Bulgarian Chemical Communications, Volume 50, Special Issue B, (pp. 110 – 114) 2018

Force line influences in a single static undetermined beam

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Received: July 24, 2017; Revised: November 12, 2017

General expressions for force line influences in a single static undetermined beam – bilateral fixed and beam fixed–joint pin are investigated. The ordinates of the line influences were obtained in an arbitrary section of the beam. The beam's elements with constructed lines influences are used for finding line influences in static undetermined systems with application of the displacement method.

Keywords: Single static undetermined beams, Line influences, Displacement method

SINGLE STATIC UNDETERMINED BEAMS - FORCE LINE INFLUENCES

The beams are with length l and constant rigidity EI. The moments and the shear forces are taken for positive, when turning a cutting beam clockwise. The basic relationships in the construction of Force line influences in the beams using the Force method are written in a general kind. The sections of the beam, in which the force influences are applied, are determined with the coordinates x_k, $x_k^I = l - x_k$. It is known that in static undetermined systems force line influences are linear curves. The ordinates of the constructed force line influences are determined in sections k, arbitrary in number and situation with the coordinates x_m, $x_m^I = l - x_m$.

Bilateral fixed beam

For a basic system a beam simply supported at the ends is chosen. The basic unknowns are the moments in the supported joints.

The matrix of displacements $[D] = [\delta_{ij}]$ is:

$$[D] = \frac{L}{3EI}\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}.$$

The corresponding matrix of the $[\beta]$ - numbers is:

$$[\beta] = [\beta_{ij}] = -[\delta_{ij}]^{-1} \qquad [\beta] = \frac{EI}{l}\begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix}.$$

In the chosen basic system the matrix influences for displacements of the applied point of each basic unknown X_i into its direction are presented as:

$$\{"D_f"\} = \begin{Bmatrix} "\Delta_{1f}" \\ "\Delta_{2f}" \end{Bmatrix},$$

$$\left| \begin{array}{l} \Delta_{1f} = \frac{1}{EI}\frac{l}{6} x_m^I \left(1 - \frac{x_m^{I\,2}}{l^2}\right) \\ \Delta_{2f} = \frac{1}{EI}\frac{l}{6} x_m \left(1 - \frac{x_m^2}{l^2}\right) \end{array} \right. ,$$

where m indicates the number of the beam's sections in which the ordinate from the $"\Delta_{if}"$ is determined.

The line influences for the basic unknowns are obtained according to the expressions:

$$\{"X"\} = [\beta]\{"D_f"\} \quad \left| \begin{array}{l} "X_1" = \beta_{11}"\Delta_{1f}" + \beta_{12}"\Delta_{2f}" \\ "X_2" = \beta_{21}"\Delta_{1f}" + \beta_{22}"\Delta_{2f}" \end{array} \right.$$

and after transformations:

$$\left| \begin{array}{l} X_1 = \frac{1}{3}\left[-2x_m^I \left(1 - \frac{x_m^{I\,2}}{l^2}\right) + x_m \left(1 - \frac{x_m^2}{l^2}\right) \right] \\ X_2 = \frac{1}{3}\left[x_m^I \left(1 - \frac{x_m^{I\,2}}{l^2}\right) - 2x_m \left(1 - \frac{x_m^2}{l^2}\right) \right] \end{array} \right. .$$

The reaction's and force line influences in the beam are generally obtained according to the principle of superposition:

$$"S_m" = "S_m^o" + \sum_i S_{m_i} "X_i".$$

The significations in the written expression are known.

For the forces they are:

$$\left| \begin{array}{l} "M_k" = "M_k^o" + \sum_i M_{k_i} "X_i" \\ "Q_k" = "Q_k^o" + \sum_i Q_{k_i} "X_i" \end{array} \right.$$

$$\left| \begin{array}{l} "M_k" = "M_k^o" + \frac{x_\kappa^I}{l}"X_1" + \frac{x_\kappa}{l}"X_2" \\ "Q_k" = "Q_k^o" - \frac{1}{l}"X_1" + \frac{1}{l}"X_2" \end{array} \right. .$$

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In an arbitrary section of the beam, determined with the coordinate x_k, the analytical expressions for line influences of reactions and forces are:

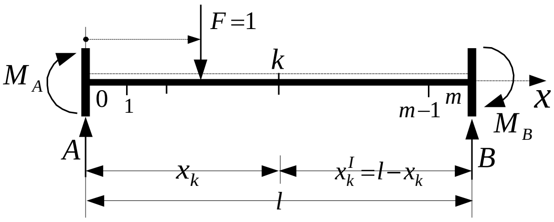


Fig. 1.

$$M_k = \begin{cases} \dfrac{x_k^I x_m}{l} + \dfrac{1}{3l}\left[x_m \left(1 - \dfrac{x_m{}^2}{l^2} \right)\left(x_k^I - 2x_k \right) + x_m^I \left(1 - \dfrac{x_m^{I\,2}}{l^2} \right)\left(x_k - 2x_k^I \right) \right] & 0 \le x_m \le x_k \\ \dfrac{x_k x_m^I}{l} + \dfrac{1}{3l}\left[x_m \left(1 - \dfrac{x_m{}^2}{l^2} \right)\left(x_k^I - 2x_k \right) + x_m^I \left(1 - \dfrac{x_m^{I\,2}}{l^2} \right)\left(x_k - 2x_k^I \right) \right] & x_k \le x_m \le l \end{cases}$$

$$, \quad Q_k = \begin{cases} -\dfrac{x_m}{l} - \dfrac{1}{l}\left[x_m \left(1 - \dfrac{x_m^2}{l^2} \right) - x_m^I \left(1 - \dfrac{x_m^{I\,2}}{l^2} \right) \right] & 0 \le x_m \le x_k \\ \dfrac{x_m^I}{l} - \dfrac{1}{l}\left[x_m \left(1 - \dfrac{x_m^2}{l^2} \right) - x_m^I \left(1 - \dfrac{x_m^{I\,2}}{l^2} \right) \right] & x_k \le x_m \le l \end{cases},$$

where m indicates the number of beam's sections in which the ordinate of line influences $"M_k", "Q_k"$ is determined.

The expressions for $"M_k", "Q_k"$ present the left and the right part of the line influences.

The ordinates of line influences for forces in a section with coordinates $x_k = 0.5l$, $x_k = 0.2l$, $x_k = 0.3l$ with $m = 10$ points of the beam are shown in Table 1.

The force line influences in the selected sections of the bilateral fixed beam are shown in Fig. 2.

Beam fixed- joint pin

The basic system is a simply supported beam. The basic unknown is the moment at the supported joint.

The matrix of deformation $[D] = [\delta_{ij}]$ is:

$$[D] = \frac{L}{3EI}[1].$$

The corresponding $[\beta]$ - matrix is:

$$[\beta] = [\beta_{ij}] = -[\delta_{ij}]^{-1} \qquad [\beta] = \frac{3EI}{l}[1].$$

The matrix influence on the displacements of the basic unknown's applied point X_i into its directions is:

$$["D_f"] = \{"\Delta_{1f}"\}, \quad \Delta_{1f} = \frac{1}{EI}\frac{l}{6}x_m^I \left(1 - \frac{x_m^{I\,2}}{l^2} \right).$$

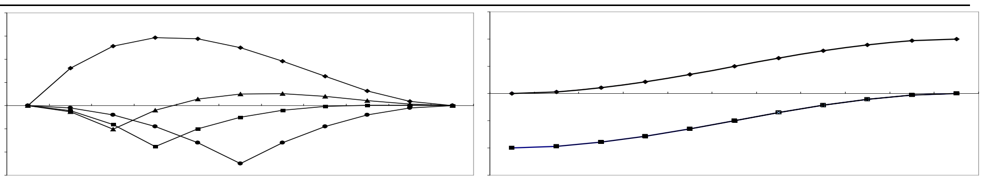
The line influences for the basic unknowns are:

$$\{"X"\} = [\beta]\{"D_f"\} \quad "X_1" = \beta_{11}"\Delta_{1f}",$$

$$X_1 = -\frac{x_m^I}{2}\left(1 - \frac{x_m^{I\,2}}{l^2} \right).$$

Table 1. The ordinates of line influences for forces in a section with coordinates *x*$_k$*=0.5l*;, *x*$_k$*=0.2l; x*$_k$*=0.3l*, with *m=10* points of the beam

m	0	1	2	3	4	5	6	7	8	9	10
						$x_k = 0,5l$					
M_{k_m}	0.0000	5.000e-3	0.0200	0.0450	0.0800	0.1250	0.0800	0.0450	0.0200	5.000e-3	0.0000
Q_{k_m}	0.0000	-0.0280	-0.1040	-0.2160	-0.3520	-0.5000 0.5000	0.3520	0.2160	0.1040	0.0280	0.0000
						$x_k = 0,2l$					
M_{k_m}	0.0000	0.0134	0.0512	0.0098	-0.0144	-0.025	-0.0256	-0.0198	-0.0112	-0.0034	0.0000
Q_{k_m}	0.0000	-0.0280	-0.104 0.8960	0.7844	0.648	0.5000	0,3520	0.2160	0.1040	0.0280	0.0000
						$x_k = 0,3l$					
M_{k_m}	0.0000	0.0106	0.0408	0.0882	0.0504	0.0250	0.0096	0.0018	-0.0008	-0.0006	0.0000
Q_{k_m}	0.0000	-0.0280	-0.0104	-0.216 0.7840	0.6498	0.5000	0.352	0.2160	0.1040	0.0280	0.0000



$"M_k"$ $\qquad$ $"Q_k"$

Fig. 2. The force line influences in the selected sections of the bilateral fixed beam

The force line influences in the beam are in general:

$$"S_k"="S_k^o"+\sum_i S_{k_i}"X_i"$$

$$\left|\begin{array}{l} "M_k"="M_k^o"+\sum_i M_{k_i}"X_i" \\ "Q_k"="Q_k^o"+\sum_i Q_{k_i}"X_i" \end{array}\right. .$$

Finally they become:

$$"M_k"="M_k^o"+\frac{x_\kappa^I}{l}"X_1"; \quad "Q_k"="Q_k^o"-\frac{1}{l}"X_1".$$

In the signified sections of the beam the ordinates of force lines influences are:

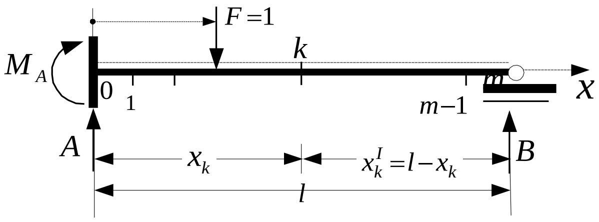


Fig. 3.

$$M_k=\left|\begin{array}{ll} \dfrac{x_k^I x_m}{l}+\dfrac{x_k^I x_m^I}{2l}\left(1-\dfrac{x_m^{I\,2}}{l^2}\right) & 0\le x_m\le x_k \\ \dfrac{x_k x_m^I}{l}+\dfrac{x_k^I x_m^I}{2l}\left(1-\dfrac{x_m^{I\,2}}{l^2}\right) & x_k\le x_m\le l \end{array}\right. ,$$

$$Q_k=\left|\begin{array}{ll} -\dfrac{x_m}{l}-\dfrac{1}{l}\left[x_m^I\left(1-\dfrac{x_m^{I\,2}}{l^2}\right)-x_m\left(1-\dfrac{x_m^2}{l^2}\right)\right] & 0\le x_m\le x_k \\ -\dfrac{x_m^I}{l}-\dfrac{1}{l}\left[x_m^I\left(1-\dfrac{x_m^{I\,2}}{l^2}\right)-x_m\left(1-\dfrac{x_m^2}{l^2}\right)\right] & x_k\le x_m\le l \end{array}\right.$$

where m indicates the number of beam's sections in which the ordinates of lines influences $"M_k","Q_k"$ are determined.

The expressions for $"M_k","Q_k"$ present the left and the right part of the line influences.

The ordinates of force line influences in the section with a coordinate $x_k=0{,}5l$, $x_k=0{,}2l$, $x_k=0{,}3l$ with $m=10$ points of the beam are shown in Table 2.

The force line influences in selected sections of the beam fixed-joint pin are shown in Fig. 4.

Line influences for the reactions in the beam's elements

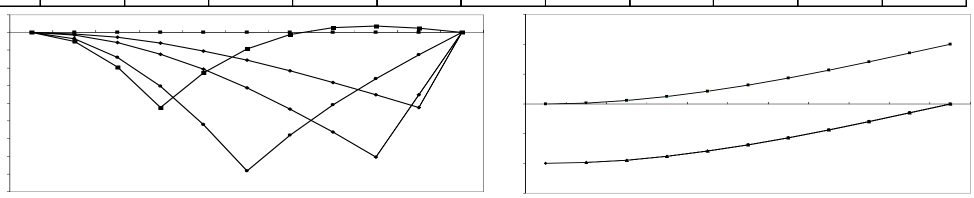
Reactions line influences in a beam's element can be obtained from the constructed force line influences and the investigated expressions.

Bilateral fixed beam

The investigated analytical expressions for the vertical reactions support are:

Table 2. The ordinates of line influences for forces in a section with coordinates *x_k=0.5l;*, *x_k=0.2l;* *x_k=0.3l,* with *m=10* points of the beam

m	0	1	2	3	4	5	6	7	8	9	10
	$x_k=0{,}5\ell$										
M_{k_m}	0.0000	0.0725	0.0280	0.06075	0.1040	0.15625	0.1160	0.08175	0.0520	0,02525	0.0000
Q_{k_m}	0.0000	-0.0145	-0.0560	-0.1215	-0.2080	$\frac{-0.3125}{0.6875}$	0.5680	0.4365	0.2960	0.1495	0.0000
	$x_k=0{,}2\ell$										
M_{k_m}	0.0000	0.0116	0.0448	-0,0028	-0,0336	-0,0500	-0,0544	-0,0492	-0,0368	-0,0196	0.0000
Q_{k_m}	0.0000	-0.0145	$\frac{-0.0560}{0.9440}$	0.8785	0.7920	0.6875	0.5680	0.4365	0.2960	0.1495	0.0000
	$x_k=0{,}3\ell$										
M_{k_m}	0.0000	0.01015	0.0392	0.08505	0.0456	0.01875	0.0024	-0.00555	-0.0072	-0.00465	0.0000
Q_{k_m}	0.0000	-0.0145	-0.0560	$\frac{-0.1215}{0.8785}$	0.7920	0.6875	0.5680	0.4365	0.2960	0.1495	0.0000



$"M_k"$ $\qquad\qquad$ $"Q_k"$

Fig. 4. The force line influences in selected sections of the beam fixed-joint pin

$$\left|\begin{array}{l} "A_v"="A_v^0"+\sum_i A_{v_i}"X_i" \\ "B_v"="B_v^0"+\sum_i B_{v_i}"X_i" \end{array}\right.$$

$$\left|\begin{array}{l} "A_v"="A_v^0"-\frac{1}{l}"X_1"+\frac{1}{l}"X_2" \\ "B_v"="B_v^0"+\frac{1}{l}"X_1"-\frac{1}{l}"X_2" \end{array}\right. .$$

After substitution the coordinates of sections support A - $x_k=0\,, x_k^I=l$, respectively B - $x_k=l\,, x_k^I=0$, the expressions for moments in the supports become:

$$"M_A"="X_1" \qquad "M_B"=-"X_2" .$$

In the signified sections of the beam the ordinates of reaction' s lines influences are:

$$A=\frac{x_m^I}{l}-\frac{x_m}{l}\left(1-\frac{x_m^2}{l^2}\right)+\frac{x_m^I}{l}\left(1-\frac{x_m^2}{l^2}\right),$$

$$B=\frac{x_m}{l}+\frac{x_m}{l}\left(1-\frac{x_m^2}{l^2}\right)-\frac{x_m^I}{l}\left(1-\frac{x_m^2}{l^2}\right);$$

$$M_A=\frac{1}{3}\left[-2x_m^I\left(1-\frac{x_m^{I\,2}}{l^2}\right)+x_m\left(1-\frac{x_m^2}{l^2}\right)\right],$$

$$M_A=\frac{1}{3}\left[-2x_m\left(1-\frac{x_m^2}{l^2}\right)+x_m^I\left(1-\frac{x_m^{I\,2}}{l^2}\right)\right].$$

The ordinates of reaction's line influences at $m=10$ points of the beam are shown in Table 3.

The ordinates of the reaction's line influences at the right end of the beam can be written mirrorly, coordinated with the accepted positive directions of the support's reactions.

Beam fixed-joint pin

In this case the expressions for the reactions in the beam, at the investigated analytical expressions are:

$$"M_A"="X_1" \qquad \left|\begin{array}{l} "A_v"="A_v^0"+A_{v_1}"X_1" \\ "B_v"="B_v^0"+B_{v_1}"X_1" \end{array}\right.$$

$$\left|\begin{array}{l} "A_v"="A_v^0"-\frac{1}{l}"X_1" \\ "B_v"="B_v^0"+\frac{1}{l}"X_1" \end{array}\right. .$$

In the beam the ordinates of the reaction's line influences are:

$$M_A=-\frac{x_m^I}{2}\left(1-\frac{x_m^{I\,2}}{l^2}\right),$$

$$A=\frac{x_m^I}{l}+\frac{x_m^I}{2l}\left(1-\frac{x_m^{I\,2}}{l^2}\right), \quad B=\frac{x_m}{l}-\frac{x_m^I}{2l}\left(1-\frac{x_m^{I\,2}}{l^2}\right).$$

The ordinates of the reaction's line influences at $m=10$ points of the beam are shown in Table 4. Reaction's line influences in a bilateral fixed beam end in a beam fixed – joint pin have a kind shown in Fig. 4.

The solutions of the ordinates of the lines influences are made with $l=1$. The force line influences, constructed at $l=1$ with a multiplier $\frac{1}{l}$ can be converted to force line influences at $l\neq 1$.

The construction of line influences is made with compound programs of PC.

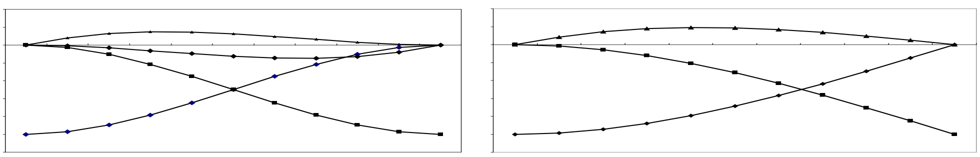
Table 3. The ordinates of reaction's line influences at *m=10* points of the beam

m	0	1	2	3	4	5	6	7	8	9	10
A	1	0.9720	0.8960	0.7840	0.6480	0.5000	0.3520	0.2160	0.1040	0.0280	0.0000
M_A	0.0000	-0.0810	-0.1280	-0.1470	-0.1440	-0.1250	-0.0960	-0.0630	-0.0320	-9.000e-3	0.0000

Table 4. The ordinates of the reaction's line influences at *m=10* points of the beam

m	0	1	2	3	4	5	6	7	8	9	10
M_A	0.0000	-0.0855	-0.1440-	-0.1785	-0.1920	-0.1875	-0.1680	-0.1365	-0.0960	-0.0495	0.0000
A	1.0000	0.9855	0.9440	0.8785	0.7920	0.6875	0.5680	0.4365	0.2960	0.1495	0.0000
B	0.0000	0.0145	0.0560	0.1215	0.2080	0.3125	0.4320	0.5635	0.7040	0.8505	1.0000

bilateral fixed beam   beam fixed-joint



$"A", "B", "M_A", "M_B"$ $"A", "B", "M_A"$

Fig. 4. Reaction's line influences in a bilateral fixed beam end in a beam fixed – joint pin

CONCLUSIONS

With the expressions investigated the ordinates of force line influences can be obtained in arbitrary sections of beams. The expressions investigated are necessary to determine line influences in indetermined systems with application of the displacement method.

The obtained ordinates of force line influences conform with the results obtained for line influences obtained using expressions for reactions in the beam's elements – bilateral fixed and beam fixed – joint pin.

The investigated expressions can be used for generating line influences for forces and reactions in the basic beam's elements with an arbitrary number and location ordinates.

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