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Modeling of light propagation in layered inhomogeneous medium

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The article considers the possibility of modeling the propagation of light in a layered inhomogeneous medium. The knowledge of students in the field of mathematics and informatics makes it possible to organize a new kind of educational activity, like mathematical and computer modeling, while studying the phenomenon of light propagation in a layered heterogeneous medium.

Keywords: optics, layered-heterogeneous medium, Snell's law, mathematical and computer modeling.

INTRODUCTION

Optics of heterogeneous medium is a quite extensive and completely not simple area of physics, which has a great scientific-practical importance. For electromagnetic waves of a certain frequency, the Earth's atmosphere introduces a layered-heterogeneous medium, the refractive index of which continuously decreases with altitude. In such environment the electromagnetic wave spreads curvilinearly, which cannot be demonstrated in laboratory conditions. Thus, familiarizing students with the physical ideas of optics of heterogeneous medium, with which they do not meet in the main course, represents a big cognitive interest.

Mathematical Methods

In heterogeneous medium, the idea of the propagation of light along the rays is preserved, and the geometric shape of the ray can be uniquely determined from Snell's law with limit transition way [1].

As an example, consider the simplest case, when the refractive index of the medium changes in only one direction and depends on one coordinate. Such propagation of light is called layered-heterogeneous medium [2].

Imagine an optically heterogeneous medium, the refractive index n of which is a function of only one coordinate y:

$$n = n(y). \quad (1)$$

Such a medium, as already noted, is called layered-heterogeneous. As mentioned above, Earth's atmosphere is an example of it. Although the dependence of the refractive index n of the medium on the coordinate y (1) is complex, the medium in the first approach for a bounded region can always be assumed to be linear:

$$n = n_0+ky \quad (2)$$

where, n_0 – refractive index of the medium in points with coordinate $y = 0$, $k = {}^{dn}/_{dy}$ - constant gradient of refraction.

Choose an arbitrary ray from an diverging ray from a point source of light going under angle $\varphi_1 < {}^{\pi}/2$ to axis y (Fig. 1). Find the trajectory of the selected ray. The optical heterogeneous medium is split into plane-parallel layers, the perpendicular axes of y are so thin that inside each of the layers the light moves rectilinearly and on the boundary between neighboring layers is refracted so that the trajectory of the light ray is a broken line.

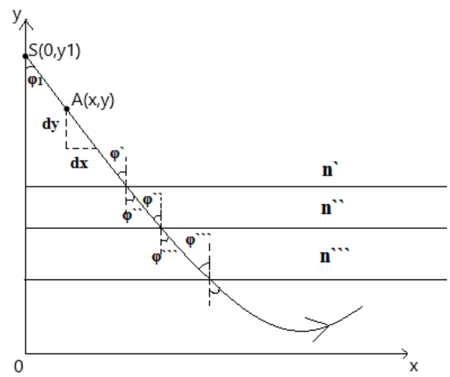

Figure 1. An arbitrary beam, a divergent beam from a point source of light.

According to the law of refraction:

$$\frac{sin\varphi_1}{sin\varphi_2} = \frac{n_2}{n_1}, \quad \frac{sin\varphi_2}{sin\varphi_3} = \frac{n_3}{n_2}, \quad \frac{sin\varphi_3}{sin\varphi_4} = \frac{n_4}{n_3}, \ , \ , \ ,$$

or $n_1 \sin\varphi_1 = n_2 \sin\varphi_2 = n_3 \sin\varphi_3 = \ , \ , \ ,$ that is $n(y) \sin\varphi = const.$

Then, in limit instead of broken, we will get curvilinear trajectory light ray. It is obvious that in this case:

$$n(y) \sin\varphi = const \quad \text{or} \quad n(y) \sin\varphi = m \quad (3)$$

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where, m is a constant. The physical meaning of the constant m is quite simple: this is the meaning of the refractive index in that plane of the medium, at the points at which $sin\varphi = 1$, that is, where the light ray is directed perpendicular to the axis y.

For an arbitrary point of a trajectory, we can write

$$tq\varphi = -\frac{dn}{dy} \qquad (4)$$

Since $tq\varphi = \frac{\sin\varphi}{\sqrt{1-\sin^2\varphi}}$, from (4) and (3) it follows that

$$\frac{dx}{dy} = -\frac{m}{\sqrt{n^2(y)-m^2}} \qquad (5)$$

Integration of the equation (5) gives

$$x = \int_0^x dx = -m\int_{y_1}^{y_2} \frac{dy}{\sqrt{n^2(y)-m^2}} \qquad (6)$$

In equation (6) we will change the variable by crossing y to n:

$$x = -m\int_{y_1}^{y_2} \frac{dy}{\sqrt{n^2(y)-m^2}} = -\frac{m}{k}\int_{n_1}^{n_2} \frac{dn}{\sqrt{n^2(y)-m^2}}$$

where n_1 – refractive index of the medium at points with coordinates y_1: $n_1 = n_0 + ky_1$.

If we use the substitution $n = mchz$, then it is not difficult to make sure that the antiderivative of the latter integral is equal to the hyperbolic arc cosine. Therefore,

$$x = -\frac{m}{k} arch\frac{n}{m} \Big|_{n_1}^{n} = -\frac{m}{n}\left(arch\frac{n}{m} - arch\frac{n_1}{n}\right).$$

From here $arch\frac{n}{m} = -\frac{k}{m}x + arch\frac{n_1}{m}$.

Taking both sides of this formula a hyperbolic cosine, we will get:

$$n = mch\left(-\frac{k}{m}x + arch\frac{n_1}{m}\right) \qquad (7)$$

Considering (2) and paying attention to the parity of the hyperbolic cosine, rewrite (7) in the form:

$$y = -\frac{n_0}{k} + \frac{m}{k}ch\left[\frac{k}{m}\left(x - arch\frac{n_1}{m}\right)\right] \qquad (8)$$

The equation (8) in an explicit form describes the trajectory of the propagation of light in a layered heterogeneous medium with a constant refractive index gradient.

Function (8) is a hyperbolic cosine. To approximately build the graph, we need to find the coordinates of the minimum of this function. Differentiating the function (8) by coordinate x, we will get:

$$\frac{dy}{dx} = sh\left(\frac{k}{m}x - arch\frac{n_1}{m}\right). \qquad (9)$$

Equating this derivative to zero, we find that this equality will be satisfied if $\frac{k}{m}x - arch\frac{n_1}{m} = 0$.

From here the abscissa of the minimum

$$x_M = \frac{k}{m}x - arch\frac{n_1}{m} \qquad (10)$$

Substituting this abscissa value in equation (8), we will find the coordinate of the same point M:

$$y_M = \frac{-n_0 + m}{k}. \qquad (11)$$

The hyperbolic cosine is symmetric relatively to the vertical axis, passing through the minimum point M. An approach of the graph of the function constructed in the Cartesian coordinate system (8) is shown in Fig . 2.

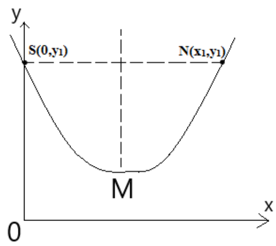

Figure 2. The graph of the function constructed in the Cartesian coordinate system.

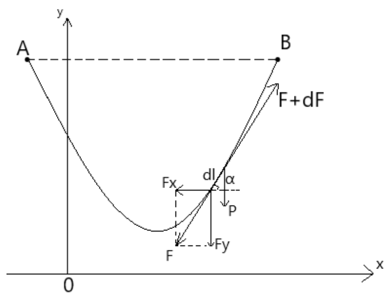

Figure 3. Homogeneous inextensible and heavy chain, the ends of which are fixed at points A and B.

To demonstrate the propagation of light in a medium with a constant refractive index gradient, we model the path of the light ray by a chain line, since modeling allows us to replace the object under study with another, specially created for this, but preserving the characteristics of the real object, necessary for its study [3]. To do this, take a break from the optics and remember the statistics. Imagine a flexible homogeneous inextensible and heavy chain, the ends of which are fixed at points A and B (Fig. 3). If the length of the thread is greater than AB, the chain will hang. Let’s find an equation that describes the position of the sagging chain.

For this we denote the mass of a unit of the chain length by ρ. Gravity acting on the element of length dl of the thread:

$$dP = \rho g\, dl, \qquad (12)$$

where g is the acceleration of gravity. Since the chain is located in equilibrium, the sum of the forces acting on any of its elements dl, is equal to zero (Fig. 3): $d\boldsymbol{P} + \boldsymbol{F} + (\boldsymbol{F} + d\boldsymbol{F}) = 0$, where $\boldsymbol{F}$ and $\boldsymbol{F} + d\boldsymbol{F}$ are the forces of chain tension. Passing from the vector equation to the equations in the projections, we obtain:

$$-F_x + (F_x + dF_x) = 0,$$
$$-F_y + (F_y + dF_y) - dP = 0.$$

From here $dF_x = 0$ (that is F_x = const) and $dF_y = dP$. It can be seen in the figure that $dy/dx = tg\alpha = F_y / F_x$. Differentiating this formula by x and given that F_x = const, $dF_y = \rho g\, dl$, we obtain:

$$\frac{d^2y}{dx^2} = \frac{1}{Fx}\frac{dFy}{dx} = \frac{\rho g}{Fx}\frac{dl}{dx}. \qquad (13)$$

Since $dl = \sqrt{dx^2 + dy^2}$ or $\frac{dl}{dx} = \sqrt{1 + (dy/dx)^2}$, then denoting $F_x / \rho g = a$, we obtain the differential equation:

$$a\frac{d^2y}{dx^2} = \sqrt{1 + (dy/dx)^2}. \qquad (14)$$

The equation can be integrated by using the substitution $dy/dx = sh\, z$. Then $d^2y/dx^2 = sh\, z \cdot dz/dx$.

Pay attention that $1 + sh^2 x = ch^2 x$, equation (14) we reduce it to the form $a \cdot dz/dx = 1$. From here $z = \frac{1}{a}x + C_1$, $dy/dx = sh\left(\frac{1}{a}x + C_1\right)$. Therefore, we finally obtain:

$$y = a\, sh\left(\frac{1}{a}x + C_1\right) + C_2. \qquad (15)$$

To determine the constant integrations C_1 and C_2 it is necessary to use some initial conditions.

Comparison of formulas (8) and (15) shows that the curve along which the chain sags is described by the same equation as the light propagation path in a layered-heterogeneous medium with a constant refractive index gradient, since both formulas are the equation of a hyperbolic function. This allows us to use the saggy chain as a physical model of the trajectory of the light ray (Fig. 4). Simple experiments show that the thread must be heavy, homogeneous, flexible and inextensible, so it is best to use a chain to demonstrate the trajectory of light in a heterogeneous medium. Therefore, the graph of the hyperbolic is called chain line.

Figure 4. The physical model of the trajectory of the light ray.

The initial “advancement” of students in the field of informatics allows organizing a new kind of educational activity like computer modeling when studying the phenomenon of light propagation in a layered heterogeneous medium. Computer modeling, carrying out of computing experiment is one of the modern methods of research of the physical phenomena.

The construction of graphs in the MathCAD system is an easy task, as the MathCAD package has a powerful mathematical device that allows performing symbolic calculations, solving algebraic and differential equations systems, operations with vectors and matrices, writing programs, drawing graphics and surfaces, etc.[4].

Use the *Graph* toolbar to construct the light propagation trajectory, i.e., the graph of the dependence of n on y [5].

CONCLUSIONS

Demonstration of the trajectory of a light ray with the help of a sagging chain and carrying out research in the form of a computer experiment makes it possible to obtain knowledge, skills (not only in the field of physics) that contribute to the generation of motivation for training activities that is so necessary in modern conditions for the formation of a whole range of competencies for future specialists.

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