Gravitational instability of a rotating streaming plasma cloud with radiation

A. A. Hasan

Basic and Applied Sciences Department, College of Engineering and Technology, Arab Academy for Science, Technology and Maritime Transport (AASTMT), Sadat Road - P.O. Box 11, Aswan, Egypt

Received January 27, 2019; Revised: July 18, 2020

The gravitational instability of an inviscid compressible rotating streaming plasma cloud with radiation is investigated. The self-gravitating force is destabilizing while the rotating force has a stabilizing influence under certain restrictions. The radiation has a strong stabilizing influence for all modes of perturbations. In the presence of radiation, the uniform streaming has no influence at all on the instability of the model and the radiation overcomes the self-gravitating instability of a plasma cloud. Stability of fluid layers or cylinders has gained considerable importance because of their applications in industries and chemical laboratories, such as medical applications of electrohydrodynamic and magnetohydrodynamic stabilities as injection of drugs inside the vessels, electric shock to treat heart attack and effect of magnetic resonance on blood flow.

Keywords: Self-gravitating, Radiation, Rotating, Plasma cloud.

INTRODUCTION

It is well known since long time ago (Jeans [1]) that a self-gravitating fluid becomes unstable with respect to small perturbations if the related wavelength exceeds a certain value. In this case the gravitational force overpowers the pressure gradient and the instability set in this process clearly plays a basic role in the initial stage of a stellar cluster formation from fragmentation of interstellar matter. It is found that the model becomes unstable for all perturbations of wave numbers less than a critical value $k^2 C_S^2 - 4\pi G \rho_o < 0$, called Jeans' criterion, where k is the net wave number of the propagated wave, C_S^2 is the sound speed in the fluid of density ρ_o, and G is the self-gravitational constant. Chandrasekhar and Fermi [2], and later on Chandrasekhar [3] made several extensions. The Jeans' model of self-gravitational medium has been elaborated with streams of variable velocity distribution by Sengar [4]. Hasan [5] has investigated the magnetodynamic stability of a fluid jet pervaded by a transverse varying magnetic field. In 2013 Kormendy and Ho [6] have analyzed the stability of the observed clumps considering that a criterion is required. However, the conventional Jeans' criterion and Roche limit become insufficient, because they regard self-gravity as the only force binding a gas cloud, which is not true in galactic nuclei. Chen *et al*. [7] have formulated a new scheme for judging the stability of the observed clumps and proved that these clumps are stable, contrary to what one would naively deduce from the Roche (tidal) limit. Hasan [8] has studied the linear stability of self-gravitating compound dielectric immiscible jets under the influence of an axial electric field. He [9] developed the magnetohydrodynamic stability of a self-gravitating rotating streaming viscous fluid medium pervaded by a general magnetic field. He [10] discussed the stability of the interface between two incompressible self-gravitating non-conducting fluids in the presence of an electric field. Hoshoudy *et al*. [11] investigated the compressibility effects on the Rayleigh-Taylor instability of two plasma layers. Morimoto *et al.* [12] clarified theoretically the suppression of three-dimensional (3D) nucleation by microscopic magnetohydrodynamic flow and formation of stream pattern of macroscopic MHD flow by the multiple nucleation of 3D nuclei in electrodeposition under a uniform parallel magnetic field. In 2019 they [13] have established the amplitude of the micro-MHD flows and concentration fluctuation on the solution side independent of the fluctuation on the electrode-surface side. Weyens *et al.* [14] have presented new expressions for the quantities of interest, namely the parallel current density, the local shear and the normal and geodesic components of the curvature. In this present work the effect of radiation on the self-gravitating instability of a rotating plasma cloud with streams of variable velocity distribution function of coordinate is discussed. There are many applications of electrohydrodynamic and magnetohydrodynamic stability in several fields of science such as:

Geophysics: the fluid of the core of the Earth and was theorized to be a huge MHD dynamo that

* To whom all correspondence should be sent:
E-mail: alfaisal772001@aast.edu

generates the Earth's magnetic field because of the motion of the liquid iron.

Astrophysics: MHD applies quite well to astrophysics since 99% of baryonic matter content of the universe is made of plasma, including stars, interplanetary medium, nebulae and jets, spiral arms of galaxies, etc. Many astrophysical systems are not in local thermal equilibrium, and therefore require an additional kinematic treatment to describe all the phenomena within the system.

Engineering applications: there are many applications in engineering sciences including oil and gas extraction processes surrounded by electric field or magnetic field, gas and steam turbines, MHD power generation systems and magneto-flow meters, etc.

Formulation of the problem

We consider an unbound plasma cloud under the combined effect of self-gravitating variable inertia, gas pressure gradient and radiation forces. The plasma medium is assumed to be homogenous, inviscid and compressible. We are interested here to identify the inertia and radiation forces effects on the self-gravitational instability of the model.

We shall utilize the Cartesian coordinates (x, y, z) for investigating this problem.

The required equations for the present problem are:

$$\rho\left(\frac{\partial \underline{u}}{\partial t}+(\underline{u}\cdot\nabla)\underline{u}\right)=-\nabla P+\rho\nabla V-2\rho(\underline{u}\wedge\underline{\Omega})+\frac{1}{2}\rho(\underline{\Omega}\wedge\underline{r})^2 \tag{1}$$

$$\frac{\partial \rho}{\partial t}+(\underline{u}\cdot\nabla)\rho=-\rho(\nabla\cdot\underline{u}) \tag{2}$$

$$\nabla^2 V=-4\pi G\rho \tag{3}$$

$$P=K\rho^{\Gamma} \tag{4}$$

Radiating equation of state:

$$\rho\frac{\partial T}{\partial t}-(\Gamma-1)T\frac{\partial \rho}{\partial t}=0 \tag{5}$$

Here ρ and $\underline{u}$ are the fluid density, velocity vector, V and G are the self-gravitating potential and constant, $\underline{\Omega}$ is the angular velocity of rotation, K and Γ are constants where Γ is the polytropic exponent, T the temperature at time t and $P\left(=P_g+P_r\right)$ the total pressure which is the sum of the gas kinetic pressure P_g and pressure P_r due to radiation:

$$P_g=R\rho T \tag{6}$$

$$P_r=\frac{1}{3}a_R T^4 \tag{7}$$

where a_R is the Boltzmann constant and R is the general constant of plasma. Equation (5) is the equation of state that indicates adiabatic changes in an enclosure containing matter and radiation. For more details concerning equation (5) we may refer to Chandrasekhar [3], where we find:

$$(\Gamma-1)(4-3b)=T_1-b \tag{8}$$

with

$$T_1-b=\left(b+12(\gamma-1)(1-b)\right)=(\gamma-1)(4-3b)^2 \tag{9}$$

$$b=\frac{P_g}{P} \tag{10}$$

where γ is the ratio of the specific heats.

In the absence of radiation:

$$P_r=0 \tag{11}$$

therefore,

$$b = 1 \tag{12}$$

$$T = T_1 = \gamma \tag{13}$$

However, if the radiation is taken into account such that its pressure P_r is much greater that gas pressure P_g (i.e. $P_r >> P_g$), we have:

$$T = T_1 = \frac{4}{3} \tag{14}$$

We assume that the medium: (i) rotates with the general uniform angular velocity:

$$\underline{\Omega} = \left(\Omega_x, \Omega_y, \Omega_z\right) \tag{15}$$

and (ii) possesses streams moving in the *x*-direction with velocity:

$$\underline{u}_o = \left(U(z), 0, 0\right) \tag{16}$$

varying along the z-direction of the Cartesian coordinates (*x*, *y*, *z*).

Perturbation analysis

For small departures from the initial state, every variable quantity Q may be expressed as:

$$Q = Q_o + Q_1 \quad , \quad |Q_1| << Q_o \tag{17}$$

where Q stands for each $\rho, P, \underline{u}$ and V.

Based on the expansion (17), the perturbation equations could be obtained from (1)-(4) in the form:

$$\rho_o\left(\frac{\partial \underline{u}_1}{\partial t} + \left(\underline{u}_o \cdot \nabla\right)\underline{u}_1 + \left(\underline{u}_1 \cdot \nabla\right)\underline{u}_o\right) = -\nabla P_1 + \rho \nabla V_1 - 2\rho_o\left(\underline{u}_1 \wedge \underline{\Omega}\right) \tag{18}$$

$$\frac{\partial \rho_1}{\partial t} + \left(\underline{u}_o \cdot \nabla\right)\rho_1 + \left(\underline{u}_1 \cdot \nabla\right)\rho_o + \rho_o\left(\nabla \cdot \underline{u}_1\right) + \rho_1\left(\nabla \cdot \underline{u}_o\right) = 0 \tag{19}$$

$$\nabla^2 V_1 = -4\pi G \rho_1 \tag{20}$$

$$\frac{dP_1}{dt} = \mathrm{C}_S^2 \frac{d\rho_1}{dt} \tag{21}$$

where $\mathrm{C}_S \left(= \sqrt{\frac{\Gamma P_o}{\rho_o}} \right)$ is the sound speed in the fluid.

By the use of the components of $\underline{u}_1$

$$\underline{u}_1 = \left(u, v, w\right) \tag{22}$$

together with the assumptions (15)-(16), the system of equations (18)-(21) may be rewritten as:

$$\rho_o\left[\frac{\partial u}{\partial t}+U_o\frac{\partial u}{\partial x}+w\frac{dU_o}{dz}\right]=-\frac{\partial P_1}{\partial x}+\rho_o\frac{\partial V_1}{\partial x}+2\rho_o\Omega_y w-2\rho_o\Omega_z v \tag{23}$$

$$\rho_o\left[\frac{\partial v}{\partial t}+U_o\frac{\partial v}{\partial x}\right]=-\frac{\partial P_1}{\partial y}+\rho_o\frac{\partial V_1}{\partial y}+2\rho_o\Omega_z u-2\rho_o\Omega_x w \tag{24}$$

$$\rho_o\left[\frac{\partial w}{\partial t}+U_o\frac{\partial w}{\partial x}\right]=-\frac{\partial P_1}{\partial z}+\rho_o\frac{\partial V_1}{\partial z}+2\rho_o\Omega_y u-2\rho_o\Omega_x v \tag{25}$$

$$\frac{\partial \rho_1}{\partial t}+U_o\frac{\partial \rho_1}{\partial x}=-\rho_o\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \tag{26}$$

$$\frac{\partial^2 V_1}{\partial x^2}+\frac{\partial^2 V_1}{\partial y^2}+\frac{\partial^2 V_1}{\partial z^2}=-4\pi G\rho_1 \tag{27}$$

$$\frac{\partial P_1}{\partial t}+U_o\frac{\partial P_1}{\partial x}=C_S^2\left(\frac{\partial \rho_1}{\partial t}+U_o\frac{\partial \rho_1}{\partial x}\right) \tag{28}$$

$$\rho_0 T_1-(\Gamma-1)T_0\,\rho_1=0 \tag{29}$$

Eigenvalue relation

Apply sinusoidal wave along the fluid interface. Consequently, from the viewpoint of the stability approaches given by Chandrasekhar [3], we assume that the space-time dependence of the wave propagation of the form:

$$Q_1\sim\exp\left[i\left(k_x x+k_y y+k_z z+\sigma t\right)\right] \tag{30}$$

Here σ is gyration frequency of the assuming wave. k_x, k_y, and k_z are (any real values) the wave numbers in the (*x*, *y*, *z*) directions. By an appeal to the time-space dependence (30), the relevant perturbation equations (23)-(29) are given by:

$$n\rho_o u+\rho_o wDU_o=-ik_x C_S^2\,\rho_1+ik_x\rho_o V_1+2\rho_o\Omega_y w-2\rho_o\Omega_z v \tag{31}$$

$$n\rho_o v=-ik_y C_S^2\,\rho_1+ik_y\rho_o V_1-2\rho_o\Omega_x w+2\rho_o\Omega_z u \tag{32}$$

$$n\rho_o w=-ik_z C_S^2\,\rho_1+ik_z\rho_o V_1+2\rho_o\Omega_y u-2\rho_o\Omega_x v \tag{33}$$

$$n\rho_1=-i\rho_o\left(k_x u+k_y v+k_z w\right) \tag{34}$$

$$k^2 V_1=-4\pi G\rho_1 \tag{35}$$

where:

$$k^2=k_x^2+k_y^2+k_z^2\qquad,\ D=\frac{d}{dz} \tag{36}$$

$$n=i\left(\sigma+k_x U_o\right) \tag{37}$$

The foregoing system equations (31)-(35) could be rewritten in the matrix form:

$$\left[a_{ij}\right]\left[b_j\right]=0 \tag{38}$$

where the elements [a_y] of the matrix are given in the Appendix while the elements of the column matrix [b_j] are u, v, w, ρ_1 and V_1. For non-trivial solution of the equations (38), setting the determinant of the matrix [a_y] equal to zero (see Appendix), we get the general eigenvalue relation of seven order in n in the form:

$$A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0 \tag{39}$$

where the compound coefficients A_i $(i = 0,1,2\ 3\ 4\ 5)$ are calculated.

RESULTS AND DISCUSSION

Equation (39) is a general eigenvalue relation of a rotating self-gravitating streaming plasma cloud. Some previously published results may be obtained as limiting cases here. That confirms the present analysis.

In absence of rotation, radiation forces and for inviscid fluid, i.e. $\underline{\Omega} = 0$, equation (39) yields:

$$k^2 n^3 + k^2 \left(k^2 C_S^2 - 4\pi G\rho_o\right) n - k_x k_z \left(k^2 C_S^2 - 4\pi G\rho_o\right) DU_o = 0 \tag{40}$$

This relation coincides with the dispersion relation of a pure self-gravitating fluid medium streaming with variable streams $\left(U_o\left(z\right), 0, 0\right)$ derived by Sengar [4]. For more details concerning the stability of this case we may refer to Sengar [4].

If $\underline{\Omega} = 0$ and $U_o = 0$, equation (39) reduces to:

$$n^2 = k^2 C_S^2 - 4\pi G\rho_o \tag{41}$$

This gives the same results given by Jeans [5]. For more details concerning the instability of this case we may refer to the discussions of Jeans [5].

The purpose of the present part is to determine the influence of rotation on Jeans' criterion (41) of a uniform streaming fluid. So in order to carry out and to facilitate the present situation we may choose $\Omega_x = 0, k_x = 0$ and $k_y = 0$, then equation (39) gives:

$$n^4 + \left(4\pi G\rho_o - C_S^2 k_z^2 - 4\Omega^2\right) n^2 + 4\Omega_z^2 \left(C_S^2 k_z^2 - 4\pi G\rho_o\right) = 0 \tag{42}$$

with

$$\Omega^2 = \Omega_y^2 + \Omega_z^2 \tag{43}$$

Equation (40) indicates that there must be two modes in which a wave can be propagated in the medium. If the roots of (40) are n_1^2 and n_2^2, then we have:

$$n_1^2 + n_2^2 = C_S^2 k_z^2 + 4\Omega^2 - 4\pi G\rho_o \tag{44}$$

$$n_1^2\, n_2^2 = 4\Omega_z^2 \left(C_S^2 k_z^2 - 4\pi G\rho_o\right) \tag{45}$$

and so we see that both the roots n^2_1 and n^2_2 are real. The discussions of (40) indicate that if the Jeans' restriction:

$$C_S^2 k_z^2 - 4\pi G\rho_o < 0 \tag{46}$$

is valid, then one of the two roots n_1^2 or n_2^2 must be negative and consequently the model will be unstable. This means that under the Jeans' restriction (46), the self-gravitating rotating fluid medium is unstable. This shows that the Jeans' criterion for a self-gravitating medium is unaffected by the influence of the uniform rotation.

If the non-streaming plasma cloud medium is acted upon by the combined effect of self-gravitation, gas pressure gradient and radiating pressure gradient forces, relation (39) reduces to:

$$\sigma^2 + \rho_0 A k^2 - 4\pi G \rho_0 = 0 \qquad (47)$$

As the radiating self-gravitating plasma cloud streams uniformly with velocity $\underline{u}_0 = (U, 0, 0)$ the general eigenvalue relation (39) becomes:

$$\sigma^2 + 2iUk\sigma - \left(U^2 k_x^2 + 4\pi G\rho_0 - \rho_0 A k^2\right) = 0 \qquad (48)$$

The quadratic equation (48) has imaginary roots if

$$4\pi G\rho_0 > A\rho_0 k^2 \qquad (49)$$

This restriction (49) for a streaming plasma cloud is exactly the same as that given by Jeans' criterion [1] Therefore, we conclude that the inertia force due to uniformly streaming plasma cloud has no influence at all on the instability of a self-gravitating plasma cloud in the presence of radiation.

CONCLUSION

The effect of radiation on the self-gravitating instability of a rotating plasma cloud with streams of variable velocity distribution function of coordinate is discussed. The self-gravitating force is destabilizing according to restrictions. The rotating force has a stabilizing influence under certain restrictions. The radiation has a strong stabilizing influence for all short and long perturbation wave lengths. In the presence of radiation, the uniform streaming has no influence at all on the instability of the model and the radiation overcomes the self-gravitating instability of a plasma cloud. The inertia force due to uniformly streaming plasma cloud has no influence at all on the instability of a self-gravitating plasma cloud in the presence of radiation.

REFERENCES

1. J. Jeans, H. Philos, *Trans. R. Soc. London*, **199,** 1 (1902).
2. S. Chandrasekhar, E Fermi, *Astrophys. J.,* **118**, 116 (1953).
3. S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Dover, New York, 1981.
4. R. S. Sengar, Proc. *Acta Sci*. India, **51**(A), 39 (1981).
5. A. Hasan, *Mathematical Problems in Engineering*, **1,** 1 (2012).
6. J. Kormendy, L. Ho, *Annual Review of Astronomy and Astrophysics*, **51**, 511 (2013).
7. X. Chen, P. Amaro-Seoane, J. Cuadra, *The Astrophysical Journal*, **819** (138), 1 (2016).
8. A. Hasan, *International Journal of Biomathematics*, **9**(3), 1 (2016).
9. A. Hasan, *Bulg. Chem. Commun*., 4**9**(2), 487 (2017).
10. A. Hasan, *Research Journal of Applied Sciences, Engineering and Technology*, **14**(10), 399 (2017).
11. G. Hoshoudy, A. Hasan, P. Kumar, *Journal of Scientific and Engineering Research*, **5**(3), 245 (2018).
12. R. Morimoto, M. Miura, A. Sugiyama, M. Miura, Y. Oshikiri, I. Mogi, S. Takagi, Y. Yamauchi, R. Aogaki, *Journal of Electroanalytical Chemistry*, **847**(113255), 1 (2019).
13. R. Morimoto, M. Miura, A. Sugiyama, M. Miura, Y. Oshikiri, I. Mogi, S. Takagi, Y. Yamauchi, R. Aogaki, *Journal of Electroanalytical Chemistry*, **848**(113254), 1 (2019).
14. T. Weyens, J. Reynolds-Barredo, A. Loarte, *Computer Physics Communications*, **242**, 60 (2019).

APPENDIX

The elements $a_{ij}\left(i=1,2,......,5 \text{ and } j=1,2,....,5\right)$ of the matrix [a_y] in equation (40) of the linear algebraic equations (32)-(36) are:

$a_{11}=\left(n\rho_o\right)$, $a_{12}=\left(2\rho_o\Omega_z\right)$, $a_{13}=\left(\rho_o DU_o-2\rho_o\Omega_y\right)$

$a_{14}=ik_x C_s^2$, $a_{15}=-i\rho_o k_x$

$a_{21}=\left(-2\rho_o\Omega_z\right)$, $a_{22}=\left(n\rho_o\right)$, $a_{23}=\left(2\rho_o\Omega_x\right)$

$a_{24}=ik_y C_s^2$, $a_{25}=-i\rho_o k_y$

$a_{31}=\left(-2\rho_o\Omega_y\right)$, $a_{32}=\left(2\rho_o\Omega_x\right)$, $a_{33}=\left(n\rho_o\right)$

$a_{34}=ik_z C_s^2$, $a_{35}=-i\rho_o k_z$

$a_{41}=i\rho_o k_x$, $a_{42}=i\rho_o k_y$, $a_{43}=i\rho_o k_z$

$a_{44}=n$, $a_{45}=0$

$a_{51}=0$, $a_{52}=0$, $a_{53}=0$

$a_{54}=4\pi G$, $a_{55}=k^2$