

Breaking features of loading key strata based on deep beam structure in shallow coal seam and its limited span-to-depth ratio

F. Wu^{1,2}, X. Sun³, J. Li^{4*}, Ch. Liu^{1,2}, Q. Chang^{1,2}, B. Chen^{1,2}

¹ Key Laboratory of Deep Coal Resource Mining, Ministry of Education of China, China University of Mining and Technology, Xuzhou, China

² School of Mines, China University of Mining and Technology, Xuzhou, China

³ ShanDong Energy Zibo Mining Group CO.,LTD, Zibo, China

⁴ Institute of Mining Engineering, Inner Mongolia University of Science and Technology, Baotou, China

Received September 14, 2017; Accepted December 19, 2017

Based on the occurrence characteristics of loading key strata in a shallow coal seam, we built the mechanical model of the deep beam structure of the loading key strata under different boundary conditions. The features of first breaking and periodic breaking as well as the breaking span of the loading key strata considered as a deep beam structure were analyzed. The analytical solutions of elastic mechanics of stress components and displacement components upon first breaking and periodic breaking of the loading key strata more conformed to the real situations of breaking of overburden rocks of shallow coal seam during the exploitation as compared with analytical solutions of material mechanics and those of elastic mechanics of general long beams. The first and periodic breaking of the loading key strata usually belongs to tensile failure. Limited span-to-thickness ratio ε characterizes the stability of loading key strata, and it is affected by load, strength of rock mass and thickness of the loading key strata.

Key words: Shallow coal seam, loading key strata, deep beam structure, breaking features, limited span-to-thickness ratio.

INTRODUCTION

Shallow coal beams tend to demonstrate unique breaking features and evolution of surface subsidence induced by mining as compared with the ordinary coal seam because of its shallow burial depth, thin bedrock and thick overburden strata [1]. In the mining of shallow coal seams, breaking of the loading key strata directly leads to the overall collapse of the overburden layers and ground surface. As a result, the ground pressure of the working face intensifies with the generation of mining cracks connecting to the ground surface. Therefore, understanding the breaking features and limited span of the loading key strata is of high importance for predicting the roof pressure of working surface in a shallow coal seam and the surface damage caused by mining.

Among various studies on the breaking features of overburden layers and breaking span in shallow coal seams, in [1,2] a theoretical expression of first and periodic breaking spans in the roof under general long-beam structure is presented. In [3,4] the formula of first and periodic weighting steps in the combinational key strata under long beam structure is derived. In [5-7], a key strata theory was applied to the roof control of shallow coal seams, and a roof structure and strata control theory for long-wall mining of shallow coal seams was

established. In [8], particular focus was given to the type and breaking instability of key strata of shallow coal seams. On this basis, it was found that single key strata structure was the fundamental geological cause of mining-induced damage of the shallow coal seams. Most studies on breaking instability of roof or key strata in the mining of shallow coal seams favored the use of mechanical model of long-beam structure, where the thickness-to-span ratio of the rock beam is below 1/5. But in real situations, the loading key strata are generally hard, thick rocks in shallow coal seams. Experimental analysis has shown that the thickness-to-span ratio of the rock beam is generally above 1/5, indicating deep beam structure in the shallow coal seam [9,10]. Therefore, breaking features of loading key strata analyzed by assuming a long-beam structure usually deviate from the real breaking features.

We built a mechanical model of deep beam structure for describing first and periodic breaking of loading key strata in the mining of shallow coal seams based on elastic mechanics. Expressions of stress components and displacement components in deep beam structure were given under different boundary conditions. The breaking features and limited span of the loading key strata were analyzed based on mechanical models.

To whom all correspondence should be sent:
E-mail: ljwcumt@cumt.edu.cn

EXPERIMENTAL

Identifying loading key strata

There may be one or several loading key strata existing in a shallow coal seam [1,2,8]. Instability and breaking of a single loading key stratum will lead to overall collapse of the overburden and ground surface. Moreover, the ground pressure of the working face intensifies. In the presence of several loading key strata, the instability and breaking of non-controlling inferior key strata will lead to increased ground pressure of the working face. As with the situation of a single loading key stratum, the instability and breaking of the main key strata can also lead to the collapse of the overburden and the ground surface with an intensification of the ground pressure in the working face. This is what we call periodic weighting phenomenon.

For shallow coal seams, the loading key stratum is defined as the rock stratum whose failure can lead to overall collapse of the overburden and thick loose layer on the ground surface, resulting in the loss of carrying capacity of the surrounding rock and happening of dynamic disasters [1,3]. Loading key strata are generally hard, thick rock layers that support the overburden and thick loose layer on the ground surface through certain mechanical structures. Breaking of loading key strata may directly lead to mining pressure, rock layer movement and surface subsidence. Loading key strata, single or multiple, can be found in nearly all shallow coal seams. Identifying the loading key strata is crucial for the study of breaking of loading key strata, weighting and safety of the advancing working face.

Fig. 1 shows the overburden distribution of the shallow coal seam with m overlying layers of bedrock. Above the bedrock is the thick loose layer.

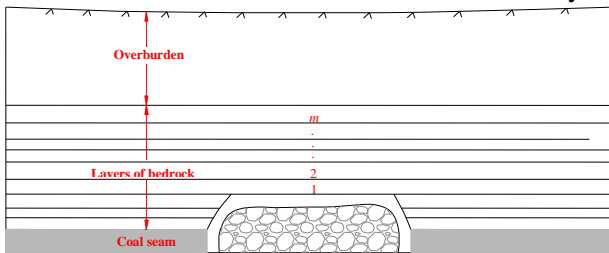


Fig. 1. Overburden distribution of shallow coal seam

Let the thickness of each rock layer be h_i , volume force being γ_i and modulus of elasticity being E_i , where $i=1, 2, 3, \dots, m$. According to the key strata theory [1], the loading key strata must satisfy three conditions:

$$\begin{cases} q_n > q_{n-1} > \dots > q_1 \\ q_n = (q_n)_0, \text{ and } (q_n)_0 > (q_n)_{n+1} > \dots > (q_n)_m \\ L_n > L_{n+1} > \dots > L_m \end{cases} \quad (1)$$

where

$$\begin{cases} (q_n)_0 = E_n h_n^3 \cdot \left(\sum_{i=n}^{i=m} \gamma_i h_i + \gamma_0 h_0 \right) / \sum_{i=n}^{i=m} E_i h_i^3 \\ (q_n)_m = E_n h_n^3 \cdot \sum_{i=n}^{i=m} \gamma_i h_i / \sum_{i=n}^{i=m} E_i h_i^3 \end{cases}$$

where $(q_n)_0$ is the load imposed by the overburden on the n^{th} rock layer; $(q_n)_m$ is the load imposed by the m^{th} rock layer on the n^{th} rock layer; q_i ($i=1, 2, 3, \dots, n$) is the load acting on the i^{th} rock layer; L_i ($i=n, n+1, \dots, m$) is the breaking step of the i^{th} rock layer.

First breaking mechanical model

As the working face advances, the mined-out area of the loading key strata increases. The stress characteristics of hard, thick loading key strata are different from those of a long-beam structure. Therefore, a mechanical model of clamped deep beam structure under uniform loading was built to analyse first breaking of loading key strata.

Fig. 2 shows the mechanical model of clamped deep beam under uniform loading. The rectangular deep beam with length l and height h is presented with two ends clamped and subjected to uniform loading q .

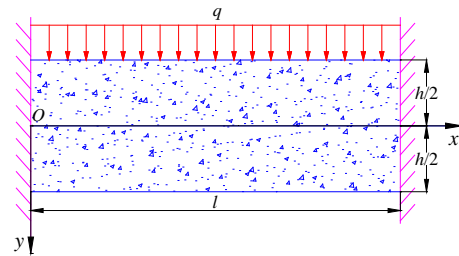


Fig.2. Mechanical model of clamped deep beam under uniform loading

Using the semi-inverse method, the biharmonic stress function is a polynomial with a degree of 5 and 7 variables [11]. Then each stress component of the deep beam structure is given by:

$$\begin{cases} \sigma_x = 2A(2y^3 - 3x^2y) + 6Bxy + 6Cy + 2D \\ \sigma_y = -2Ay^3 + 2Ey + 2G \\ \tau_{xy} = 6Axy^2 - 3By^2 - 2Ex - F \end{cases} \quad (2)$$

From the displacement-stress relationship [12,13] the displacement component is derived:

$$\begin{cases} u = [2A(2+\mu)xy^3 - (2Ax^3 - 3Bx^2 - 6Cx + 2\mu Ex)y + 2Dx \\ \quad - 2\mu Gx] / E' - [B(2+\mu)y^3 + 2(1+\mu)Fy] / E' + \omega y + v_0 \\ v = [-A(1+2\mu)y^4 / 2 - (2\mu Ax^2 - 3\mu Bx - 3\mu Cx + E)y^2 \\ \quad - 2\mu Dy + 2Gy + Ax^4 / 2 - Bx^3 - (2+\mu)Ex^2] / E' - \omega x + v_0 \end{cases} \quad (3)$$

where ω , u_0 and v_0 are arbitrary integral constants, determined by boundary conditions; E' is modulus of elasticity.

If treated as a plane stress problem in elastic mechanics, the boundary conditions for clamped deep beam are usually simplified into displacements $u=0$ and $v=0$ for the fixed end at the mid-point of the end surface in two directions and rotational angle $\partial v/\partial x=0$ or $\partial u/\partial y=0$, as is done in some studies [2,4,12]. Under the above simplified boundary conditions for clamped beam, the analytical solution of the plane stress is more accurate for a shallow beam structure. But great errors may occur for deep beam structure [14-15]. Therefore, simplification of boundary conditions for clamped deep beam should be done carefully and according to the real stress status.

Loading key strata in shallow coal seams bear considerable static load imposed by the overburden layer and the thick loose layer of ground surface. Before first breaking, if the upper surface of the clamped beam undergoes vertical displacement, the entire rock strata will break and lose stability under static load of the overburden. Therefore, for clamped deep beam, changing the boundary condition of rotational angle $\partial v/\partial x=0$ or $\partial u/\partial y=0$ at the central point of the fixed end to vertical displacement $u=0$ at the vertex of the fixed end will better conform to the real situation of shallow coal seam mining [11].

The boundary conditions for the clamped deep structure before the first breaking of the loading key strata are:

$$\begin{cases} y = -h/2, & \sigma_y = -q, \tau_{xy} = 0 \\ y = h/2, & \sigma_y = 0, \tau_{xy} = 0 \\ x = 0, y = 0, & u = 0, v = 0 \\ x = 0, y = -h/2, & u = 0 \\ x = l, y = 0, & u = 0, v = 0 \\ x = l, y = -h/2, & u = 0 \end{cases} \quad (4)$$

Although the loading key strata are thick, the thickness is still smaller than that of the controlling key strata and overburden layer. The loading key strata are under great external load in this sense [1, 16, 17]. By transforming the dead weight of the loading key strata into uniform loading, the calculation can be simplified and better conforms to the real situation.

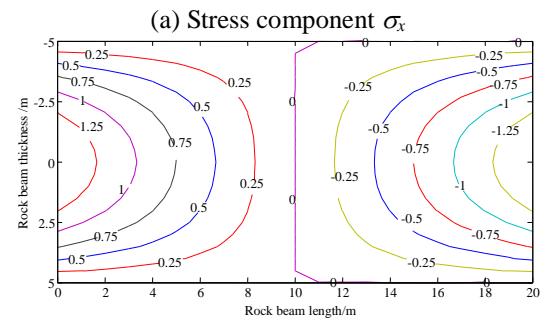
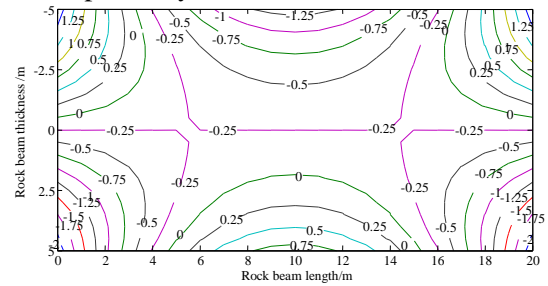
Using the expression of stress component (2), displacement component (3) and boundary conditions (4), the stress component and displacement component before first breaking of the loading key strata are given as follows:

$$\begin{cases} \sigma_x = \frac{4q}{h^3}y^3 - \frac{6q}{h^3}x^2y + \frac{6ql}{h^3}xy - \frac{q(l^2 + h^2 - \mu h^2)}{h^3}y - \frac{\mu q}{2} \\ \sigma_y = \frac{-2q}{h^3}y^3 + \frac{3q}{2h}y - \frac{q}{2} \\ \tau_{xy} = \frac{6q}{h^3}xy^2 - \frac{3ql}{h^3}y^2 - \frac{3q}{2h}x + \frac{3ql}{4h} \end{cases} \quad (5)$$

where q is the sum of loading from the overburden and the dead weight of the loading key strata; μ is the Poisson's ratio of the loading key strata.

$$\begin{cases} u = \left[\frac{2q}{h^3}(2+\mu)xy^3 - \frac{2q}{h^3}x^3 - \frac{3ql}{h^3}x^2 - \frac{q(-l^2 - h^2 + \mu h^2)}{h^3}x + \frac{3\mu q}{2h}xy \right] / E' \\ \quad - \left[\frac{ql(2+\mu)}{h^3}y^3 - \frac{3ql(1+\mu)}{2h}y \right] / E' - \frac{ql(4+5\mu)}{4hE'}y \\ v = \left[-\frac{q(1+2\mu)}{2h^3}y^4 - \frac{2\mu q}{h^3}x^2 - \frac{3\mu ql}{h^3}x - \frac{\mu q(-l^2 - h^2 + \mu h^2)}{2h^3}x + \frac{3q}{4h}y^2 \right. \\ \quad \left. + \frac{\mu^2 q}{2}y - \frac{q}{2}y + \frac{q}{2h^3}x^4 - \frac{ql}{h^3}x^3 - \frac{(2+\mu)3q}{4h}x^2 \right] / E' + \frac{ql(4+5\mu)}{4hE'}x \end{cases} \quad (6)$$

The values were $q=1.0\text{MPa}$, $\mu=0.2$ and $E'=30\text{GPa}$ in this paper. From equations (5) and (6), the distribution curves of stress components σ_x and τ_{xy} and displacement components u and v of the rock beam under the rock beam length $l=20$ m and thickness $h=10$ m were plotted, as shown in Figs. 3 and 4, respectively.



(a) Stress component σ_x
(b) Stress component τ_{xy}
Fig. 3. Stress component distribution of rock beam under clamped conditions

It can be seen that for clamped deep beam under uniform loading, the stress component and displacement component distributions are symmetrical about the central line $x=10$ m of the deep beam, respectively.

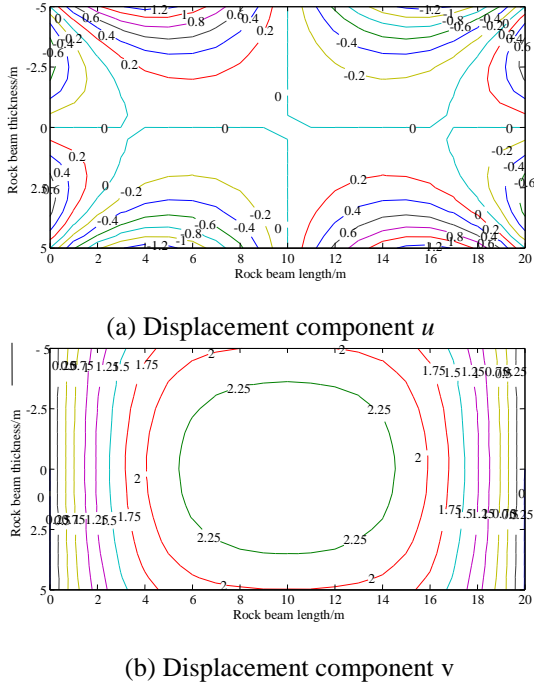


Fig. 4. Displacement component distribution of rock beam under clamped conditions

Periodic breaking mechanical model

After the first breaking, as the working face advances, periodic breaking of the loading key strata will occur. Since the shallow coal seam has higher working thickness, the loading key strata may enter the caving zone and assume a cantilevered beam structure [18]. In light of this, we built a mechanical model of cantilevered deep beam under uniform loading for the analysis of periodic breaking.

Fig. 5 shows the mechanical model of cantilevered deep beam under uniform loading. The rectangular deep beam with length l and height h is presented and subjected to uniform loading q on the upper surface of the beam.

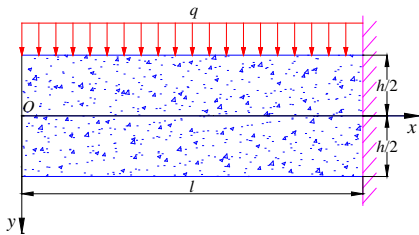


Fig. 5. Mechanical model of cantilevered deep beam under uniform loading

Referring to the method for determining the boundary conditions for clamped deep beam under uniform loading, we considered the shear force and bending moment of the fixed end for the cantilevered conditions before periodic breaking [19]. The boundary conditions for the cantilevered deep beam are:

$$\left\{ \begin{array}{l} y = -h/2, \sigma_y = -q, \tau_{xy} = 0 \\ y = h/2, \sigma_y = 0, \tau_{xy} = 0 \\ x = 0, y = 0, u = 0, v = 0 \\ x = 0, y = -h/2, u = 0 \\ \int_{-h/2}^{h/2} (\tau_{xy})_{x=0} dy = -ql \\ \int_{-h/2}^{h/2} (\sigma_x)_{x=0} y dy = -ql^2/2 \\ \int_{-h/2}^{h/2} (\sigma_x)_{x=0} dy = 0 \end{array} \right. \quad (7)$$

Using the semi-inverse method, the stress components and the displacement components before periodic breaking are given by:

$$\left\{ \begin{array}{l} \sigma_x = \frac{4q}{h^3} y^3 - \frac{6q}{h^3} x^2 y - \frac{12ql}{h^3} xy - \frac{2q(5l^2 - h^2)}{5h^3} y \\ \sigma_y = \frac{-2q}{h^3} y^3 + \frac{3q}{2h} y - \frac{q}{2} \\ \tau_{xy} = \frac{6q}{h^3} xy^2 + \frac{6ql}{h^3} y^2 - \frac{3q}{2h} x - \frac{3ql}{2h} \end{array} \right. \quad (8)$$

where q is the sum of loading from the overburden and dead weight of the loading key strata.

$$\left\{ \begin{array}{l} u = \left[\frac{2q}{h^3} (2 + \mu) xy^3 - \frac{2q}{h^3} x^3 + \frac{6ql}{h^3} x^2 + \frac{2q(5l^2 - h^2)}{5h^3} x + \frac{3\mu q}{2h} xy + \frac{\mu q}{2} x \right] / E' \\ \quad + \left[\frac{2ql(2 + \mu)}{3h^3} y^3 - \frac{3ql}{2h} y \right] / E' + \frac{ql(7 - 5\mu)}{6hE'} y \\ v = \left[-\frac{q(1 + 2\mu)}{2h^3} y^4 - \frac{3\mu q}{h^3} x^2 + \frac{6\mu ql}{h^3} x + \frac{3\mu q(5l^2 - h^2)}{15h^3} x + \frac{3q}{4h} y^2 + \frac{q}{2} y \right. \\ \quad \left. + \frac{q}{2h^2} x^4 + \frac{2ql}{h^3} x^3 + \frac{q(20l^2 + 34h^2 + 30\mu h^2)}{20h^3} x^2 \right] / E' - \frac{ql(7 - 5\mu)}{6hE'} x \end{array} \right. \quad (9)$$

The values were $q=1.0$ MPa, $\mu=0.2$ and $E'=30$ GPa. From equations (8) and (9), the distribution curves of stress components σ_x and τ_{xy} and displacement components u and v of the rock beam under the rock beam length $l=20$ m and thickness $h=10$ m were plotted, as shown in Figs. 6 and 7, respectively.

It can be seen that for cantilevered deep beam under uniform loading, the stress component and displacement component distributions are symmetrical about the central line $x=10$ m of the deep beam, respectively.

RESULTS AND DISCUSSION

In equation (5), let $x=ml$ ($0 \leq m \leq 1$), $y=nh$ ($-0.5 \leq n \leq 0.5$) and $l/h=\varepsilon$. Then the shear stress of the clamped deep beam is:

$$\tau_{xy} = \left| \frac{3\varepsilon q}{4} (2m - 1)(4n^2 - 1) \right| \quad (10)$$

where ε is span-to-thickness ratio of deep beam, i.e., the ratio of span l to thickness h .

From equation (10) and according to the analysis of probable site of breaking for clamped deep beam, when $m=0.5$ and $n=0.5$, $\tau_{xy}=0$. That is, the shear stress of the lower boundary ($l/2, h/2$) of the central

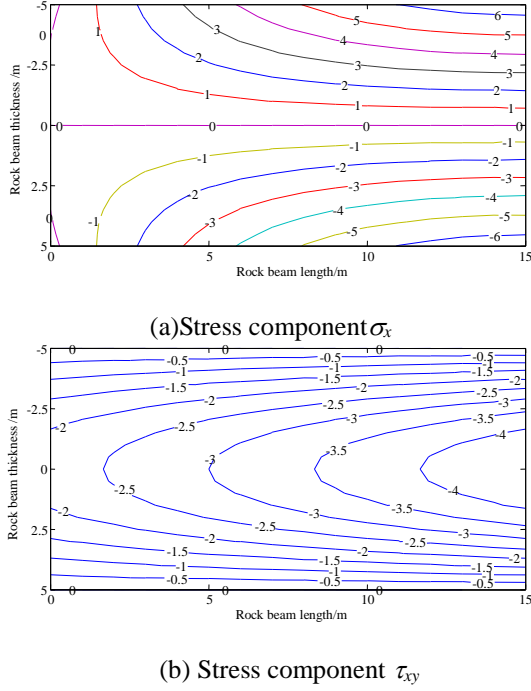


Fig. 6. Stress component distribution of rock beam under cantilevered conditions

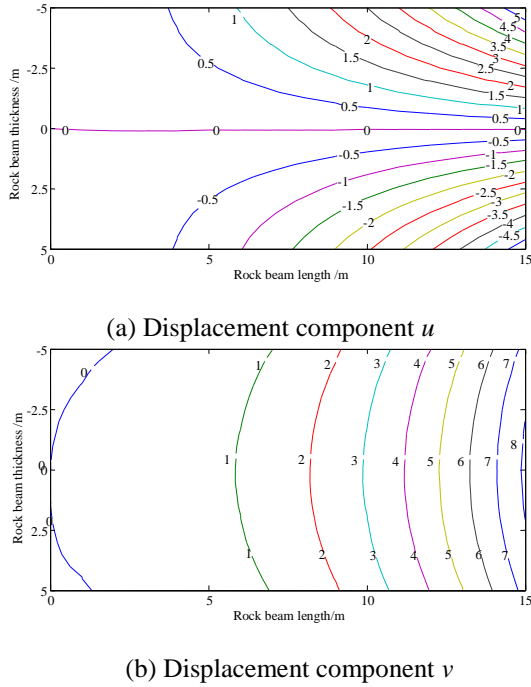


Fig. 7. Displacement component distribution of rock beam under cantilevered conditions

cross-section of the clamped deep beam is 0 (Fig. 3b). At this site, the tensile stress σ_x in the horizontal direction is the maximum principal stress (Fig. 3a). From the above we calculate the tensile stress at this site:

$$\sigma_{x\max} = \sigma_x \Big|_{\left(\frac{l}{2}, \frac{h}{2}\right)} = \frac{ql^2}{2h^2} \quad (11)$$

From equation (10), τ_{xy} reaches the maximum when $m=0$ and $n=0$ or when $m=1$ and $n=0$. That is,

the maximum shear stress is at the fixed end $(0, 0)$ or $(l, 0)$ in the clamped deep beam (Fig. 3b). The shear stress at this site is:

$$|\tau_{\max}| = |(\tau_{xy})_{(0,0)}| = |(\tau_{xy})_{(l,0)}| = \frac{3ql}{4h} \quad (12)$$

When the normal stress at this site reaches the ultimate tensile strength of the rock strata, i.e., when $\sigma_{x\max} = R_T$, the strata will undergo tensile failure. Considering the heterogeneity and brittle fracture of the rock strata, the coefficient η of the rock strata on the verge of failure is used to calculate the limited span upon rupture of the deep beam:

$$L_{iT} = h \cdot \sqrt{\frac{2R_T}{\eta q}} \quad (13)$$

When the shear stress of the clamped deep beam reaches the ultimate tensile strength of the rock strata at this site, i.e., when $\tau_{\max} = R_S$, the rock strata will undergo tensile failure. Then the limited span upon rupture of the deep beam is:

$$L_{iS} = h \cdot \frac{4R_S}{3\eta q} \quad (14)$$

With span-to-thickness ratio $\varepsilon = l/h$ and comparing equations (10) and (11), the limited span-to-thickness ratio upon tensile failure of the deep beam is $\varepsilon_{iT} = \sqrt{2R_T/\eta q}$, and that upon shear failure is $\varepsilon_{iS} = 4R_S/3\eta q$. Thus equations (13) and (14) are respectively changed into:

$$L_{iT} = \varepsilon_{iT} h \quad (15)$$

$$L_{iS} = \varepsilon_{iS} h \quad (16)$$

Similarly, for equation (8), let $x = ml$ ($0 \leq m \leq 1$), $y = nh$ ($-0.5 \leq n \leq 0.5$) and $l/h = \varepsilon$. Then the shear stress of the cantilevered deep beam is calculated by:

$$\tau_{xy} = \left| \frac{3\varepsilon q}{2} (m+1)(4n^2 - 1) \right| \quad (17)$$

From equation (8) and according to the analysis of the probable site of breaking for clamped deep beam, when $m=1$ and $n=-0.5$, $\tau_{xy}=0$. That is, the shear stress of the upper boundary $(l, -h/2)$ of the fixed end of a cantilevered deep beam is 0 (Fig. 6b). At this site, the tensile stress σ_x in horizontal direction is the maximum principal stress (Fig. 6a). From the above the limited span upon tensile failure of the cantilevered deep beam is given by:

$$L'_{iT} = h \cdot \sqrt{\frac{R_T}{4\eta q} + \frac{7}{40}} \quad (18)$$

The limited span upon shear failure of the cantilevered deep beam is:

$$L'_{iS} = h \cdot \frac{2R_S}{3\eta q} \quad (19)$$

The limited span-to-thickness ratio upon tensile failure is $\varepsilon'_{iT} = \sqrt{R_T/4\eta q + 0.175}$ and that upon shear failure is $\varepsilon'_{iS} = 2R_S/3\eta q$.

Loading key strata in a shallow coal seam are generally sandstone rocks. According to ref. [3], the shear strength of sandstone is 2.22-5.28 times that of the tensile strength (average 3.26 times). With $R_s = 3R_T$ and $\eta=1.0$, we calculate the limited span-to-thickness ratios upon first breaking and periodic breaking of the loading key strata for deep beam structure, as shown in Fig. 8.

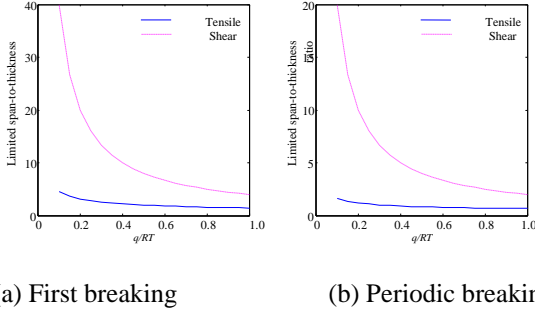


Fig. 8. Limited span-to-thickness ratio of deep beam structure of loading key strata in first breaking and periodic breaking ($R_T=3.0\text{MPa}$)

As shown in Fig. 8, the limited span-to-thickness ratio of loading key strata for deep beam structure upon tensile failure will be smaller than that upon shear failure. Thus the limited spans for first breaking and periodic breaking are respectively given by:

$$L_{IT} = h \cdot \sqrt{\frac{2R_T}{\eta q}} \quad (\text{First breaking})$$

$$L'_{IT} = h \cdot \sqrt{\frac{R_T}{4\eta q} + \frac{7}{40}} \quad (\text{Periodic breaking})$$

The limited span-to-thickness ratios are $\varepsilon_{IT} = \sqrt{2R_T/\eta q}$ and $\varepsilon'_{IT} = \sqrt{R_T/4\eta q + 0.175}$, respectively.

According to the theory of material mechanics [1, 19], the spans of first and periodic breaking for the clamped deep beam are $L_{IT} = h \cdot \sqrt{2R_T/q}$ and $L'_{IT} = h \cdot \sqrt{R_T/3q}$, respectively. Then using the theory of elastic mechanics [20], the span of first breaking for the clamped deep beam and the span of periodic breaking for the cantilevered beam are respectively given by: $L_{IT} = 2h \cdot \sqrt{R_T/\eta q - 0.2}$ and $L'_{IT} = h \cdot \sqrt{R_T/3\eta q + 0.27}$.

The values were taken as $R_T=3\text{MPa}$ and $q=1.5\text{MPa}$ in this paper. Considering the heterogeneity and brittle fracture of rock strata, the coefficient $\eta=2.0$ was used. Thus the curves of spans of first and periodic breaking vs. thickness of rock beam were plotted using the three calculation methods, as shown in Fig. 9.

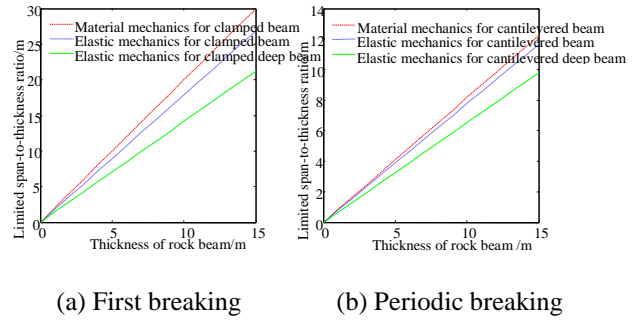
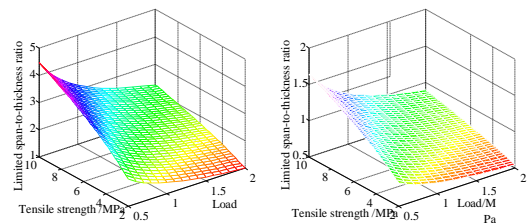


Fig. 9. Comparison of calculation results of three methods

It can be seen from Fig. 9 that as the thickness of the rock beam increases, the breaking span also increases. All three methods achieve small relative errors at a low thickness of the rock beam. But the relative errors increase with the increasing thickness of rock beam. Since the solutions of material mechanics are based on the plane section assumption [19], the calculation does not consider the stress and deformation conditions of the clamped or cantilevered beam boundary. This leads to an overestimation as compared with the other two methods. Solutions of elastic mechanics for general rock beams only consider the stress conditions of the boundary while ignoring the displacement boundary [20]. Consequently, the calculated result will be larger than that based on elastic mechanics for a deep beam. The analytical solutions of elastic mechanics for a deep beam fully consider the stress and displacement boundary conditions as the thickness of rock beam increases. The calculated result better conforms to the real situation of breaking of overburden layer in the mining of shallow coal seam.

Limited span-to-thickness ratio ε of the loading key stratum for clamped deep beam is affected by rock mass strength R_T and load q . Relations of ε to R_T and q under different breaking features are shown in Fig. 10. It can be seen that ε decreases with increasing q and it increases with increasing strength. ε characterizes the stability of the loading key strata.



(a) First breaking ($\eta=2.0$) (b) Periodic breaking ($\eta=2.0$)

Fig.10. Relationship between ε and R_T and q

CONCLUSIONS

We built mechanical models for loading key strata in a shallow coal seam based on deep beam structure. The analytical solutions of elastic mechanics for stress components and displacement components upon first and periodic breaking of the loading key strata were calculated.

Both first and periodic breaking can occur as tensile failure or shear failure in the loading key strata. They are affected by limited span-to-thickness ratio and load. For a shallow coal seam, tensile failure is more common upon first and periodic breaking.

By comparing the analytical solutions of material mechanics and elastic mechanics for a general long beam, we found that elastic mechanics more conforms to the real situation of breaking in shallow coal seam as it fully considers the stress and displacement boundary conditions.

The limited span-to-thickness ratio of the loading key strata increases with the increasing strength and thickness of rock mass, but decreases with increasing load. The limited span-to-thickness ratio characterizes the stability of the loading key strata.

Acknowledgments: *Financial support for this research was provided by the National Natural Science Foundation of China (51574220); Fundamental Research Funds for the Central Universities, by China University of Mining and Technology (20142DPY21); the Special Funds of Taishan Scholar Construction Project of China(201309); project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (SZBF2011-6-B35).*

REFERENCES

1. M.G. Qian, X.X. Miao, J.L. Xu, X.B. Mao, Key Strata Theory in Strata Control. China University of Mining and Technology Press, Xuzhou, 2003.
2. M.G. Qian, P.W. Shi, Ground Pressure and Strata Control. China University of Mining and Technology Press, Xuzhou, 2003.
3. Z.J. Hou, *J. China Coal Soc.*, **24** (4), 359 (1999).
4. Z.J. Hou, J. Zhang, *J. Liaoning Tech. Univ.*, **23** (5), 577 (2004).
5. Q.X. Huang, *Chin. J. Rock Mech. Eng.*, **21** (8), 1174 (2002).
6. Q.X. Huang: Study on Roof Structure and Ground Control in Shallow Seam Longwall Mining. China University of Mining and Technology Press, Xuzhou, 2000.
7. Q.X. Huang, Roof Structure *J. Coal Sci. Eng.*, **9**(2), 21 (2003).
8. J.L. Xu, W.B. Zhu, X.Z. Wang, M.S. Yin. *J. China Coal Soc.*, **34** (7), 865 (2009).
9. G.F. Wang, *J. Chengdu Uni. Sci. Techno.*, **70**(3), 70 (1997).
10. F.L. Mei, D.S. Zeng. *Mech. Pract.*, **24** (3), 58 (2002).
11. Y. Dai, X. Ji, *J. Tongji Univ. (Nat. Sci. Ed)*, **36** (7), 890 (2008).
12. S.P. Timoshenko, J.N. Goodier, Theory of Elasticity. People's Education Press, Beijing, 1964.
13. H.J. Ding, D.J. Huang, H.M. Wang, *J. Zhejiang Univ.*, **6** (8), 779 (2005).
14. G.F. Wang, Applied Elasticity. Chengdu University of Science and Technology Press, Chengdu, 1995.
15. S.R. Ahmed, A.B.M. Idris, M.W. Uddin, *Comput. Struct.*, **61** (1), 21 (1996).
16. G.F. Wang, *J. Sichuan Univ. (Eng. Sci. Ed.)*, **32** (2), 8 (2000).
17. Q.X. Huang, *J. China U. Min. Technol.*, **34** (3), 289 (2005).
18. J.L. Xu, J.F. Ju, *Chin. J. Rock Mech. Eng.*, **30** (8), 1547 (2011).
19. H.W. Liu, Material Mechanics. Higher Education Press, Beijing, 2004.
20. Z.L. Xu, Elastic Mechanics. Higher Education Press, Beijing, 2006.