the iteration scheme is obtained as

$$f'_{r+1} = p_r, \quad f_{r+1}(0) = 0 \quad (15)$$
$$p''_{r+1} + f_{r+1}p'_{r+1} - Mp_{r+1} = p_r^2 - M - 1, \quad (16)$$
$$\theta''_{r+1} + Prf_{r+1}\theta'_{r+1} + \epsilon B^*\theta_{r+1} = -[PrNb\theta'_r\phi'_r + PrNt\theta_r'^2 + \epsilon A^* f'_{r+1}], \quad (17)$$
$$\phi''_{r+1} + Lef_{r+1}\phi'_{r+1} = -\frac{Nt}{Nb}\theta''_{r+1}. \quad (18)$$

The boundary conditions for the above iteration scheme are

$$p_{r+1}(0) = \epsilon, \quad p_{r+1}(\infty) \to 1, \quad (19)$$
$$\theta'_{r+1}(0) = -Bi\{1-\theta_{r+1}(0)\}, \ \theta_{r+1}(\infty) \to 0, \quad (20)$$
$$\phi_{r+1}(0) = 1, \quad \phi_{r+1}(\infty) \to 0. \quad (21)$$

In order to solve the decoupled equations (15)-(18), we use the Chebyshev spectral collocation method. The computational domain [0, L] is transformed to the interval [1, 1] using $\eta = L(\xi + 1)/2$ on which the spectral method is implemented. Here L is used to invoke the boundary conditions at . The basic idea behind the spectral collocation method is the introduction of a differentiation matrix which is used to approximate the derivatives of the unknown variables at the collocation points as the matrix vector product of the form

$$\frac{df_{r+1}}{d\eta} = \sum_{k=0}^{N} D_{lk} f_r(\xi_k) = Df_r,$$
$$l = 0, 1, 2, \dots, N, \quad (22)$$

where $N + 1$ is the number of collocation points (grid points), $D = 2/L$, and $f = [f(\xi 0), f(\xi 1), \dots, f(\xi N)]T$ is the vector function at the collocation points. Higher-order derivatives are obtained as powers of D, that is,

$$f_r^{(p)} = D^p f_r, \quad (23)$$

where p is the order of the derivative.

Applying the spectral method to equations (15)-(18), we obtain

$$A_1 f_{r+1} = B_1, \quad f_{r+1}(\xi_N) = 0, \quad (24)$$
$$A_2 p_{r+1} = B_2, p_{r+1}(\xi_N) = \epsilon, \quad p_{r+1}(\xi_0) = 1 \quad (25)$$
$$A_3\Theta_{r+1} = B_3, \quad \theta'_{r+1}(\xi_N) = -Bi\{1-\theta_{r+1}(\xi_N)\}, \quad \theta_{r+1}(\xi_0) = 0 \quad (26)$$
$$A_4\Phi_{r+1} = B_4, \quad \phi_{r+1}(\xi_N) = 1, \phi_{r+1}(\xi_0) = 0 \quad (27)$$

where,

$$A_1 = D, \quad B_1 = p_r, \quad (28)$$
$$A_2 = D^2 + diag(f_r)D + diag(-M)I,$$
$$B_2 = p_r^2 - (M-1), \quad (29)$$
$$A_3 = D^2 + diag(P_r f_r)D + diag(\epsilon B^*)I,$$
$$B_3 = -[P_r Nb\phi'_r\theta'_r + PrNt\theta_r'^2 + \epsilon A^* f'_{r+1}], \quad (30)$$
$$A_4 = D^2 + diag(Lef_2)D, \ B_4 = -\frac{Nt}{Nb}\theta''_{r+1}, (31)$$

In equations (28)-(31), **I** is an identity matrix and *diag*[.] is a diagonal matrix, all of size (N + 1) (N + 1) where N is the number of grid points, **f** , **p**, Θ, and Φ are the values of the functions f ,, p, θ and φ, respectively, when evaluated at the grid points and the subscript r denotes the iteration number.

The initial guesses, to start the SRM scheme for equations (15)-(18), are chosen as

$$f_0(\eta) = \eta + e^{-\eta} - e^{-\epsilon\eta},$$
$$p_0(\eta) = 1 - e^{-\eta} + \epsilon e^{-\epsilon\eta},$$
$$\theta_0(\eta) = \frac{1}{2}e^{-\eta B_i}\phi_0(\eta) = e^{-\eta}. \quad (32)$$

ERROR ANALYSIS

The error in the iteration scheme is assessed by taking the norm of the difference in the values of functions between two successive iterations. For each iteration scheme, we define the following maximum error Ed at the (r + 1)*th* iteration level

$$E_d = max\left(\left\|z_{1,r+1} - z_{1,r}\right\|_\infty, \left\|z_{2,r+1} - z_{2,r}\right\|_\infty, \dots, \left\|z_{m,r+1} - z_{m,r}\right\|_\infty\right), \quad (33)$$

where zi; i = 1, 2, ..., m are the governing unknown functions in the nonlinear system. It is observed from Fig 1 that the error Ed decreases rapidly with an increase in the number of iterations, which ascertain us the convergence of iteration schemes. It may be noted from

Fig 1 that about 50 iterations are required to obtain an accuracy of up to 10−12 in nanofluid velocity, temperature, and species concentration. The unknowns are calculated, for a given number of collocation points N , until the following criteria for convergence is satisfied at iteration r

$$E_d \le \epsilon,$$

where E is the convergence tolerance level. The effect of the number of collocation points N is examined in order to select the smallest value of N which gives a consistent solution to the error level E.

This is achieved by repeatedly solving the governing equations using the above iteration schemes with different values of N until the consistent solution is reached.

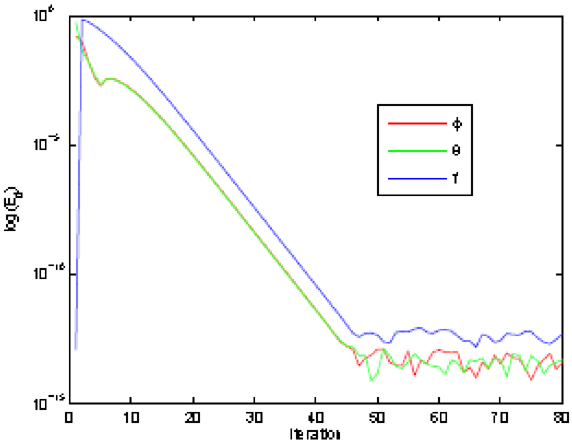

Fig. 1. Maximum error in f, θ, and φ versus number of iterations.

RESULTS AND DISCUSSION

The steady two dimensional laminar stagnation point boundary layer flow of a viscous, incompressible, and electrically conducting nanofluid past a stretching sheet in the presence of an external magnetic field is investigated with a view to analyze the effects of non-uniform heat generation/absorption and Newtonian heating. In order to analyze the effects of several flow parameters viz. the magnetic parameter M, Brownian motion parameter Nb, thermophoresis parameter Nt, heat generation/absorption parameter $A*$ and $B*$, and the Biot number Bi, the profiles of nanofluid velocity $f\,/$, nanofluid temperature θ, and nano-particle concentration φ are depicted graphically in Figs. 2- 6 while the values of coefficient of skin-friction, Nusselt number, and Sherwood number are tabulated in Table 1.

In Figs. 2 and 3, the effects of Brownian motion parameter Nb and thermophoresis parameter Nt on the nanofluid temperature and nanoparticle concentration are visualized. These are the two important effects which perturb the heat transfer characteristics of the fluid significantly due to the presence of nanoparticles in the flow field. The increase in Brownian motion parameter Nb measures an increase in the Brownian motion of the nano-sized particles present in the fluid. It is observed that the increase in Brownian motion of the nanoparticles causes an increase in the nanofluid temperature and a decrease in nanoparticle concentration within the boundary layer region. This is the cumulative effect due to transfer of heat from the surface to the fluid and the kinematic energy gained by the nanoparticle due to Brownian motion. The Brownian motion of nanoparticles transfers the surface heat to the fluid while the nanoparticles gain higher kinematic energy which contributes to the thermal energy of the fluid. Also there is a movement of the nanoparticles from the high temperature region towards the low temperature region and as a result the concentration of the nanoparticles within the boundary layer region decrease with the increase Brownian motion of the nanoparticles. It is concluded from Fig. 3, that the increase in thermopheretic parameter Nt increases the nanofluid temperature. The increase in the temperature is viewed as a result of the thermophoresis force by which a nanoparticle pushes the other nanoparticles away from the heated surface which in turn generates thermal energy due to the collision of nanoparticles. On the other hand, the effect of thermopheretic force on nanoparticle concentration is only significant in a region away from the surface where it increases with the increase in thermopheretic force.

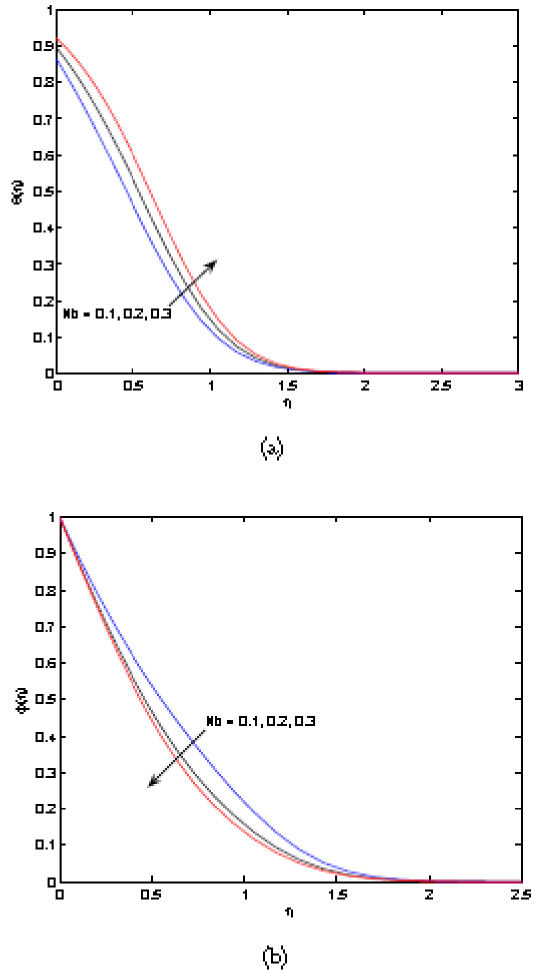

Fig. 2. The effect of Brownian motion parameter Nb on (a) the fluid temperature θ and (b) the species concentration φ, when M = 2, E = 0.1, Le = 5, Pr = 5, Bi = 5, Nt = 0.1, $A*$ = 0.5, and $B*$ = 0.5.

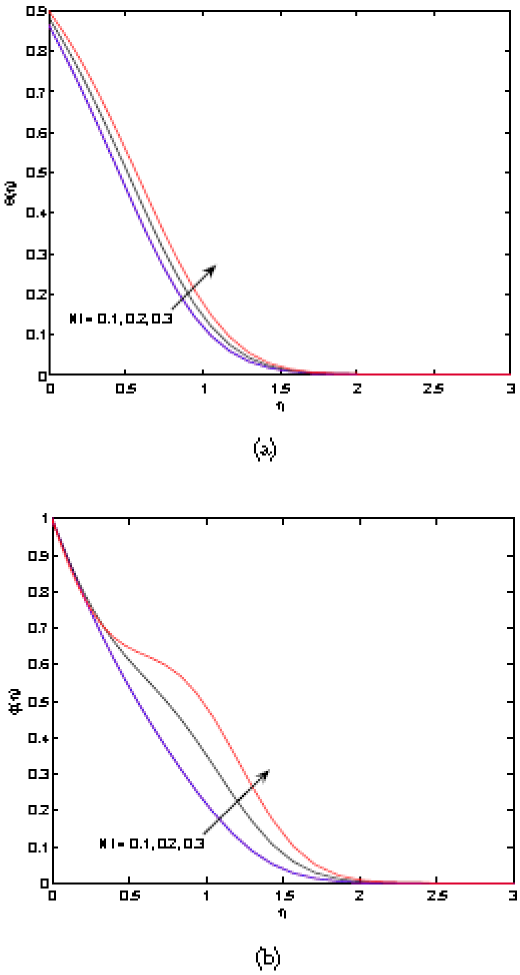

Fig. 3. The effect of Thermophoresis parameter *Nt* on (a) the fluid temperature θ and (b) the species concentration φ, when $M = 2$, $E = 0.1$, $Le = 5$, $Bi = 5$, $Nb = 0.1$, $Pr = 5$, and $B* = 0.05$.

Fig. 4 displays the effect of space dependent heat generation/absorption parameter $A*$ and temperature dependent heat generation/absorption parameter $B*$ on the nanofluid temperature and nanoparticle concentration within the boundary layer region. An increase in $A*$ and $B*$ cumulatively decides up to what extent the temperature will rise or fall within the boundary layer. Increase in the values of $A* > 0$, $B* > 0$ contribute in the generation of thermal energy and hence the nanofluid temperature increases while an increase in $A* < 0$, $B* < 0$ takes the thermal energy from the system and the nanofluid temperature decreases. As the nanofluid temperature increases, with increasing $A*$and $B*$, diffusion of nanoparticles take place from the high temperature region to the low temperature region and as a result the nanoparticle concentration within the boundary layer decreases which is depicted in Fig. 4(b).

The surface of stretching sheet is convectively heated by a hot fluid via Newtonian heating process. The effect of this heating is measured by the non-dimensional parameter *Bi*, and is captured in Fig. 5. It is clearly observed that the nanofluid temperature increases with the increase in convective heating of the surface. Also there is an increase in the nanoparticle concentration with the increase in convective heating of the surface.

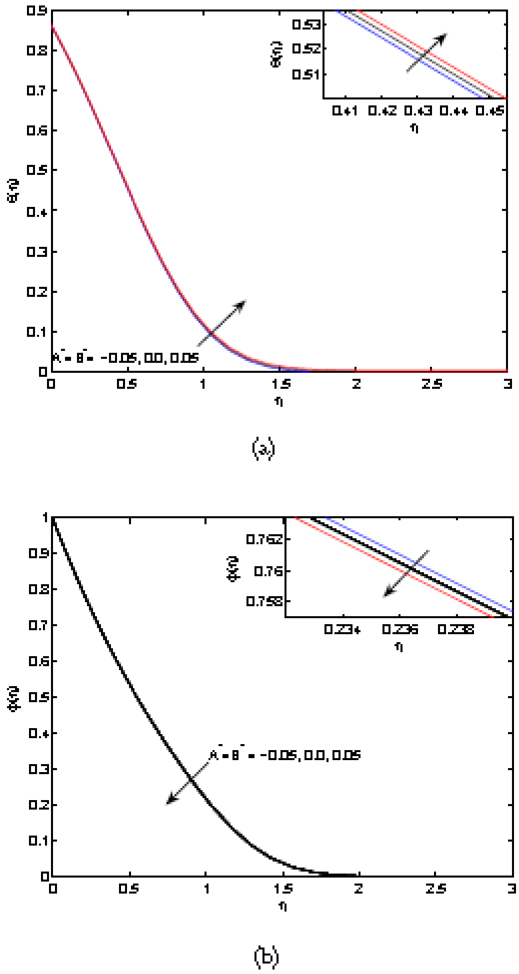

Fig. 4. The effect of space dependent heat source/sink $A*$ and temperature dependent heat source/sink $B*$ on (a) the fluid temperature θ and (b) the species concentration φ, when $M = 2$, $E = 0.1$, $Le = 5$, $Bi = 5$, $Nt = Nb = 0.1$, and $Pr = 5$.

The effect of magnetic parameter *M* is presented in Fig. 6. An increase in magnetic parameter measures the increage in the strength of the externally applied magnetic field. Interestingly, it is observed that the incasing in magnetic parameter causes a flow acceleration while it decreases the nanofluid temperature and nano-particle concentration. This is an opposite effect with the usual effect of magnetic field because in most of the cases the application of an external magnetic field gives rise to a resistive force, known as Lorentz force. However the obtained results are in agreement with the results of Ibrahim et al. [9]. Thus, in order to delay the boundary layer formation the magnetic field strength should be decreased appropriately.

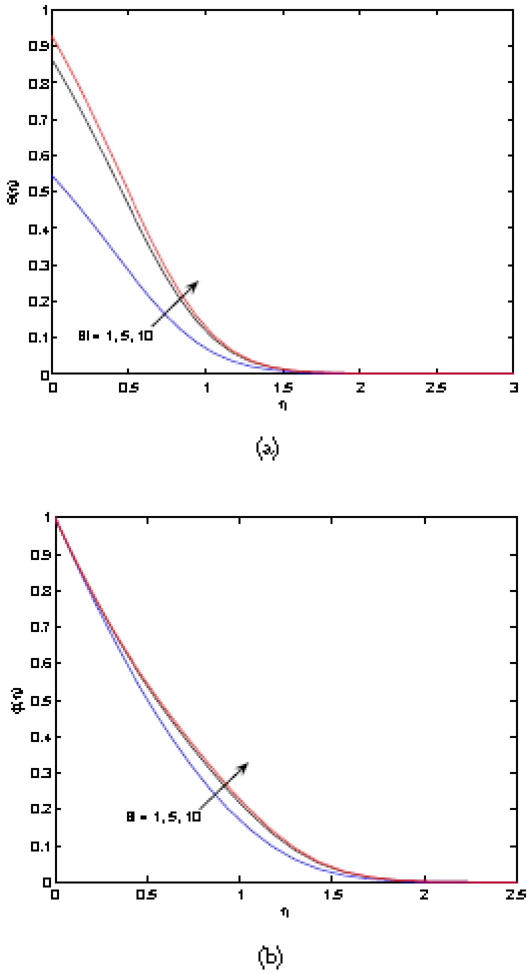

Fig. 5. The effect of Biot number *Bi* on (a) the fluid temperature θ and (b) the species concentration φ, when $M = 2$, $E = 0.1$, $Le = 5$, $Nb = Nt = 0.1$, $Pr = 5$, $A* = 0.05$, and $B* = 0.05$.

It is perceived from Table 1 that an increase in the magnetic field causes an increase in the coefficient of skin-friction which is due to the increase in the tangential force taking place as a result of increasing velocity. It is observed that the Nusselt number, which measures the rate of heat transfer from the surface, increases with increasing values of M and Bi, while it decreases with increasing values of Nb, Nt and $A* = B*$. This implies that the rate of heat transfer from the surface increases with an increases in the magnetic field and convective heating of the surface while the effects of Brownian motion, thermopheresis and heat generation/absorption is to decrease the rate of heat transfer from the surface. It may also be noted that the nanoparticle mass transfer increases with the increase in magnetic field, Brownian motion, thermophoresis, and heat generation/absorption while it is reversely affected by an increase in the convective heating at the surface.

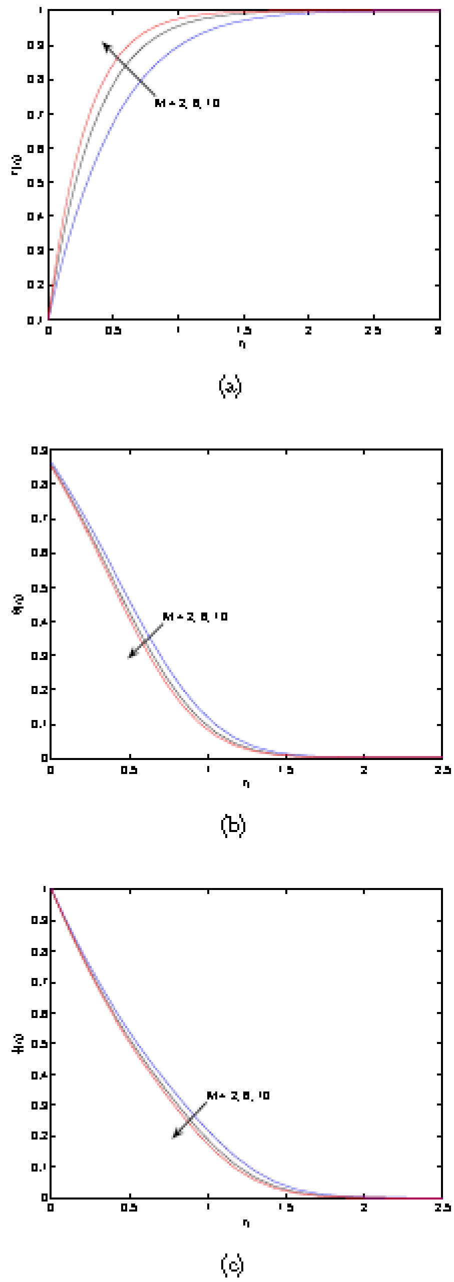

Fig. 6. The effect of magnetic parameter M on (a) the fluid velocity f', (b) the fluid temperature θ and (c) species concentration φ when $Le = 5$, $Pr = 5$, $Bi = 5$, $E = 0.1$, $Nb = Nt = 0.1$, $A* = 0.05$, and $B* = 0.05$.

CONCLUSIONS

The effects of space/temperature dependent non-uniform heat generation/absorption and Newtonian heating on the flow of a viscous, incompressible, and electrically conducting nanofluid past a stretching sheet is studied. The space dependent and temperature dependent heat generation have increasing effect on the nanofluid temperature while the heat absorption has a decreasing effect. However, the observed effects were found to be very less in these cases. The Newtonian heating at the surface of the sheet has an increasing effect on the nanofluid temperature and nanoparticle concentration and the effect on the nanofluid temperature is significant. One of the interesting results of the present study is the increasing effect of the magnetic field on the nanofluid velocity and an opposite one on the nanofluid temperature where these two physical quantities behave opposite to their usual trend with a change in the strength of the applied magnetic field. Thus in physical problems of the type as presented in this study, it is worthwhile to have a lesser applied magnetic field, as this will delay the boundary layer formation in the flow-field and will also decrease the skin-friction coefficient and rate of heat transfer from the surface.

Table 1. Effects of different parameters on coefficient of local skin-friction C_{fx}, local Nusselt number Nu_x, and local Sherwood number Sh_x.

M	BI	NB	NT	$A* = B*$	$C_{fx}\sqrt{RE_x}$	$NU_x/\sqrt{RE_x}$	$SH_x/\sqrt{RE_x}$
2	5	0.1	0.1	0.05	1.7104942	0.682568	1.09760339
6					2.482135450	0.725279740	1.16350495
10					3.065835750	0.749335331	1.19956408
	1				1.7104942	0.455541371	1.12193154
	5				1.7104942	0.682568	1.09760339
	10				1.7104942	0.72469558	1.095271
		0.1			1.7104942	0.682568	1.09760339
		0.2			1.7104942	0.527392751	1.22560301
		0.3			1.7104942	0.397449151	1.25879947
			0.1		1.7104942	0.682568	1.09760339
			0.2		1.7104942	0.589630761	1.13147173
			0.3		1.7104942	0.5069898	1.28114771
				0.05	1.7104942	0.691489461	1.09310392
				0	1.7104942	0.687039391	1.09534757
				0.05	1.7104942	0.682568	1.09760339

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НАНОФЛУИДЕН ПОТОК С ТОЧКА НА СТАГНАЦИЯ ПО ПРОТЕЖЕНИЕ НА РАЗТЕГЛЯЩА СЕ ПЛАСТИНА С НЕЕДНАКВО ГЕНЕРИРАНЕ / ПОГЛЪЩАНЕ НА ТОПЛИНА И НЮТОНОВО НАГРЯВАНИ

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(Резюме)

Разглеждат се неравномерното генериране / абсорбция на топлина и ефектите на Нютоново нагряване върху постоянния двумерен ламинарен граничен слой на стагнация на нанофлуид преминаващ разтягаща се пластина в присъствието на външно унифицирано магнитно поле . Предполага се, че нанофлуидът е вискозен, несвиваем и електрически проводим. Ефектите от Брауновото движението и термофоретичната дифузия са взети под внимание. Управляващите нелинейни частни диференциални уравнения се трансформират в набор от обикновени диференциални уравнения, които се решават с помощта на спектралния метод на релаксация (SRM). Ефектите от съответните параметри на потока, топлината и концентрацията на наночастици се изследват с помощта на графики и таблици. Представеният нанофлуиден модел има значителни приложения при инженерни процеси с течение, при които се осъществява едновременен процес на генериране на топлина и конвективно нагряване на граничната повърхност, като топлообменници и охлаждане на ядрен реактор.