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Effect of heat absorption on Cu-water based magneto-nanofluid over an impulsively moving ramped temperature plate

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The aim of the present paper is to explore the effects of heat absorption on a transient free convective boundary-layer flow of an electrically conducting, viscous and incompressible magneto-nanofluid over an impulsively moving vertical ramped temperature plate. Water-based nanofluids with added nanoparticles of titanium oxide, aluminium oxide and copper are taken into account. The mathematical model of the problem is obtained using the model of nanoparticle volume fraction. The governing model is solved analytically by making use of Laplace transform technique. The expressions for nanofluid velocity, nanofluid temperature, skin friction and Nusselt number were obtained for both the cases of ramped and isothermal conditions. The effects of various physical parameters on the nanofluid velocity and nanofluid temperature are shown by various graphs whereas the numerical values of skin friction and Nusselt number are presented in tables. The numerical results of the problem were compared for both ramped and isothermal conditions and it is observed that the numerical values in case of isothermal conditions are lower in magnitude as compared to ramped conditions.

Keywords: Nanofluid, Free convection, Heat absorption, Ramped temperature

INTRODUCTION

The study of nanofluids has attracted several researchers due to their huge applications in real life problems and industries. Initially, the term nanofluid, pioneered by Choi [1], refers to a suspension of nanoparticles having diameter less than 100 nm in a base fluid such as water, ethylene glycol, etc. Normally, conventional base fluids do not have an adequate thermal conductivity for many practical applications. Therefore, to augment the thermal conductivity of the base fluid, metal nanoparticles having higher thermal conductivity than the conventional base fluid are added to the fluid. Nanofluids being a mixture of nanoparticles and a base fluid are an innovative type of energy transport fluid and their novel property makes them tremendously useful in different processes of heat transfer including microelectronics, automobiles, hybrid-powered engines, fuel cells, domestic refrigerator, nuclear reactor coolants, pharmaceutical processes, etc. Choi [1] was the first who pointed out that the thermal conductivity of a base fluid can be radically improved by uniform dispersion of nano-sized particles into a fluid. This concept prompted many researchers towards nanofluids, and abundant studies, analyzing the thermal properties of nanofluids, have been worked out. Keblinski *et al*. [2] explored the possible mechanism for the augmentation of thermal conductivity of the fluid by a uniform suspension of nano-sized particles in the fluid. Buongiorno [3] proposed a non-homogeneous equilibrium model which states that the thermal conductivity of a fluid can be significantly enhanced due to the presence of Brownian diffusion and thermophoretic diffusion effects of nanoparticles. Remarkable research studies reporting the improvement in the thermal conductivity of fluids due to a suspension of nanoparticles and its frequent applications have been presented by Jang and Choi [4], Daungthongsuk and Wongwises [5], Seyyedi *et al*. [6], Rashidi *et al*. [7], Garoosi *et al*. [8,9] and Malvandi and Ganji [10,11].

The hydromagnetic nanofluids possess both liquid and magnetic characteristics and are known to have enthralling relevance to magneto-optical wavelength filters, ink float separation, optical switches, optical gratings, nonlinear optical materials, etc. The magneto-nanofluids have wide applications in drug delivery for cancer treatment as they play a significant role in guiding the drug particles up the blood stream to a tumor because the magnetic nanoparticles are known to be more adhesive to tumor cells than to non-malignant cells. The magneto-nanofluids are also known to have applications in the treatment of hyperthermia, contrast enhancement in magnetic resonance imaging and magnetic cell separation. Following this, Sheikholeslami *et al*. [12-14] reported their

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investigations on the problems of MHD convective flow of nanofluids considering different geometries and configurations. Sheikholeslami and Ganji [15] theoretically analyzed the problem of hydromagnetic flow in a permeable channel infused with nanofluid. They concluded that due to the increasing values of Reynolds number and nanoparticle volume fraction the nanofluid velocity boundary-layer thickness shrinks whereas it amplifies with the augment of Hartmann number. Recently, Hayat *et al*. [16] examined the unsteady hydromagnetic two-dimensional squeezing flow of nanofluids confined between two parallel walls under the influence of Brownian motion and thermophoresis effects. More recently, Dhanai *et al*. [17] investigated the influence of thermal slip on hydromagnetic mixed convective flow of nanofluid with heat transfer along an inclined cylinder taking into account the effects of thermophoresis, Brownian motion and viscous dissipation. They concluded that an increase in mass transfer parameter leads to a raise in the rate of heat transfer whereas it reduces due to thermal slip parameter. The heat generation/absorption has an important impact on the heat transfer characteristics in the various physical phenomena involved in the industry such as dissociating fluids in packed-bed reactors, post accident heat removal, fire and combustion, underground disposal of radioactive waste material, storage of food stuffs, etc. This encouraged many researchers to undertake the investigation of hydromagnetic convective flow over bodies with different geometries under the influence of heat generation/absorption. Some relevant studies dealing with the influence of heat generation or absorption have been reported by Acharya and Goldstein [18], Vajravelu and Nayfeh [19], Chamkha [20], Leea *et al*. [21], Sheikh and Abbas [22], Mehmood and Saleem [23], Mondal *et al*. [24] and Nandkyeolyar *et al*. [25]. Hamad and Pop [26] investigated the unsteady free convective nanofluid flow over an oscillatory moving vertical permeable flat plate under the influence of constant heat source and magnetic field in a rotating frame of reference. Chamkha and Aly [27] studied the two-dimensional steady hydromagnetic free convective boundary-layer nanofluid flow of an incompressible pure base fluid suspended with nanoparticles over semi-infinite vertical permeable plate in the presence of magnetic field, heat generation or absorption, thermophoresis and Brownian diffusion effects. The analysis of MHD free convective flow of Al_2O_3 -water based nanofluid in an open cavity considering uniform thermal boundary condition in the presence of uniform heat absorption/generation was performed by Mahmoudi *et al*. [28].

In all the aforesaid studies, the solutions were analyzed by considering simplified conditions, where velocity and temperature at the plate are continuous and defined. But, numerous problems of practical interest require the velocity and temperature to satisfy non-uniform, discontinuous or arbitrary conditions at the plate. Following this, several researchers, namely, Chandran *et al*. [29], Seth *et al*. [30-32], Nandkyeolyar *et al*. [33,34] and Hussain *et al*. [35] studied the problems of convective flow past a moving plate considering the ramped temperature. Khalid *et al*. [36] obtained the exact solution for a natural convective flow of a nanofluid past an oscillating moving vertical plate with ramped temperature, using the Laplace transform technique. Hussain *et al*. [37] explored the combined effects of Hall current and rotation on a natural convective flow with heat transfer over an accelerated moving ramped temperature in the presence of heat absorption and homogenous chemical reaction employing the Laplace transform technique. Recently, Hussain *et al*. [38,39] discussed the impact of thermal radiation on the magneto free convective nanofluid over a moving uniformly accelerated ramped temperature plate with and without considering Hall effects. Subsequently, Sharma *et al*. [40] extended this problem in a rotating medium by making use of Laplace transform technique.

Objective of the present research work is to analyze the consequence of heat absorption on the natural convective flow of an incompressible, viscous and electrically conducting magneto-nanofluid over an impulsively moving ramped temperature plate. It is expected that the present findings will be useful in biological and physical sciences, transportation, electronics cooling, environment and national security.

MATHEMATICAL ANALYSIS

Formulation of problem and its solution

In the present study, we considered unsteady hydromagnetic natural convective flow of an electrically conducting, viscous and incompressible nanofluid over an impulsive moving vertical infinite plate. The coordinate system is chosen as follows: x' -axis is considered along the length of the plate in upward direction and y' -axis is normal to the plane of plate. A uniform transverse magnetic field B_0 is applied in a direction which is parallel to y' -axis.

Table 1. Thermophysical properties of base fluid and nanoparticles [38]

	ρ (kg/m^3)	c_p (J/kgK)	k (W/mK)	$\beta\times10^5$ (K^{-1})	ϕ	σ (S/m)
Water (Base fluid)	997.1	4179	0.613	21	0.00	5.5×10^{-6}
Cu (Copper)	8933	385	401	1.67	0.05	59.6×10^{6}
Al_2O_3 (Alumina)	3970	765	40	0.85	0.15	35×10^{6}
TiO_2 (Titanium oxide)	4250	686.2	8.9538	0.90	0.20	2.6×10^{6}

The fluid and plate are at rest and are maintained at a uniform temperature α'_∞ at time $t'\le 0$. At time $t'>0$, the plate starts moving in x'-direction with uniform velocity U_0 in its own plane. The temperature of the plate is raised or lowered to $\alpha'_\infty+(\alpha'_w-\alpha'_\infty)t'/t_0$ when $0<t'\le t_0$ and it is maintained at uniform temperature α'_w when $t'>t_0$ (t_0 being characteristic time). The schematic diagram of the model is presented in Fig. 1. The water-based nanofluid is considered which contain three types of nanoparticles: Cu, Al_2O_3 and TiO_2. The nanoparticles are assumed to have uniform shape and size. Moreover, it is also assumed that both the base fluid and the nanoparticles are in thermal equilibrium state and no slip takes place between them. The thermophysical properties of base fluid and nanoparticles are given in Table 1. The plate is considered to be of infinite extent in x' and z' directions and is electrically non-conducting so all physical quantities except pressure are functions of y' and t' only. Since the magnetic Reynolds number of the flow is taken to be very small, the induced magnetic field is neglected so that the magnetic field $\vec{B}\equiv(0, B_0, 0)$. No electric field is applied, so the electric field due to polarization of charges is negligible; that is, $\vec{E}\equiv(0,0,0)$. This corresponds to the case where no energy is added or extracted from the fluid by electrical means.

Under the assumptions made above and Boussinesq approximation, the governing equations for natural convective flow of an electrically conducting, viscous and incompressible magneto-nanofluid taking into account the effects of heat absorption are given by:

$$\rho_{nf}\frac{\partial U}{\partial t'}=\mu_{nf}\frac{\partial^2 U}{\partial y'^2}-\sigma_{nf}B_0^2U+g(\rho\beta)_{nf}(\alpha'-\alpha'_\infty), \quad (1)$$

$$\frac{\partial \alpha'}{\partial t'}=\frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2\alpha'}{\partial y'^2}-\frac{Q_0}{(\rho c_p)_{nf}}(\alpha'-\alpha'_\infty), \quad (2)$$

where

$U, \alpha', \rho_{nf}, \mu_{nf}, \sigma_{nf}, g, \beta_{nf}, (\rho c_p)_{nf}, Q_0$ and k_{nf} are: component of nanofluid velocity in x'-direction, temperature of the nanofluid, density of the nanofluid, dynamic viscosity of the nanofluid, electrical conductivity of the nanofluid, acceleration due to gravity, thermal expansion coefficient of the nanofluid, heat capacitance of the nanofluid, heat absorption coefficient and thermal conductivity of the nanofluid, respectively.

Initial and boundary conditions for the nanofluid flow problem are:

$$\left.\begin{aligned}
&U=0,\ \alpha'=\alpha'_\infty \ \text{ for } y'\ge 0 \text{ and } t'\le 0,\\
&U=U_0 \ \text{ at } y'=0 \text{ for } t'>0,\\
&\alpha'=\alpha'_\infty+(\alpha'_w-\alpha'_\infty)t'/t_0 \ \text{ at } y'=0 \text{ for } 0<t'\le t_0,\\
&\alpha'=\alpha'_w \ \text{ at } y'=0 \ \text{ for } t'>t_0,\\
&U\to 0,\ \alpha'\to\alpha'_\infty \ \text{ as } y'\to\infty \text{ for } t'>0.
\end{aligned}\right\} \quad (3)$$

For the nanofluids, the expressions for $\rho_{nf}, \mu_{nf}, \sigma_{nf}, (\rho\beta)_{nf}$ and $(\rho c_p)_{nf}$ are given as:

$$\left.\begin{aligned}
&\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\ \mu_{nf}=\mu_f(1-\phi)^{-2.5},\\
&(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s,\\
&(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s,\\
&\sigma_{nf}=\sigma_f\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right],\ \sigma=\frac{\sigma_s}{\sigma_f},
\end{aligned}\right\} \quad (4)$$

where

$\phi, \rho_f, \rho_s, \mu_f, \beta_f, \beta_s, (\rho c_p)_f, (\rho c_p)_s, \sigma_f$ and σ_s are: solid volume fraction of nanoparticle, density of the base fluid, density of the nanoparticle, viscosity of the base fluid, thermal expansion coefficient of the base fluid, thermal expansion coefficient of the nanoparticle, heat capacitance of the base fluid, heat capacitance of the nanoparticle, electrical conductivity of the base fluid and electrical conductivity of the nanoparticle, respectively. The expressions presented in equation (4) are limited to spherical nanoparticles, and are not valid for other shapes of nanoparticles. The model for effective thermal conductivity of the nanofluid, i.e., k_{nf} for the spherical nanoparticles, given by Hamilton and Crosser model followed by Oztop and Abu-Nada [38], is expressed as:

$$k_{nf}=k_f\left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right], \quad (5)$$

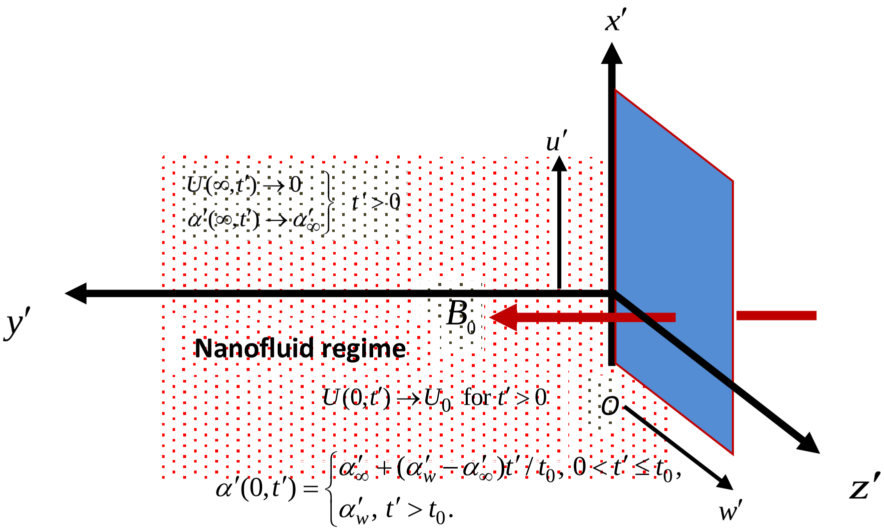

Fig. 1. Schematic diagram of the model

where k_f is the thermal conductivity of the base fluid and k_s is the thermal conductivity of the nanoparticles.

In order to convert the equations (1)-(3) in dimensionless form, now we introduce the following non-dimensional variables and parameters:

$$y=\frac{y'}{U_0t_0},u=\frac{U}{U_0},t=\frac{t'}{t_0},\alpha=\frac{\alpha'-\alpha'_\infty}{\alpha'_w-\alpha'_\infty}. \tag{6}$$

Equations (1) and (2), in dimensionless form, reduce to:

$$\frac{\partial u}{\partial t}=K_1\frac{\partial^2 u}{\partial y^2}-K_2M^2u+K_3G_r\alpha, \tag{7}$$

$$\frac{\partial \alpha}{\partial t}=\frac{1}{K_5}\frac{\partial^2\alpha}{\partial y^2}-K_6\alpha, \tag{8}$$

where,

$$\left.\begin{aligned}
&\lambda_1=\left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right],\ \lambda_2=\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right],\ \sigma=\frac{\sigma_s}{\sigma_f},\\
&\lambda_3=\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right],\lambda_4=\left[(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right],Q=\frac{\upsilon_fQ_0}{(\rho c_p)_fU_0{}^2}\\
&K_1=\frac{1}{(1-\phi)^{2.5}\lambda_1},K_2=\frac{\lambda_2}{\lambda_1},\ K_3=\frac{\lambda_3}{\lambda_1},\ K_4=\frac{k_{nf}}{k_f},\ K_5=\frac{P_r}{K_4},K_6=\frac{Q}{\lambda_4}\\
&M=\frac{\sigma_fB_0^2\upsilon_f}{\rho_fU_0^2},G_r=\left[\frac{g\beta_f\upsilon_f(\alpha'_w-\alpha'_\infty)}{U_0^3}\right],\quad P_r=\frac{(\rho\upsilon c_p)_f}{k_f}.
\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}
&u=0,\ \alpha=0\ \text{for}\ y\ge 0\ \text{and}\ t\le 0,\\
&u=1\ \text{at}\ y=0\ \text{for}\ t>0,\\
&\alpha=t\ \text{at}\ y=0\ \text{for}\ 0<t\le 1,\\
&\alpha=1\ \text{at}\ y=0\ \text{for}\ t>1,\\
&u\to 0,\ \alpha\to 0\ \text{as}\ y\to\infty\ \text{for}\ t>0.
\end{aligned}\right\} \tag{10}$$

The characteristic time t_0 may be defined according to the non-dimensional process mentioned above as $t_0=\upsilon_f/U_0^2$, where U_0 is characteristic velocity.

The initial and boundary conditions (3) in dimensionless form reduce to:

$$u(y,t)=\frac{1}{2}g(y,t)+\lambda_3\left[f_1(y,t)-H(t-1)f_1(y,t-1)\right], \tag{11}$$

$$\alpha(y,t)=\alpha_1(y,t)-H(t-1)\ \alpha_1(y,t-1), \tag{12}$$

where,

$$g(y,t)=\left[e^{y\sqrt{\lambda_1\lambda_2}}erfc\left(\sqrt{\lambda_2t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)+e^{-y\sqrt{\lambda_1\lambda_2}}erfc\left(-\sqrt{\lambda_2t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)\right], \tag{13}$$

$$\alpha_1(y,t)=\frac{1}{2}\left[\left(t+\frac{y}{2}\sqrt{\frac{K_5}{K_6}}\right)e^{y\sqrt{K_5K_6}}erfc\left(\sqrt{K_6t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)+\left(t-\frac{y}{2}\sqrt{\frac{K_5}{K_6}}\right)e^{-y\sqrt{K_5K_6}}erfc\left(-\sqrt{K_6t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)\right], \tag{14}$$

$$f_1(y,t)=\frac{e^{\lambda_4 t}}{2\lambda_4{}^2}\Big[e^{y\sqrt{\lambda_1(\lambda_2+\lambda_4)}}erfc\left(\sqrt{(\lambda_2+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)+e^{-y\sqrt{\lambda_1(\lambda_2+\lambda_4)}}erf\left(-\sqrt{(\lambda_2+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)$$

$$-e^{y\sqrt{K_5(K_6+\lambda_4)}}erf\,(\sqrt{(K_6+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}-e^{-y\sqrt{K_5(K_6+\lambda_4)}}erf\,(-\sqrt{(K_6+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\Big]$$

$$-\frac{1}{2\lambda_4}\left[\left\{\frac{1}{\lambda_4}+\left(t+\frac{y}{2}\sqrt{\frac{\lambda_1}{\lambda_2}}\right)\right\}\left\{e^{y\sqrt{\lambda_1\lambda_2}}erf\left(\sqrt{\lambda_2 t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)\right\}\right.$$

$$+\left\{\frac{1}{\lambda_4}+\left(t-\frac{y}{2}\sqrt{\frac{\lambda_1}{\lambda_2}}\right)\right\}\left\{e^{-y\sqrt{\lambda_1\lambda_2}}erf\left(-\sqrt{\lambda_2 t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)\right\}$$

$$-\left\{\frac{1}{\lambda_4}+\left(t+\frac{y}{2}\sqrt{\frac{K_5}{K_6}}\right)\right\}\left\{e^{y\sqrt{K_5K_6}}erf\left(\sqrt{K_6 t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)\right\}$$

$$\left.-\left\{\frac{1}{\lambda_4}+\left(t-\frac{y}{2}\sqrt{\frac{K_5}{6K_2}}\right)\right\}\left\{e^{-y\sqrt{K_5K_6}}erf\left(-\sqrt{K_6 t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)\right\}\right]. \tag{15}$$

Here $H(t-1)$ and $erfc(x)$ are Heaviside step and complementary error functions, respectively.

Solution for the case of isothermal plate

The expressions (11) to (15) represent the solution for nanofluid velocity and temperature for the time-dependent natural convective flow of electrically conducting, viscous and incompressible magneto-nanofluids over an impulsively moving vertical ramped temperature plate under the influence of heat absorption. In order to analyze the effect of ramped temperature on the flow-field, it is worthwhile to compare such a flow with the one near an impulsively moving vertical uniform temperature plate. Owing to the assumptions made in the initial and boundary condition (10), the equations (7) and (8) are solved analytically with the help of Laplace transform technique and the exact solutions for nanofluid velocity $u(y,t)$ and nanofluid temperature $\alpha(y,t)$ are obtained as aforesaid, the solutions for nanofluid velocity and temperature for natural convective magneto-nanofluids flow over an impulsively moving vertical plate with isothermal condition are obtained and are expressed in the following forms:

$$u(y,t)=\frac{1}{2}\left[e^{y\sqrt{\lambda_1\lambda_2}}erfc\left(\sqrt{\lambda_2 t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)+e^{-y\sqrt{\lambda_1\lambda_2}}erfc\left(-\sqrt{\lambda_2 t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)\right]$$

$$+\frac{\lambda_3}{\lambda_4}\left[\frac{e^{\lambda_4 t}}{2}\left\{e^{y\sqrt{\lambda_1(\lambda_2+\lambda_4)}}erfc\left(\sqrt{(\lambda_2+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)+e^{-y\sqrt{\lambda_1(\lambda_2+\lambda_4)}}erfc\left(-\sqrt{(\lambda_2+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)\right.\right.$$

$$\left.\left.-e^{y\sqrt{K_5(K_6+\lambda_4)}}erfc\left(\sqrt{(K_6+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)-e^{-y\sqrt{K_5(K_6+\lambda_4)}}erfc\left(-\sqrt{(K_6+\lambda_4)t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)\right\}\right]$$

$$-\frac{\lambda_3}{\lambda_4}\left[\frac{1}{2}\left\{e^{y\sqrt{\lambda_1\lambda_2}}erfc\left(\sqrt{\lambda_2 t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)+e^{-y\sqrt{\lambda_1\lambda_2}}erfc\left(-\sqrt{\lambda_2 t}+\frac{y}{2}\sqrt{\frac{\lambda_1}{t}}\right)\right.\right.$$

$$\left.\left.-e^{y\sqrt{K_5K_6}}erfc\left(\sqrt{K_6 t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)+e^{-y\sqrt{K_5K_6}}erfc\left(-\sqrt{K_6 t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)\right\}\right], \tag{16}$$

$$\alpha(y,t)=\frac{1}{2}\left[e^{y\sqrt{K_5K_6}}erfc\left(\sqrt{K_6 t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)+e^{-y\sqrt{K_5K_6}}erfc\left(-\sqrt{K_6 t}+\frac{y}{2}\sqrt{\frac{K_5}{t}}\right)\right]. \tag{17}$$

Skin friction and Nusselt number

The expressions for Nusselt number *Nu*, which measures the rate of heat transfer at the plate and skin friction τ, which measures the shear stress at the plate are presented in the following forms for both ramped and isothermal conditions:

For a ramped temperature plate:

$$Nu=-\frac{1}{2}\left[\left(\sqrt{\frac{K_5}{K_6}}+2t\sqrt{K_5K_6}\right)\left\{erfc(\sqrt{K_6t})-1\right\}-2\sqrt{\frac{K_5}{t\pi}}\,e^{-K_6t}\right]\quad-\frac{1}{2}H(t-1)\left[\left(\sqrt{\frac{K_5}{K_6}}+2(t-1)\sqrt{K_5K_6}\right)\left\{erfc(\sqrt{K_6(t-1)})-1\right\}-2\sqrt{\frac{K_5}{(t-1)\pi}}\,e^{-K_6(t-1)}\right], \tag{18}$$

$$\tau=\sqrt{a_1a_2}\left\{erfc(\sqrt{a_2t})-1\right\}-\sqrt{\frac{a_1}{t\pi}}\,e^{-a_2t}+a_3\left[f_2(t)-H(t-1)\,f_2(t-1)\right], \tag{19}$$

$$f_2(t)=\frac{e^{a_4t}}{a_4}\Bigg[\sqrt{a_1(a_2+a_4)}\left\{erfc\left(\sqrt{(a_2+a_4)t}\right)-1\right\}-\sqrt{K_5(K_6+a_4)}\left\{erfc\left(\sqrt{(K_6+a_4)t}\right)-1\right\}$$
$$-\sqrt{\frac{a_1}{t\pi}}\,e^{-(\lambda_2+\lambda_4)t}+\sqrt{\frac{K_5}{t\pi}}\,e^{-(K_6+a_4)t}\Bigg]$$

where,

$$-\frac{1}{2a_4}\Bigg[\sqrt{\frac{a_1}{a_2}}\left\{erfc\left(\sqrt{a_2t}\right)-1\right\}+\left(\frac{1}{a_4}+t\right)\sqrt{a_1a_2}\,2\left\{erfc\left(\sqrt{a_2t}\right)-1\right\}-\left(\frac{1}{a_4}+t\right)\sqrt{\frac{a_1}{t\pi}}\,2e^{-a_2t}$$
$$+\sqrt{\frac{K_5}{K_6}}\left\{erfc\left(\sqrt{K_6t}\right)-1\right\}+\left(\frac{1}{a_4}+t\right)\sqrt{K_5K_6}\,2\left\{erfc\left(\sqrt{K_6t}\right)-1\right\}-\left(\frac{1}{a_4}+t\right)\sqrt{\frac{K_5}{t\pi}}\,2e^{-K_6t}\Bigg],$$

$$a_1=\frac{1}{K_1},\ a_2=MK_2,\ a_3=\frac{G_r\cdot a_1\cdot K_3}{K_5-a_1}\ \text{and}\ a_4=\frac{K_5\cdot K_6+a_1\cdot a_2}{K_5-a_1}.$$

For an isothermal temperature plate,

$$Nu=\sqrt{\frac{K_5}{t\pi}}\,e^{-K_6t}-\sqrt{K_5K_6}\left\{erfc(\sqrt{K_6t})-1\right\}, \tag{20}$$

$$\tau=\left[\sqrt{a_1a_2}\left\{erfc\left(\sqrt{a_2t}\right)-1\right\}-\sqrt{\frac{a_1}{t\pi}}\,e^{-a_2t}\right]+\frac{a_3}{a_4}\Bigg[e^{a_4t}\Bigg\{\sqrt{a_1(a_2+a_4)}\left\{erfc\left(\sqrt{(a_2+a_4)t}\right)-1\right\}$$
$$-\sqrt{\frac{a_1}{t\pi}}\,e^{-(a_2+a_4)t}-\sqrt{K_5(K_6+a_4)}\left\{erfc\left(\sqrt{(K_6+a_4)t}\right)-1\right\}-\sqrt{\frac{K_5}{t\pi}}\,e^{-(K_6+a_4)t}\Bigg\}\Bigg]$$
$$-\frac{a_3}{a_4}\left[\sqrt{a_1a_2}\left\{erfc\left(\sqrt{a_2t}\right)-1\right\}-\sqrt{\frac{a_1}{t\pi}}\,e^{-a_2t}-\sqrt{K_5K_6}\left\{erfc\left(\sqrt{K_6t}\right)-1\right\}+\sqrt{\frac{K_5}{t\pi}}\,e^{-K_6t}\right]. \tag{21}$$

RESULTS AND DISCUSSION

In order to emphasize the effects of various physical parameters on the flow-fluid, the numerical computations were performed and numerical results for the nanofluid velocity and temperature were described with the help of various graphs. For the engineering aspects, the numerical values of skin friction and Nusselt number are presented in different tables. Three distinct types of water-based nanofluids with nanoparticles of copper (Cu), aluminum oxide (Al_2O_3) and titanium oxide (TiO_2) were taken into account. The numerical values of copper-water based nanofluid velocity $u(y,t)$, computed from the analytical solutions reported in this paper are shown by different graphs against the boundary layer coordinate y in Figs. 2-6 for different values of magnetic parameter M, heat absorption parameter Q, Grashof number G_r, nanoparticle volume fraction ϕ and time t taking Prandtl number $P_r=6.2$. The values of ϕ are considered in the range $0<\phi\le 0.25$. In addition, the spherical nanoparticles with thermal conductivity and their dynamic viscosity are given in Table 1. The values of other physical parameters are shown in the respective figures. It can be observed from Figs. 2 and 3 that the nanofluid velocity gets reduced due to reduced magnetic and heat absorption parameters M and Q, respectively. This may be endorsed to the fact that the existence of magnetic field in the presence of an electrically conducting nanofluid generates a resistive type of body force, termed as Lorentz force which has a tendency to impede the motion of a fluid in the boundary layer region. Fig. 4 shows that augmentation in G_r results in significant rise in the nanofluid velocity which is persistent to the fact that the Grashof number G_r behaves as a gratifying pressure gradient which accelerates the nanofluid velocity in the entire boundary layer region. It can be seen from Fig. 5 that the volume fraction of nanoparticle ϕ slows down the nanofluid velocity near the proximity of the plate while it has an annulment consequence away from the moving plate.

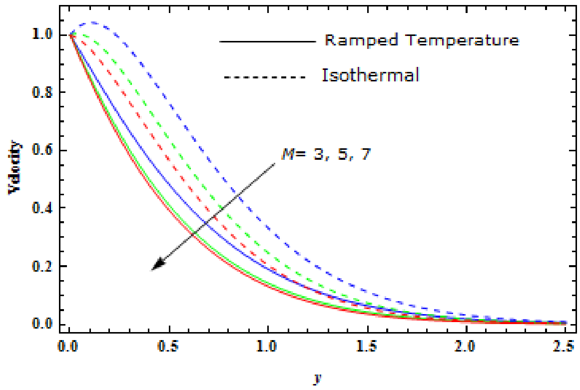

Fig. 2. Effect of *M* on velocity profiles

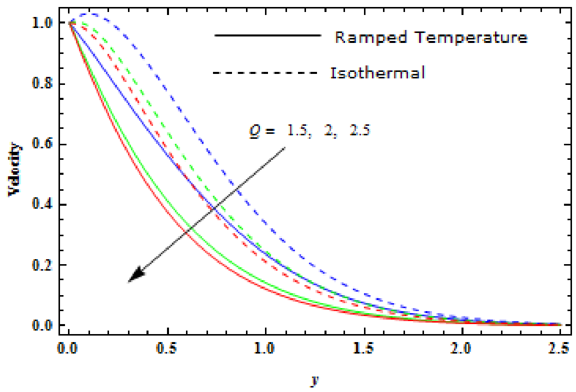

Fig. 3. Effect of Q on velocity profiles.

This happens due to the reason that the increasing value of volume fraction of nanoparticle reduces the thermal conductivity of the fluid, which in turn causes the thickness of the boundary layer to reduce and the viscosity to increase, thereby reducing the nanofluid velocity in the proximity of the moving plate. Fig. 6 reveals that as time passes, the fluid velocity gets accelerated. The consequences of heat absorption parameter Q, nanoparticle volume fraction ϕ and time t on copper-water based temperature profiles are shown in Figs. 7-9, taking Prandtl number $P_r = 6.2$. In Fig. 7 it is observed that as heat absorption parameter gradually increases, a significant reduction results in the nanofluid temperature for both ramped and isothermal conditions. From Figs. 8 and 9 it is evident that the nanofluid temperature rises with the increase of nanoparticle volume fraction ϕ and time t for both ramped and isothermal conditions. Physically, it is described as the nanoparticles volume fraction has a tendency to raise the nanofluid temperature throughout the boundary layer region; also nanofluid temperature gets increased with the progress of time for both ramped and isothermal cases.

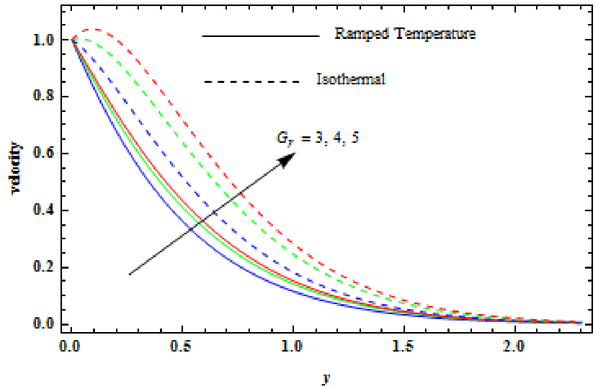

Fig. 4. Effect of G_r on velocity profiles.

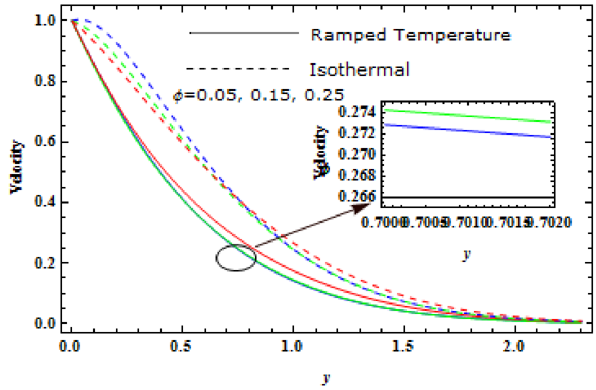

Fig. 5. Effect of ϕ on velocity profiles.

The comparison of nanofluid temperature profiles in the case of isothermal and ramped conditions for different nanofluids with nanoparticles of Cu, Al_2O_3 and TiO_2 is depicted in Fig. 10. It is seen from Fig. 10 that for both isothermal and ramped conditions, the temperature of TiO_2-water based nanofluid is higher in magnitude followed by the temperature of Al_2O_3-water and Cu-water based nanofluids.

The numerical values of skin friction τ for both cases of ramped and isothermal temperature plates considering the copper-water based nanofluid, evaluated from equations (19) and (21) are given in Table 2 for different values of M, G_r, ϕ, Q and t keeping Prandtl number $P_r = 6.2$.

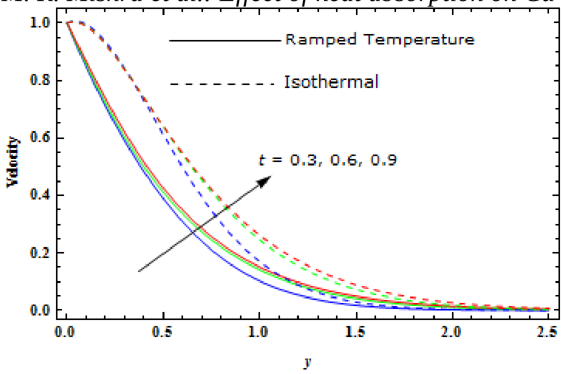

Fig. 6. Effect of *t* on velocity profiles.

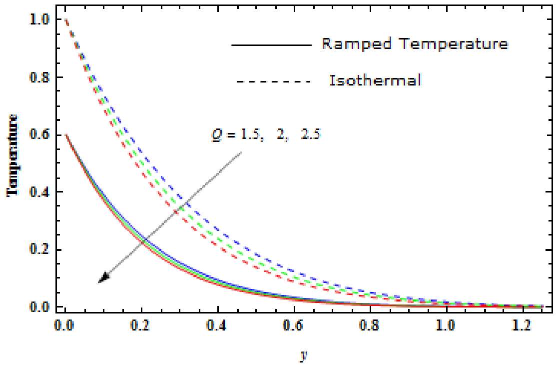

Fig. 7. Effect of Q on temperature profiles.

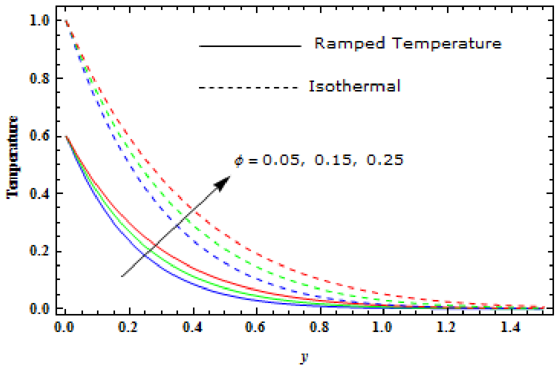

Fig. 8. Effect of ϕ on velocity profiles.

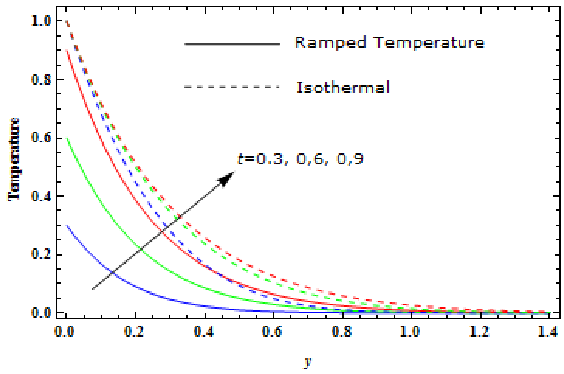

Fig. 9. Effect of *t* on temperature profiles.

It follows from Table 2 that the shear stress increases due to the increase of M, G_r, ϕ, Q and t. This infers that magnetic field, thermal buoyancy force, volume fraction of nanoparticle, heat absorption parameter have tendency to increase the shear stress at the plate, also it gets augmented as time passes for both ramped and isothermal conditions.

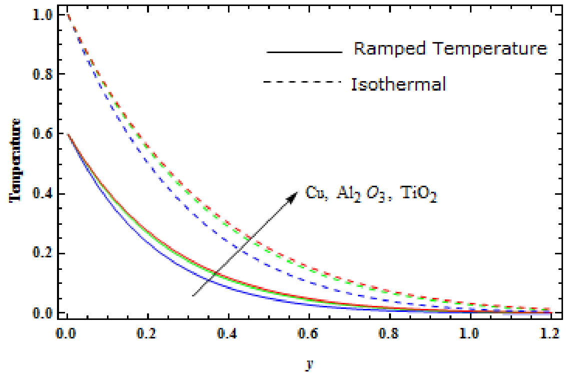

Fig. 10. Comparison of temperature profiles for different nanofluids.

It is reckoned from Table 3 that the Nusselt number *Nu* for both ramped and isothermal conditions, increases for increasing value of Q and decreases with the increase of ϕ, whereas for the ramped case it increases and decreases for isothermal case with the increase in *t*. This suggests that for both cases of ramped and isothermal conditions, the rate of heat transfer at the plate gets raised with the augmentation of heat absorption parameter whereas the volume fraction of nanoparticles has an adverse effect on it. As time passes the rate of heat transfer gets improved for the ramped case whereas it gets reduced for the isothermal case.

Validation of the obtained results

The obtained results were validated by comparing the nanofluid velocity profiles with the results obtained by Khalid *et al*. [36] considering similar assumptions but in the absence of magnetic field and heat absorption. This comparison demonstrates an excellent conformity of our results as it is revealed from Fig. 11.

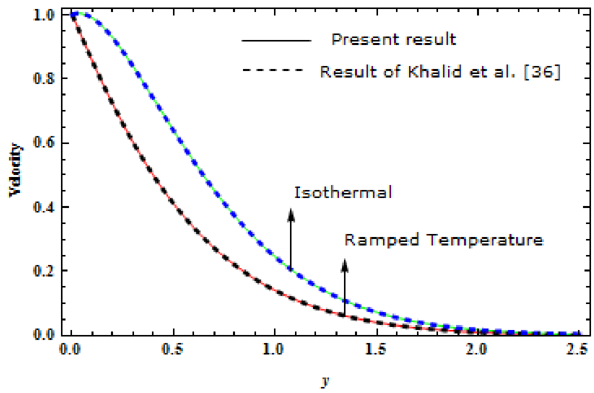

Fig. 11. Comparison of nanofluid velocity profiles in the present work considering *M=Q=0* with those obtained by Khalid *et al.* [36] when ω =0

Table 2. Skin friction τ for Cu-water based nanofluid when $P_r = 6.2$

M	G_r	ϕ	Q	t	τ for ramped temperature	τ for isothermal plate
3	4	0.05	2	0.6	6.15463	5.80615
5	4	0.05	2	0.6	6.29641	6.14205
7	4	0.05	2	0.6	6.58365	6.58457
5	**3**	0.05	2	0.6	4.99342	4.82386
5	**4**	0.05	2	0.6	6.29641	6.14205
5	**5**	0.05	2	0.6	7.59940	7.46024
5	4	**0.05**	2	0.6	6.29641	6.14205
5	4	**0.15**	2	0.6	9.81332	9.37154
5	4	**0.25**	2	0.6	19.1638	18.7232
5	4	0.05	**1.5**	0.6	4.32815	4.35649
5	4	0.05	**2**	0.6	6.29641	6.14205
5	4	0.05	**2.5**	0.6	8.98351	8.71207
5	4	0.05	2	**0.3**	3.52347	2.84424
5	4	0.05	2	**0.6**	6.29641	6.14205
5	4	0.05	2	**0.9**	17.8309	17.7829

Table 3. Nusselt number Nm for Cu-water based nanofluid

ϕ	Q	T	Nu for ramped temperature	Nu for isothermal plate
0.05	2	0.6	2.94341	3.38195
0.15	2	0.6	2.54640	2.94062
0.25	2	0.6	2.21385	2.56973
0.05	**1.5**	0.6	2.84580	3.00798
0.05	**2**	0.6	2.94341	3.38195
0.05	**2.5**	0.6	3.05186	3.72975
0.05	2	**0.3**	2.60289	3.68166
0.05	2	**0.6**	2.94341	3.38195
0.05	2	**0.9**	3.76742	3.31097

CONCLUSIONS

The exact solution of the governing equations was obtained by using Laplace transform technique to explore the consequences of various relevant parameters on the natural convective flow of an incompressible, viscous and electrically conducting magneto-nanofluid over an impulsively moving ramped temperature plate. The important findings are summarized below for both the cases of ramped temperature and isothermal plates:

(i) The existence of magnetic field in the presence of an electrically conducting nanofluid generates a resistive type of body force, which has a tendency to impede the motion of fluid in the boundary layer region.

(ii) The thermal buoyancy force behaves as a gratifying pressure gradient which accelerates nanofluid velocity in the regime of boundary layer.

(iii) The increasing value of the volume fraction of nanoparticles reduces the thermal conductivity of fluid, which in turn reduces the thickness of the boundary layer and increases the viscosity of fluid and as result the nanofluid velocity gets reduced in the proximity of the moving plate. The nanofluid velocity gets accelerated with progress of time.

(iv) Heat absorption has a tendency to decrease the nanofluid temperature whereas the nanoparticle volume fraction has a tendency to enhance the nanofluid temperature. As time progresses the nanofluid temperature is enhanced.

(v) Magnetic field, thermal buoyancy force, volume fraction of nanoparticles and heat absorption parameter have a tendency to increase the shear stress at the plate.

(vi) The rate of heat transfer at the plate gets raised with the augmentation of heat absorption parameter whereas the volume fraction of nanoparticles has an adverse effect on it. As time passes, the rate of heat transfer gets improved for the ramped case whereas it gets reduced for the isothermal case.

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ВЛИЯНИЕ НА АБСОРБЦИЯТА НА ТОПЛИНА ВЪРХУ МЕД-СЪДЪРЖАЩ МАГНИТО-НАНОФЛУИД НАД ИМПУЛСНО ДВИЖЕЩА СЕ ПЛАСТИНА С ПРОМЕНЯЩА СЕ ТЕМПЕРАТУРА

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(Резюме)

Изследвано е влиянието на абсорбцията на топлина върху транзиентния свободен конвективен поток на граничния слой на електропроводима, вискозна и несвиваема магнито-нанотечност над импулсно движеща се вертикална пластина с променлива температура. Използвана е нанотечност на основата на вода с добавени наночастици от титанов оксид, алуминиев оксид и мед. Математическият модел е получен с използване на модела на обемната фракция на наночастиците. Управляващият модел е решен аналитично с помощта на трансформационната техника на Laplace. Изразите за скоростта и температурата на течността, повърхностното триене и числото на Nusselt са получени за случаите на променяща се и на постоянна температура. Влиянието на различните физични параметри върху скоростта на нанотечността е илюстрирано графично, а числените стойности на повърхностното триене и числото на Nusselt **с**а представени в таблици. Получените резултати при променяща се и при постоянна температура са сравнени и е установено, че стойностите при изотермни условия са по-ниски от тези при променлива температура.